Correlation-Fueled Quantum Engine with Acoustic Cooling: Exact-Stroke Protocol and Build Guide

EchoKey Asks: Correlated Engine Demo

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Abstract

We provide an exact-stroke experimental protocol that realizes a correlation-fueled quantum engine/refrigerator on a superconducting platform with an acoustic leg. The working medium is two transmon qubits (S_h, S_c); the hot environment is a finite reservoir (B_{h1}, B_{h2}); the cold environment B_c comprises a mechanical mode M (SAW/HBAR) plus a dump mode D (over-coupled cavity or phonon sink). All cycle strokes are unitary (piecewise-constant Hamiltonians, exact exponentials), so energetics follow from quench-work identities without master-equation approximations. We define cycle heat/work, an entropic resource ledger σ, and practical observables (mechanical occupancy, sideband thermometry). The document maps every parameter to deployable hardware and gives a minimal bill of materials. A reference implementation `ek_engine_acoustic.py` accompanies this note.

1 Architecture and Hilbert Space

Subsystem order is $(S_h, S_c, B_{h1}, B_{h2}, M, D)$ with dimensions $(2, 2, 2, 2, n_M, n_D)$. Bare (embedded) Hamiltonians:

$$H_S = \tfrac{\omega_h}{2}\sigma_z^{(S_h)} + \tfrac{\omega_c}{2}\sigma_z^{(S_c)}, \tag{1}$$

$$H_{B_h} = \tfrac{\omega_{h1}}{2}\sigma_z^{(B_{h1})} + \tfrac{\omega_{h2}}{2}\sigma_z^{(B_{h2})}, \tag{2}$$

$$H_M = \omega_M\left(a_M^\dagger a_M + \tfrac{1}{2}\right), \qquad H_D = \omega_D\left(a_D^\dagger a_D + \tfrac{1}{2}\right), \tag{3}$$

$$H_{B_c} = H_M + H_D, \qquad H_0 = H_S + H_{B_h} + H_{B_c}. \tag{4}$$

Work-channel (XY) coupling:

$$H_g = \tfrac{1}{2}\left[\sigma_x^{(S_h)}\sigma_x^{(S_c)} + \sigma_y^{(S_h)}\sigma_y^{(S_c)}\right]. \tag{5}$$

Contacts:

$$H_{SB}^{(h)} = \sigma_x^{(S_h)}\sigma_x^{(B_{h1})} + \sigma_x^{(S_h)}\sigma_x^{(B_{h2})}, \tag{6}$$

$$H_{SB}^{(c)} = \sigma_x^{(S_c)}X^{(M)}, \quad X^{(M)} = a_M + a_M^\dagger. \tag{7}$$

Cooling-leg couplings:

$$H_{\rm bs} = \sigma_-^{(S_c)}a_M^\dagger + \sigma_+^{(S_c)}a_M \quad \text{(beam-splitter)}, \tag{8}$$

$$H_{\rm md} = a_M a_D^\dagger + a_M^\dagger a_D \quad \text{(mode–mode)}. \tag{9}$$

2 Cycle Strokes (Exact Unitaries)

With durations $(t_{\rm th}, t_{\rm work}, t_{\rm cool}, t_{\rm rlx})$ and couplings $(\kappa_h, \kappa_c, g_{\rm on}, \chi, g_{\rm bs}, g_{\rm md})$, define stroke Hamiltonians

$$H_{\rm th} = H_0 + \kappa_h H_{SB}^{(h)} + \kappa_c H_{SB}^{(c)}, \tag{10}$$

$$H_{\rm work} = H_0 + g_{\rm on} H_g + \chi \left(H_{SB}^{(h)} + H_{SB}^{(c)} \right), \tag{11}$$

$$H_{\rm cool} = H_0 + g_{\rm bs} H_{\rm bs} + g_{\rm md} H_{\rm md}, \tag{12}$$

$$H_{\rm rlx} = H_0 + \tfrac{1}{2}\kappa_h H_{SB}^{(h)} + \tfrac{1}{2}\kappa_c H_{SB}^{(c)}. \tag{13}$$

Each stroke uses an exact propagator $U = \exp(-iHt)$; quenches between strokes account for work. Initial state is a tensor product of thermal references at (T_h, T_c):

$$\rho_0 = \rho_{S_h}^{\rm th}(T_h) \otimes \rho_{S_c}^{\rm th}(T_c) \otimes \rho_{B_h}^{\rm th}(T_h) \otimes \rho_M^{\rm th}(T_c) \otimes \rho_D^{\rm th}(T_c).$$

3 Energetics and Resource Ledger

Heat and work. Evaluated at cycle boundary:

$$Q_h = -\Delta \langle H_{B_h} \rangle \quad (+ \text{ means energy from hot}), \tag{14}$$

$$Q_c = +\Delta \langle H_{B_c} \rangle = Q_M + Q_D, \qquad Q_M = +\Delta \langle H_M \rangle\,, \quad Q_D = +\Delta \langle H_D \rangle\,, \tag{15}$$

$$W_{\rm out} = -\sum_{\rm quenches} {\rm Tr}[\rho\,(H_{\rm after} - H_{\rm before})]\,. \tag{16}$$

Engine efficiency is reported only when in engine mode:

$$\eta = \frac{W_{\rm out}}{\max(Q_h, 0)} \quad \text{else “n/a”}, \qquad \eta_C = 1 - \tfrac{T_c}{T_h}.$$

Entropic resource. With $R = B_{h1} B_{h2} M D$ and $B_c = MD$,

$$I(S_h S_c : R) = S(\rho_{S_h S_c}) + S(\rho_R) - S(\rho), \tag{17}$$

$$D(\rho \| \sigma) = {\rm Tr}[\rho(\log \rho - \log \sigma)], \tag{18}$$

and

$$\sigma = I(S_h S_c : R) + D\Big(\rho_{B_h} \Big\| \rho_{B_h}^{\rm th}\Big) + D\Big(\rho_{B_c} \Big\| \rho_{B_c}^{\rm th}\Big)\,, \qquad \Delta\sigma = \sigma_{\rm final} - \sigma_{\rm initial}. \tag{19}$$

$\Delta\sigma < 0$ indicates consumption of correlational/athermal “fuel” (athermal cycle).

Energy checks. The script prints

$$(\langle H_{\rm th} \rangle_f - \langle H_{\rm th} \rangle_i) + W_{\rm out} \approx 0, \tag{20}$$

$$W_{\rm out} - (Q_h - Q_c) + (\Delta E_S + \Delta E_{\rm int}) \approx 0, \tag{21}$$

which are satisfied to machine precision in the faithful (unitary) path.

4 Control, Timing, and Tuning

Strokes and drives

- **Pre-thermalize** (H_{th}): weak microwave tones realizing $\kappa_h H_{SB}^{(h)}$; piezo (or parametric) tone realizing $\kappa_c H_{SB}^{(c)}$.
- **Work** (H_{work}): XY gate via tunable coupler or cross-resonance (effective $g_{\mathrm{on}} H_g$). Keep a small χ fraction of contacts to allow $I(S_h S_c : R)$ to drop during work (spend resource).
- **Cooling** (H_{cool}): red-sideband tone for $S_c \leftrightarrow M$ beam-splitter ($g_{\mathrm{bs}} H_{\mathrm{bs}}$); parametric bridge for $M \leftrightarrow D$ ($g_{\mathrm{md}} H_{\mathrm{md}}$).
- **Relax** (H_{rlx}): soft contacts to settle before boundary quench.

Two presets (as in the code)

Engine-burst: larger $(g_{\mathrm{on}}, |\chi|, t_{\mathrm{work}})$; slightly weaker cooling leg. Expect at least one cycle with $W_{\mathrm{out}} > 0$ and $\Delta\sigma < 0$.

Deep-fridge: larger $(g_{\mathrm{bs}}, g_{\mathrm{md}}, t_{\mathrm{cool}})$; expect $Q_M < 0$ and $W_{\mathrm{out}} < 0$.

5 Measurement Plan

Mechanical occupancy. From $\langle H_M \rangle$ we define $n_M = \langle H_M \rangle / \omega_M - \frac{1}{2}$. In practice, use sideband thermometry:

1. Calibrate S_c–M red/blue sidebands; measure asymmetry to infer n_M.
2. Cross-check by fitting Lorentzian area under the mechanical response with known line width.

Heat splits. Report Q_M and Q_D per cycle; refrigeration is indicated by $Q_M < 0$ while typically $Q_D > 0$ as entropy is exported to the dump.

Work and resource. W_{out} follows from programmed quenches (exact). $I(S_h S_c : R)$ is bounded via tomography on S_h, S_c and coarse-grained thermometry on R; the relative entropies use thermal references at (T_h, T_c). We recommend:

- Full two-qubit tomography on S_h, S_c at cycle boundaries to compute $S(\rho_{S_h S_c})$.
- Independent thermometry of (B_h, B_c) to set $\rho_{B_h}^{\mathrm{th}}, \rho_{B_c}^{\mathrm{th}}$; changes in $\langle H_{B_h} \rangle, \langle H_{B_c} \rangle$ provide consistent heat estimates.

6 Calibration

1. **Single-qubit:** ω_h, ω_c by spectroscopy; Rabi and T_1/T_2.
2. **Coupler/CR:** calibrate effective g_{on}; extract XX/YY weights; minimize spurious ZZ.
3. **Contacts:** set κ_h, κ_c by weak, off-resonant exchange rates (swap tests).
4. **Sidebands:** amplitude and detuning for H_{bs}; Rabi rate $\propto g_{\mathrm{bs}}$.

5. **Mode bridge:** tune parametric pump for H_{md}; verify energy flow $M \to D$.
6. **Timing:** align stroke durations to the code's $(t_{\mathrm{th}}, t_{\mathrm{work}}, t_{\mathrm{cool}}, t_{\mathrm{rlx}})$.

7 Recommended Hardware (examples)

All brand names are examples; equivalent alternatives are fine.

- **Cryostat:** dilution refrigerator (base $\lesssim 20.000$ mK), $\gtrsim 1.000$ mW cooling at 100.000 mK.
- **Qubits:** two fixed-frequency transmons (5–7.000 GHz), tunable coupler or CR-capable drive path.
- **Hot reservoir:** two lossy CPW resonators or helper qubits around 5.000 5.500 GHz with engineered Q.
- **Mechanical mode** M**:** SAW/HBAR device (GHz), piezo transduction to S_c; coupling g_{bs} in the few–tens of MHz regime.
- **Dump** D**:** overcoupled 1-port CPW cavity (GHz) or phononic waveguide, external Q set for fast extraction.
- **Microwaves:** 2–3 low-phase-noise sources, vector signal generators for pulses/sidebands; IQ mixers or direct-SDM.
- **AWGs/FPGA:** multi-channel AWG (≥ 1 GS/s) or FPGA-based sequencer for synchronized strokes.
- **Readout:** JPA/TWPA amplification chain, heterodyne digitizer, VNA for characterization.
- **Wiring:** standard cryo attenuator chain (20/10/6.000 dB at 4K/Still/MX), circulators/isolators on readout, Eccosorb filters on pumps.

8 Safety and Practical Notes

- High-power pumps (for H_{md}) can heat the mixing chamber; ramp amplitudes slowly, monitor fridge thermometry.
- Avoid spurs/IMD near qubit and mechanical frequencies; use filters and calibrate IQ imbalance.
- Enforce inter-stroke blanking to prevent overlap beyond the intended χ fraction.

9 Parameter Map (code $\leftrightarrow$ lab)

Script flag	Meaning	Hardware knob
`--fh, --fc`	ω_h, ω_c (GHz)	qubit design / flux bias
`--fbh1, --fbh2`	ω_{h1}, ω_{h2} (GHz)	reservoir resonator frequencies
`--fm, --fd`	ω_M, ω_D (GHz)	mech/cavity design, pump detunings
`--Th, --Tc`	hot/cold refs	physical stage temps / effective temps
`--kappa_h, --kappa_c`	contact strengths	weak drive amplitudes
`--g_on`	XY strength	coupler bias / CR amplitude
`--chi`	residual contact in work	small simultaneous contact drive
`--g_bs`	$S_c \leftrightarrow$M	red-sideband amplitude
`--g_md`	M$\leftrightarrow$D	parametric bridge amplitude
`--t_th, --t_work, --t_cool, --t_rlx`	stroke times	sequencer durations
`--n_m, --n_d`	truncations	simulation only

10 Expected Signatures

- **Engine-burst cycle:** $W_{\text{out}} > 0$ and $\Delta\sigma < 0$ on at least one cycle; Q_M may be small or positive.
- **Refrigeration cycle:** $Q_M < 0$, n_M strictly decreases; typically $W_{\text{out}} < 0$, $\Delta\sigma \geq 0$.
- **Energy checks:** faithful path residuals $\sim 10^{-13}$ in simulation; target consistency (within error bars) in experiment.

11 Procedure (minimal reproducible run)

1. Calibrate all frequencies, sidebands, and couplings to match chosen preset.
2. Program the four-stroke schedule with non-overlapping windows (except for the intended χ in work).
3. Acquire per-cycle: readout of S_h, S_c; sideband thermometry of M; power/phase monitors for pumps.
4. Compute Q_h, Q_M, Q_D, Q_c, W_{out}, n_M, and η (engine mode only).
5. Repeat over a grid of $(g_{\text{on}}, \chi, g_{\text{bs}}, g_{\text{md}}, t.)$; identify athermal bursts or refrigeration plateaus.

12 Optional Diagnostic Control

For comparison only, one may decorrelate at the boundary: $\rho \mapsto \rho_{S_h S_c} \otimes \rho_{B_h} \otimes \rho_{B_c}$. This is non-unitary and not free; to compare fairly, charge the minimum erasure work $W_{\text{reset}} \geq T_{\text{reset}}(\sigma_{\text{pre}} - \sigma_{\text{post}})$.