Quantum Gravity Lab (QGL) Core Engine: Theoretical and Computational Specification

Internal Documentation – Quantum Gravity Lab

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1 Purpose and Scope

The *Quantum Gravity Lab* (QGL) core engine is a semi–classical simulation framework designed to study how quantum probes respond to curved spacetime backgrounds generated by classical gravitational fields.

The current version implements:

- A test particle orbiting a central mass in an effective 1PN (first post–Newtonian) Schwarzschild–like potential.
- A scalar *complexity field* that modulates the strength of relativistic corrections as a function of radius.
- A co–moving quantum sensor described by a Jaynes–Cummings (JC) Hamiltonian, with parameters tied directly to the instantaneous orbital geometry.
- Relativistic quantum information diagnostics such as a phenomenological Bell parameter, entanglement entropy, and coherence.

This document specifies the mathematics actually implemented in the `unified_sandbox_core` / QGL codebase. It is not a claim of a fundamental quantum gravity theory. Rather, it is a controlled, reproducible testbed to explore:

1. how small relativistic corrections alter classical orbits, and
2. how a quantum probe, evolved in proper time along that orbit, encodes those corrections in its internal dynamics and correlations.

2 Gravitational Sector: Orbital Dynamics

We consider a point–like test mass m moving in the gravitational field of a central mass M. All dynamics are evolved in coordinate time t (time at infinity). The position vector is $\mathbf{r}(t) = (x, y, z)$, with $r = \|\mathbf{r}\|$ and velocity $\mathbf{v} = \dot{\mathbf{r}}$.

2.1 Orbital Geometry and Units

For a bound orbit, we use the standard Kepler parameters:

$$\begin{aligned} a &= \text{semi–major axis}, && (1)\\ e &\in [0,1) \quad \text{eccentricity}, && (2)\\ p &= a(1-e^2) \quad \text{semi–latus rectum}, && (3)\\ r_{\text{apo}} &= a(1+e) \quad \text{apoapsis radius}. && (4) \end{aligned}$$

The code typically works in units where G and c are explicit, but M may be normalized to unity in some runs. The gravitational radius is

$$r_g = \frac{GM}{c^2}, \tag{5}$$

and many expressions are written in terms of dimensionless ratios r_g/r.

2.2 Complexity Scalar Field

The QGL engine introduces a dimensionless scalar field $C_Q(r)$ that acts as a *control knob* between purely Newtonian dynamics and an effective relativistic correction. It is defined as

$$C_Q(r) = \frac{1}{1 + \left(\dfrac{r}{r_{\text{trans}}}\right)^p}, \tag{6}$$

where:

- $p > 0$ is a phenomenological *complexity power*, calibrated from orbital precession.
- r_{trans} is a transition radius tied to the orbit:

$$r_{\text{trans}} = k_{rq}\, r_{\text{apo}}, \qquad r_{\text{apo}} = a(1 + e), \tag{7}$$

  with k_{rq} a dimensionless factor of order unity.

By construction:

- $C_Q(r) \to 1$ as $r \ll r_{\text{trans}}$ (deep potential),
- $C_Q(r) \to 0$ as $r \gg r_{\text{trans}}$ (asymptotically Newtonian),

so C_Q behaves like a smooth switch that turns relativistic corrections on near periapsis and off at large radii.

The complementary field

$$C_N(r) = 1 - C_Q(r) \tag{8}$$

is sometimes used in diagnostics as a Newtonian weight, but the central force law is written directly in terms of C_Q.

2.3 Decomposition of the Acceleration

The total acceleration is written as

$$\mathbf{a}_{\text{tot}} = \mathbf{a}_{\text{Newt}} + \mathbf{a}_{\text{GR}} + \mathbf{a}_{\text{LT}}, \tag{9}$$

where:

- $\mathbf{a}_{\text{Newt}}$ is the standard Newtonian monopole term,
- $\mathbf{a}_{\text{GR}}$ is a phenomenological 1PN–like correction tuned to reproduce Schwarzschild perihelion precession,
- $\mathbf{a}_{\text{LT}}$ is an optional Lense–Thirring (frame–dragging) contribution for a spinning central body.

2.3.1 Newtonian Term

The Newtonian acceleration is

$$\mathbf{a}_{\text{Newt}} = -\frac{GM}{r^3}\,\mathbf{r}. \tag{10}$$

2.3.2 Effective Relativistic Correction

In Schwarzschild spacetime, the effective potential for a test mass with specific angular momentum L contains a $1/r^3$ term:

$$V_{\text{eff}}(r) = -\frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GML^2}{c^2 r^3} + \cdots, \tag{11}$$

and this cubic contribution is responsible for the standard 1PN perihelion precession.

In QGL, we model this as an explicit radial correction to the force with strength controlled by $C_Q(r)$. We define

$$\alpha_{\text{GR}} = \frac{3GML^2}{c^2}, \tag{12}$$

which is the coefficient that reproduces the known 1PN precession formula when the correction is fully "on". The relativistic contribution in the engine is then

$$\mathbf{a}_{\text{GR}} = -\Lambda_Q(r,v)\,\frac{\mathbf{r}}{r^4}, \qquad \Lambda_Q(r,v) = \alpha_{\text{GR}}\,C_Q(r)\,\gamma(v), \tag{13}$$

where

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{14}$$

is the usual Lorentz factor and $v^2 = \mathbf{v}\cdot\mathbf{v}$.

Comments.

- The factor $1/r^4$ in (13) yields a net $1/r^3$ contribution to the effective potential, matching the radial structure of the Schwarzschild correction.
- The $\gamma(v)$ factor allows high–velocity segments of the orbit (near periapsis) to feel a slightly enhanced correction, while remaining negligible in the non–relativistic limit.
- The scalar field $C_Q(r)$ ensures that at large r the dynamics smoothly reduce to the Newtonian form (10).

2.3.3 Lense–Thirring (Frame Dragging)

For a central body with dimensionless spin parameter a_{spin}, the code implements the leading gravitomagnetic correction. The prefactor is calculated as:

$$\Omega_{\text{pref}} = \frac{2a_{\text{spin}}G^2M^2}{c^3} \tag{15}$$

The resulting acceleration vector is:

$$\mathbf{a}_{\text{LT}} = \frac{\Omega_{\text{pref}}}{r^3}\big(\mathbf{v}\times\hat{\mathbf{z}}\big), \tag{16}$$

where $\hat{\mathbf{z}}$ is the spin axis direction.

In the implementation this is applied componentwise as:

$$a_x \mathrel{+}= -\Omega_z v_y, \tag{17}$$
$$a_y \mathrel{+}= \quad \Omega_z v_x, \tag{18}$$
$$\Omega_z = \frac{2GM a_{\text{spin}}}{c^2 r^3}. \tag{19}$$

This assumes a fixed spin axis along $+\hat{\mathbf{z}}$ and is intended for controlled experiments with frame dragging, not a full Kerr metric.

3 Clocks and Spacetime Metrics

3.1 Coordinate Time vs. Proper Time

The orbit is parameterized by coordinate time t (time measured by an observer at infinity). A local clock co–moving with the test mass ticks proper time τ. In the Schwarzschild metric, ignoring transverse motion for the redshift factor, the relation is

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2GM}{c^2 r}}. \tag{20}$$

This factor is used to map classical timestep Δt to local quantum evolution time:

$$\Delta\tau = \left(\frac{d\tau}{dt}\right) \Delta t. \tag{21}$$

3.2 Gravitational Redshift Between Radii

Given an emitter at radius r_e and an observer at radius r_o, the frequency shift is computed via the ratio of proper time rates:

$$1 + z = \frac{\nu_o}{\nu_e} = \frac{(d\tau/dt)_{r_e}}{(d\tau/dt)_{r_o}}, \tag{22}$$

where $(d\tau/dt)_r$ is given by (20). This relation is used internally for diagnostics and for mapping quantum frequencies to an "observer at infinity" when needed.

4 Quantum Sector: Jaynes–Cummings Sensor

Along the classical trajectory, QGL evolves a co–moving quantum probe modeled as a two–level system (qubit) coupled to a single bosonic mode (cavity). This is described by a Jaynes–Cummings Hamiltonian, with parameters slaved to the instantaneous orbital state.

4.1 Hamiltonian in the Rotating–Wave Approximation

In the rotating–wave approximation (RWA), the local Hamiltonian is

$$\hat{H}_{\text{JC}} = \hbar\omega_c(r)\, \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_a(r)}{2} \hat{\sigma}_z + \hbar g(v) \big(\hat{a}^\dagger \hat{\sigma}_- + \hat{a}\hat{\sigma}_+\big), \tag{23}$$

where:

- $\omega_c(r)$ is the cavity mode frequency,
- $\omega_a(r)$ is the qubit transition frequency,
- $g(v)$ is the cavity–qubit coupling,
- $\hat{a}$, $\hat{a}^\dagger$ are bosonic ladder operators,
- $\hat{\sigma}_\pm$, $\hat{\sigma}_z$ are Pauli operators for the two–level atom.

4.2 Mapping Geometry to Quantum Parameters

To tie the quantum sector to the gravitational environment, QGL maps local geometric quantities into frequency shifts and coupling strengths. We introduce three dimensionless sensitivity parameters s_ρ, s_ϕ, s_v and define

$$\omega_c(r) = \omega_{c,0}\left[1 + s_\rho\,\frac{\rho(r)}{\rho_{\rm ref}}\right], \tag{24}$$

$$\omega_a(r) = \omega_{a,0}\left[1 + s_\phi\,\frac{\Phi(r)}{\Phi_{\rm ref}}\right], \tag{25}$$

$$g(v) = g_0\left[1 + s_v\,\frac{|\mathbf{v}|}{v_{\rm ref}}\right]. \tag{26}$$

Here:

- $\omega_{c,0}$ and $\omega_{a,0}$ are reference frequencies,
- g_0 is the baseline coupling strength,
- $\rho(r)$ is a simple density proxy
$$\rho(r) \propto \frac{1}{r^2}, \tag{27}$$
representing a spherically symmetric falloff,
- $\Phi(r)$ is the Newtonian potential
$$\Phi(r) = -\frac{GM}{r}, \tag{28}$$
- $\rho_{\rm ref}$, $\Phi_{\rm ref}$, and $v_{\rm ref}$ are reference scales (chosen once per run) used to make the ratios dimensionless.

Thus:

- As the probe dips deeper into the potential well (more negative $\Phi(r)$), the atomic transition frequency shifts according to (25).
- As the probe moves to smaller radii (larger $\rho(r)$), the cavity frequency (24) responds to the effective "density" environment.
- Faster motion (larger $|\mathbf{v}|$) modifies the coupling $g(v)$ via (26), allowing Doppler and kinetic effects to leave a signature in the sensor dynamics.

4.3 Quantum Time Evolution

The quantum state is represented in the engine as a density matrix $\rho(\tau)$ on a truncated JC Hilbert space $\mathcal{H} = \mathrm{span}\{|g,n\rangle, |e,n\rangle\}$ with $n = 0, 1, \ldots, n_{\max}$. For a classical timestep Δt, the corresponding proper time step is

$$\Delta\tau = \left(\frac{d\tau}{dt}\right)\Delta t, \tag{29}$$

with $d\tau/dt$ from (20). The unitary update is

$$\rho(t+\Delta t) = U\rho(t)U^\dagger, \qquad U = \exp\left(-\frac{i}{\hbar}\hat{H}_{\rm JC}\Delta\tau\right). \tag{30}$$

The implementation:

- Recomputes $\hat{H}_{\mathrm{JC}}$ along the trajectory using the instantaneous $\omega_c(r)$, $\omega_a(r)$, and $g(v)$.
- Applies U via a matrix exponential (e.g. `expm`).
- Enforces Hermiticity and trace normalization after each step.

Optional dissipation channels (e.g. cavity decay κ and atomic relaxation γ) can be added in a Lindblad step following the unitary, but the core engine keeps the dynamics closed unless explicitly configured otherwise.

5 Relativistic Quantum Information Diagnostics

QGL includes several diagnostics to quantify how the environment and the complexity field imprint themselves on the quantum sensor.

5.1 Phenomenological Bell Parameter

To model how non–local correlations degrade in curved spacetime, the engine computes a phenomenological Bell parameter $\beta(r)$ with the correct classical and quantum limits:

$$\beta(r) = \beta_{\mathrm{cl}} + C_Q^{\mathrm{eff}}(r)\big(\beta_{\mathrm{qm}} - \beta_{\mathrm{cl}}\big), \tag{31}$$

where

$$\beta_{\mathrm{cl}} = 2, \qquad \beta_{\mathrm{qm}} = 2\sqrt{2} \tag{32}$$

are the CHSH bounds for classical and maximally entangled quantum systems.

The effective correlation coefficient combines the complexity field, gravitational redshift, and special–relativistic time dilation:

$$C_Q^{\mathrm{eff}}(r) = C_Q(r)\sqrt{1 - \frac{2GM}{c^2 r}}\left(2 - \frac{1}{\gamma(v)}\right), \tag{33}$$

with $\gamma(v)$ as before.

By construction:

- In weak fields and low velocities ($r \gg r_{\mathrm{trans}}$, $v \ll c$), $C_Q^{\mathrm{eff}} \to 0$ and $\beta \to 2$.
- In strongly curved regions with high complexity activation, $C_Q^{\mathrm{eff}} \to 1$ and $\beta \to 2\sqrt{2}$.

Equation (31) is explicitly labeled as an *ansatz*: it is not derived from a specific QFT in curved spacetime, but is a controlled way to encode environmental suppression of Bell violations into a single scalar observable.

5.2 Entanglement Entropy of the Qubit

Let ρ be the full JC density matrix and $\rho_q = \mathrm{Tr}_{\mathrm{cavity}}(\rho)$ the reduced state of the qubit. The von Neumann entropy

$$S(\rho_q) = -\mathrm{Tr}(\rho_q \log_2 \rho_q) = -\sum_i \lambda_i \log_2 \lambda_i \tag{34}$$

is computed from the eigenvalues λ_i of ρ_q. This measures the entanglement (for a pure global state) or, more generally, the mixedness of the probe induced by its interaction with the cavity mode and the environment.

5.3 Coherence and Concurrence Proxy

In the qubit basis $\{|g\rangle, |e\rangle\}$, we define a coherence measure

$$\mathcal{C}_{\text{coh}} = |(\rho_q)_{ge}|, \tag{35}$$

and a simple concurrence–like proxy

$$C_{\text{proxy}} = 2\,\mathcal{C}_{\text{coh}}, \tag{36}$$

which is numerically clamped to $[0,1]$ in the implementation. These provide a fast way to track how off–diagonal quantum coherence responds to orbital phase and curvature.

6 Numerical Implementation and Calibration

6.1 Integration Scheme

The classical equations of motion with acceleration (9) are integrated using an explicit embedded Runge–Kutta method of order 8(5,3) (DOP853). This scheme is chosen because:

- The relativistic term (13) is small but highly sensitive; resolving its effect on precession requires high local accuracy over many orbital periods.
- The adaptive timestep control allows the integrator to refine near periapsis (where $C_Q(r)$ is large and curvature effects are strongest) without wasting effort at large radii.

Energy and angular momentum are monitored along the trajectory to ensure that numerical artifacts remain small compared to the intended relativistic perturbation.

6.2 Complexity Power Calibration

The complexity power p in (6) is not chosen by hand. Instead, QGL calibrates p by matching the measured perihelion precession to the 1PN prediction.

For an orbit with semi–major axis a and eccentricity e, general relativity predicts a per–orbit advance

$$\Delta\omega_{\text{GR}} = \frac{6\pi GM}{c^2 a(1-e^2)}. \tag{37}$$

In the engine, the precession measured from the integrated orbit is denoted $\Delta\omega_{\text{measured}}(p)$. The calibration loop minimizes the scalar error

$$\epsilon(p) = \left|\Delta\omega_{\text{measured}}(p) - \frac{6\pi GM}{c^2 a(1-e^2)}\right|. \tag{38}$$

The algorithm performs:

- A coarse scan over a range of candidate p values.
- A refinement step (e.g. golden–section search) around the best coarse candidate.
- Storage of the resulting (p, e) calibration pair in a CSV table for later reuse.

As a result, for each (p, e) pair used in production runs, the relativistic correction (13) is numerically tied to the 1PN Schwarzschild precession via (37). This is the sense in which the gravitational sector of QGL is “anchored” to known physics.

7 Summary

The Quantum Gravity Lab core engine combines:

- A calibrated semi–classical orbital model with Newtonian and effective 1PN corrections,
- A geometric complexity scalar $C_Q(r)$ that controls the transition between regimes,
- A co–moving Jaynes–Cummings quantum sensor whose parameters are direct functions of the local gravitational environment,
- Relativistic quantum information diagnostics such as a phenomenological Bell parameter and entanglement measures.

Everything in this document corresponds directly to code paths in the QGL repository. The goal is to provide a transparent, theory–heavy, but computationally grounded specification that can be:

- reproduced by other researchers,
- extended to more complex field configurations, and
- eventually adapted to model more extreme environments (e.g. plasma and fusion conditions in tokamak geometries).