FPV Tutorübung

Woche 3

MiniJava 2.0, Loop Invariants

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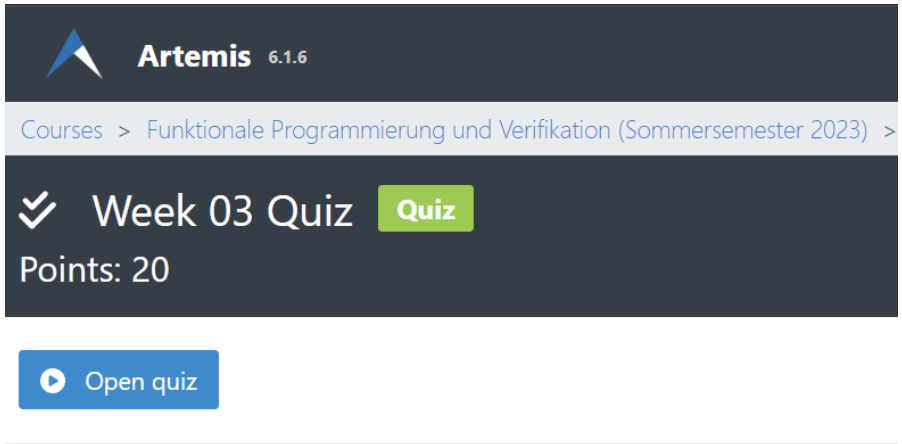
09.05.2023

Quiz



Passwort:

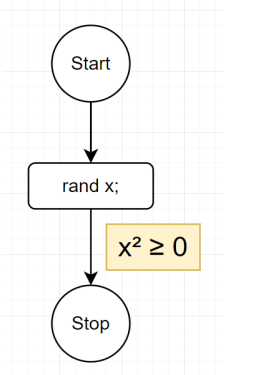
T01: MiniJava 2.0

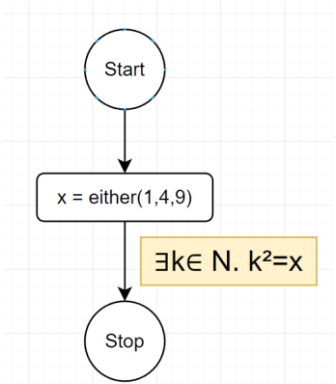
In the lecture, the weakest precondition operator has been defined for all statements of MiniJava. In this assignment, we consider an extension of the MiniJava language, which provides four new statements:

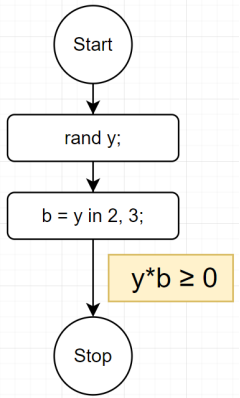
1. `rand x`:
   Assigns a random value to variable x,
2. `x = either` $e_0, \ldots, e_k$:
   Assigns one of the values of the expressions $e_0, \ldots, e_k$ to variable `x` non-deterministically,
3. `x = e in a, b`:
   Assigns the value 1 to variable `x`, if the value of expression `e` is in the range $[a, b]$ and 0 if `e` is not in the range or the range is empty $(a > b)$,
4. `stop`:
   Immediately stops the program.

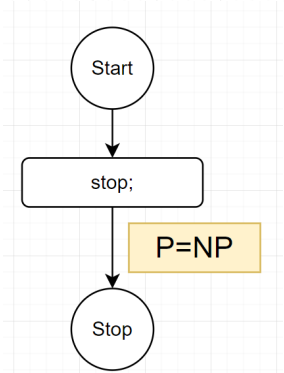
Define the weakest precondition operator $\mathbf{WP}[\![\ldots]\!](B)$ for each of these statements. (In terms of B)

Beispiele zum Testen:



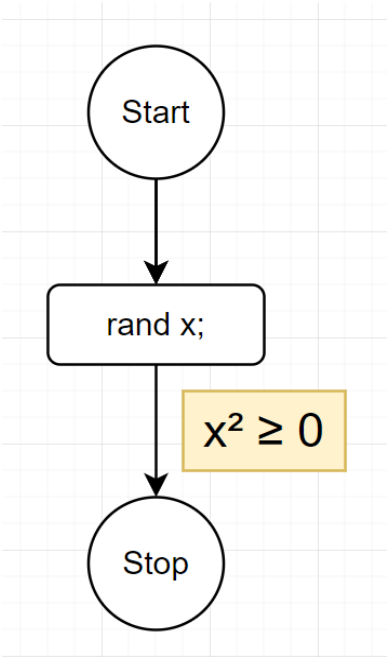






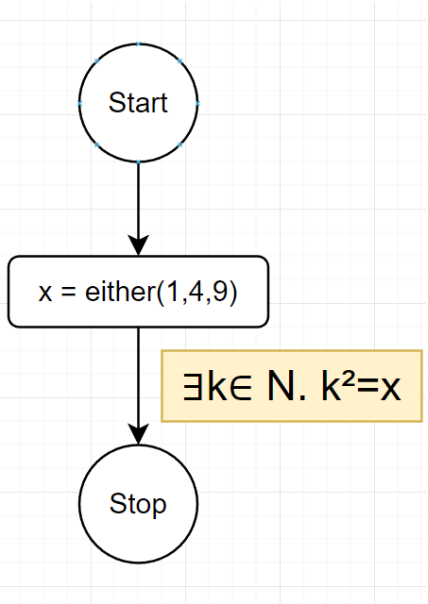
T01: MiniJava 2.0

WP[rand x;](B) =



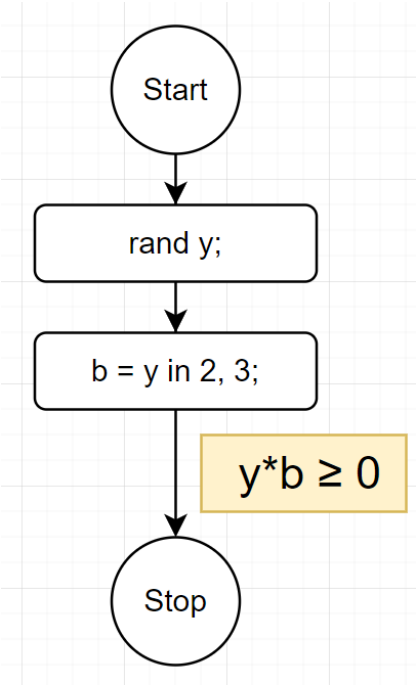
T01: MiniJava 2.0

WP[x = either e_0, e_1 … e_k](B) =



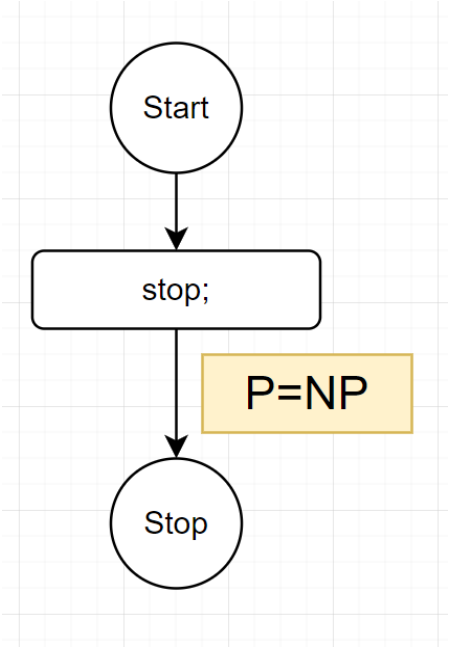
T01: MiniJava 2.0

WP[x e in a, b](B) =



T01: MiniJava 2.0

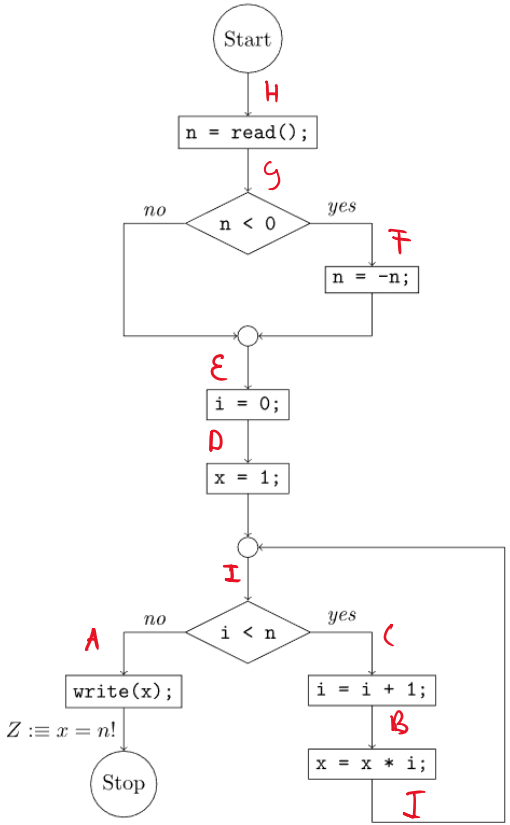
WP[stop](B) =



T02: Loop Invariants

1. Discuss the problem that arises when computing weakest preconditions to prove Z.
2. How can you use weakest preconditions to prove Z anyway?
3. Try proving Z using the the loop invariants $x \geq 0$ and $i = 0 \land x = 1 \land n = 0$ **at the end of the loop body** and in particular discuss these questions:
   - a) How has a useful loop invariant be related to Z?
   - b) What happens if the loop invariant is chosen too strong?
   - c) What happens if the loop invariant is chosen too weak?
   - d) Can you give a meaningful lower and upper bound for useful loop invariants?
4. Retry proving Z using the loop invariant $x = i!$ (again at the end of the loop body) and improve this invariant until the proof succeeds.

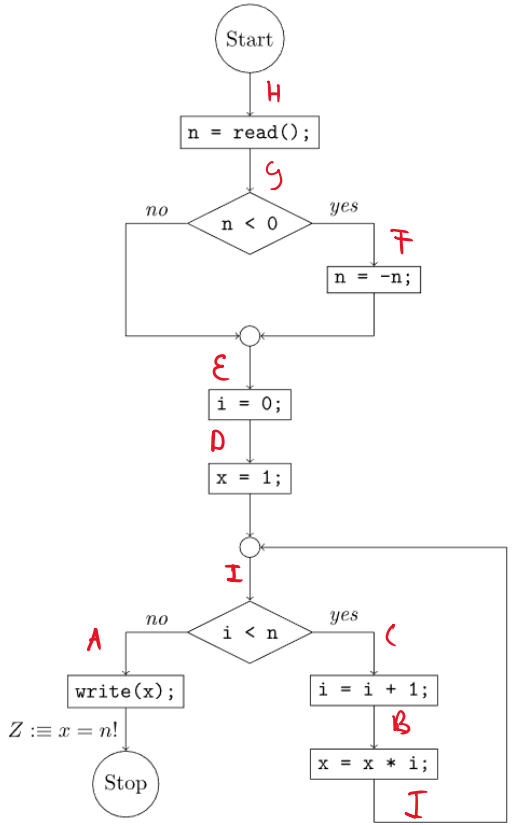
A program computes the factorial of its input:



T02: Loop Invariants 1

3. Try proving Z using the the loop invariants $x \geq 0$ and $i = 0 \wedge x = 1 \wedge n = 0$ **at the end of the loop body** and in particular discuss these questions:

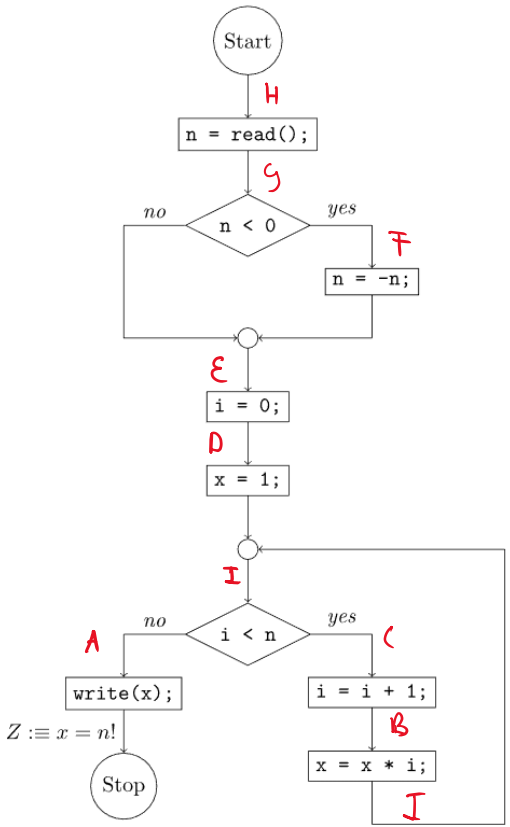
A program computes the factorial of its input:



T02: Loop Invariants 2

3. Try proving Z using the the loop invariants $x \geq 0$ and $i = 0 \wedge x = 1 \wedge n = 0$ **at the end of the loop body** and in particular discuss these questions:

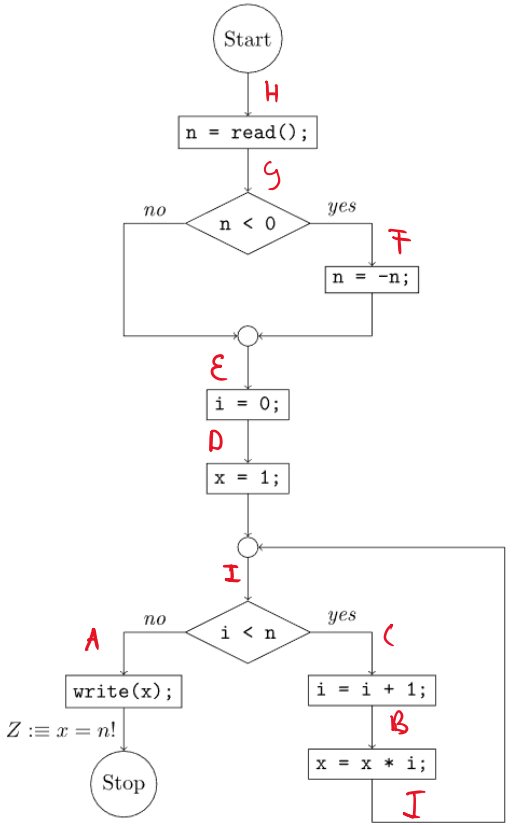
A program computes the factorial of its input:



T02: Loop Invariants 3

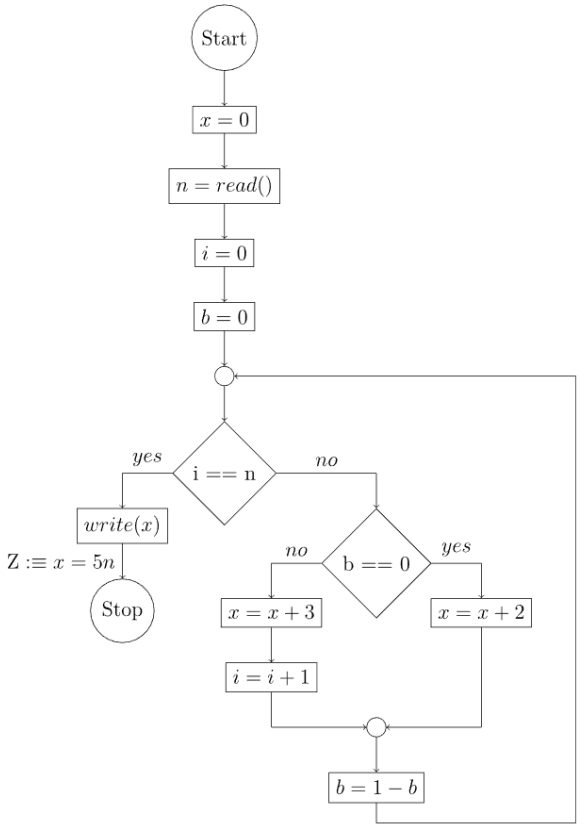
4. Retry proving Z using the loop invariant $x = i!$ (again at the end of the loop body) and improve this invariant until the proof succeeds.

A program computes the factorial of its input:



T03: Two b, or Not Two b

Prove Z using weakest preconditions.



T03: Two b, or Not Two b

Tipps zum finden von Loop Invarianten:
https://ttt.in.tum.de/recordings/Info2_2017_11_24-1/Info2_2017_11_24-1.mp4

Beispieltrace: n=3

Variable \ Schleifendurchgang	0	1	2	3	4	5	6
x	**0**	**2**	**5**	**7**	**10**	**12**	**15**
i	**0**	**0**	**1**	**1**	**2**	**2**	**3**
b	**0**	**1**	**0**	**1**	**0**	**1**	**0**