FPV Tutorübung

Woche 1

Implications, Assertions and Conditions

Manuel Lerchner

20.04.2023

Organisatorisches

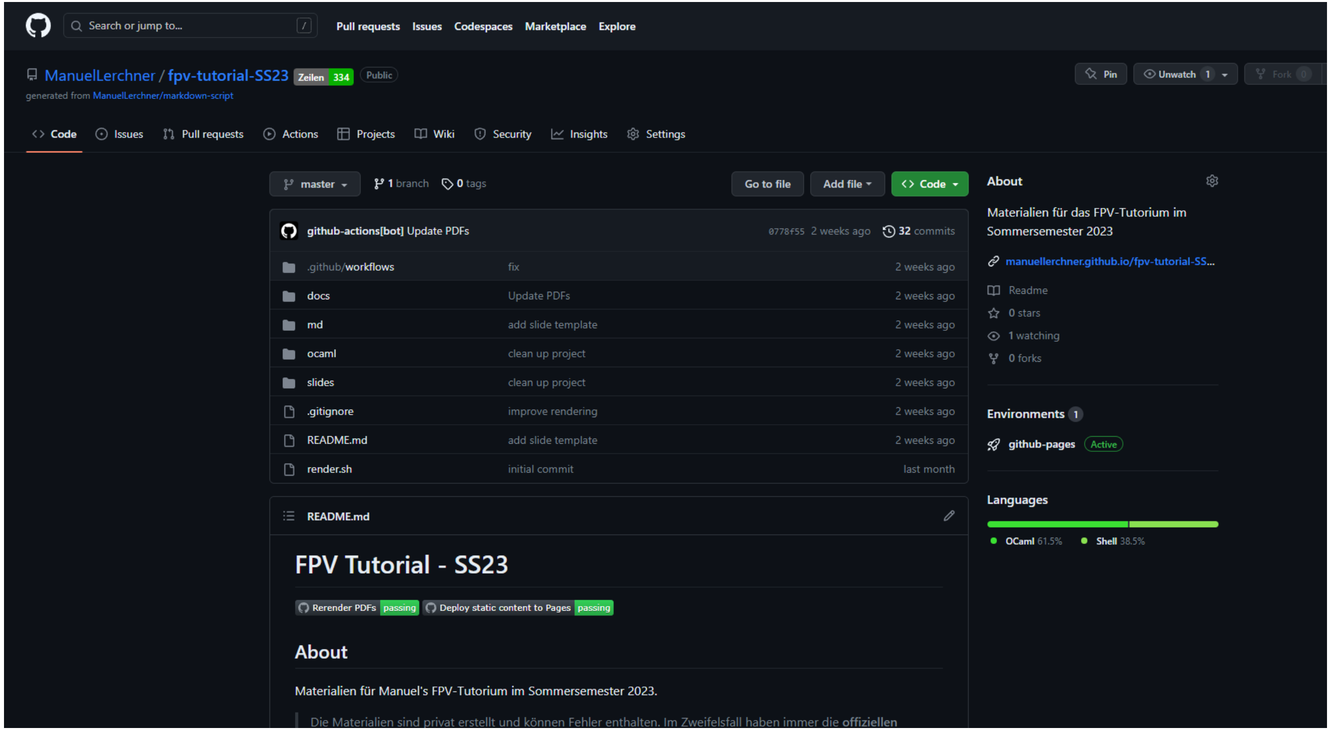
Grade Bonus

- Successful participation ($\geq 70\%$) in quizzes and programming tasks will lead to a bonus of 0.3 in the final exam, provided that you passed the exam.
- Programming homework and quizzes are to be submitted individually.
- Discussing solutions before the end of the week is considered plagiarism.
- Plagiarism will not be tolerated and will (at the very least) lead to exclusion from the bonus system

Changes

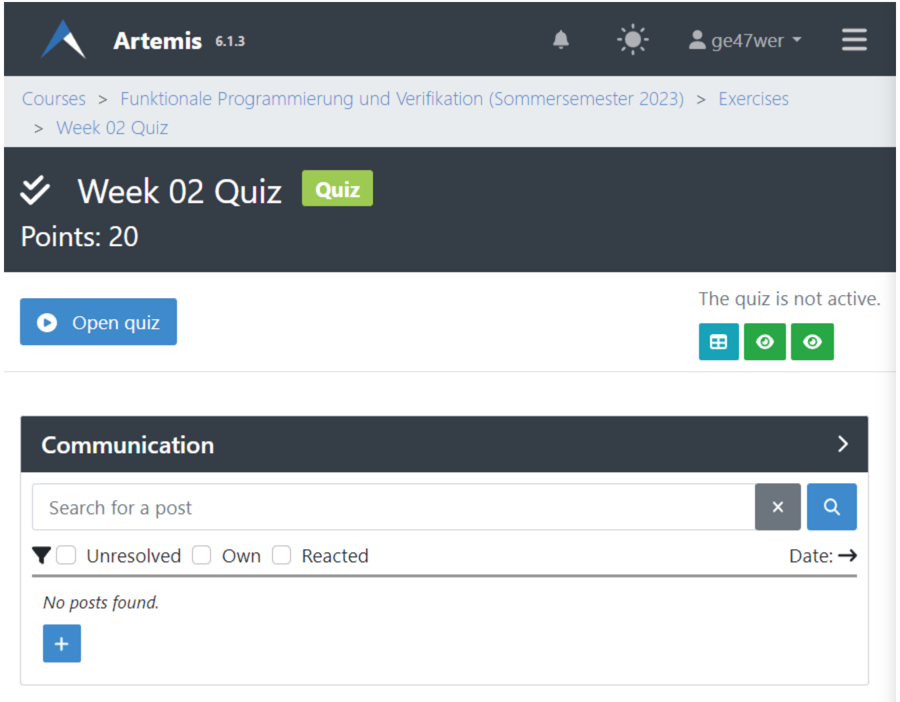
- Manual correction of homework not possible. However, non-programming exercises remain crucial for the exam
- 20% of the exam will be Single-Choice
- To receive points in the exam, your code needs to compile
- We currently anticipate an in-person exam using Artemis

Materialien



https://github.com/ManuelLerchner/fpv-tutorial-SS23

Quiz

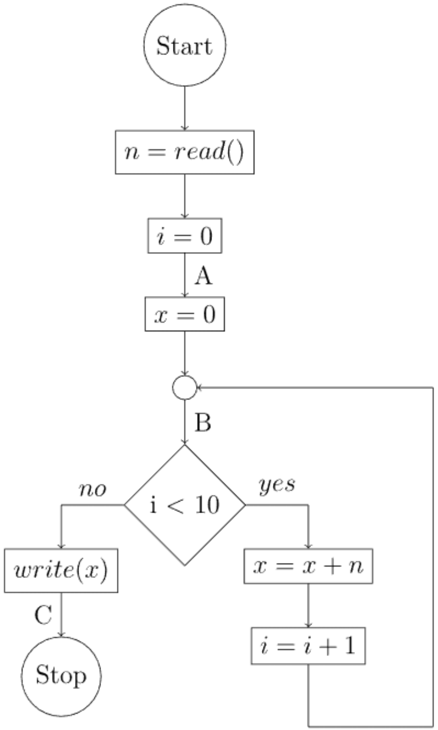


Passwort:

T01: Recap Implications

1. $x = 1 \implies 0 < x$
2. $x < 6 \implies x = 3$
3. $x > 0 \implies x \geq 0$
4. $x = -2 \implies x < -1 \lor x > 1$
5. $x = 0 \lor x = 7 \implies 4 \neq x$
6. $x = 1 \implies x \leq 3 \land y > 0$
7. $x < 8 \land y = x \implies y \neq 12$
8. $x = 1 \lor y = 1 \implies x > 0$
9. $x \neq 5 \implies false$
10. $true \implies x \neq y$
11. $false \implies x = 1$
12. $x \geq 1 \implies 2x + 3 = 5$
13. $A \land x = y \implies A$
14. $B \implies A \lor B$
15. $A \implies (B \implies A)$
16. $(A \implies B) \implies A$

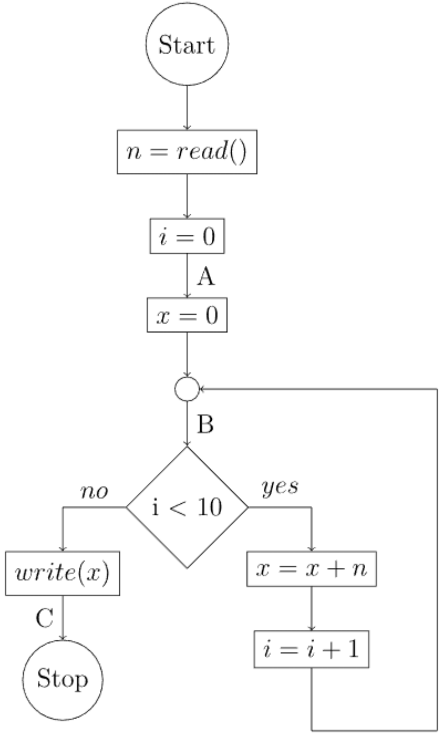
T02: Assertions



1. Which of the following assertions hold at point A?
   - a) $i \geq 0$
   - b) $x = 0$
   - c) $i \leq 10 \wedge x \neq 0$
   - d) $true$
   - e) $i = 0$
   - f) $x = i$
2. Which of the following assertions hold at point B?
   - a) $x = 0 \wedge i = 0$
   - b) $x = i$
   - c) $i < x$
   - d) $0 \leq i \leq 10$
   - e) $i \geq 0 \wedge x \geq 0$
   - f) $n = 1 \implies x = i$
3. Which of the following assertions hold at point C?
   - a) $i \geq 0$
   - b) $i = 10$
   - c) $i > 0$
   - d) $x \neq n$
   - e) $x = 10n$
   - f) $x = i * n \wedge i = 10$

T03: The Strong and the Weak

3. Which of the following assertions hold at point C?
   - a) $i \geq 0$ ✓
   - b) $i = 10$ ✓
   - c) $i > 0$ ✓
   - d) $x \neq n$ ✗
   - e) $x = 10n$ ✓
   - f) $x = i * n \wedge i = 10$ ✓



Again consider the assertions that hold at point C of assignment 2. Discuss the following questions:

1. When annotating the control flow graph, can you say that one of the given assertions is "better" than the others?
2. Can you arrange the given assertions in a meaningful order?
3. How can you define a *stronger than* relation formally?
4. How do $true$ and $false$ fit in and what is their meaning as an assertion?
5. What are the strongest assertions that still hold at A, B and C?

T04: Strongest Postconditions 1

1.

$$true$$
$$\boxed{x = 5}$$
$$B$$

2.

$$x = 0 \land y = 2$$
$$\boxed{x = 18}$$
$$B$$

3.

$$x \geq 7 \land y = -2$$
$$\boxed{x = y + x}$$
$$B$$

4.

$$x = 3 * i \land i \geq 0$$
$$\boxed{i = i + 1}$$
$$B$$

5.

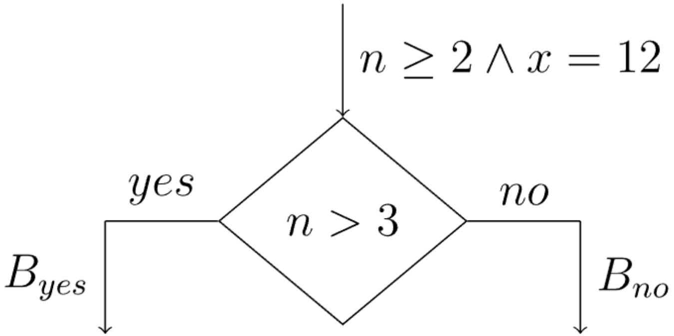
$$y = 2 + x$$
$$\boxed{x = read()}$$
$$B$$

6.

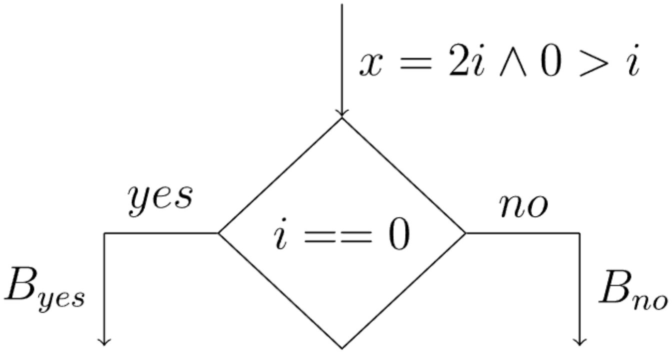
$$i = i + 1$$
$$\boxed{i = 0}$$
$$B$$

T04: Strongest Postconditions 2

7.

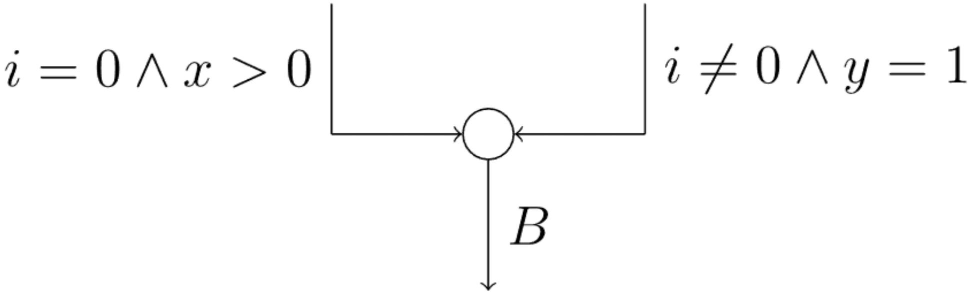


8.



T04: Strongest Postconditions 3

9.



FPV Tutorübung

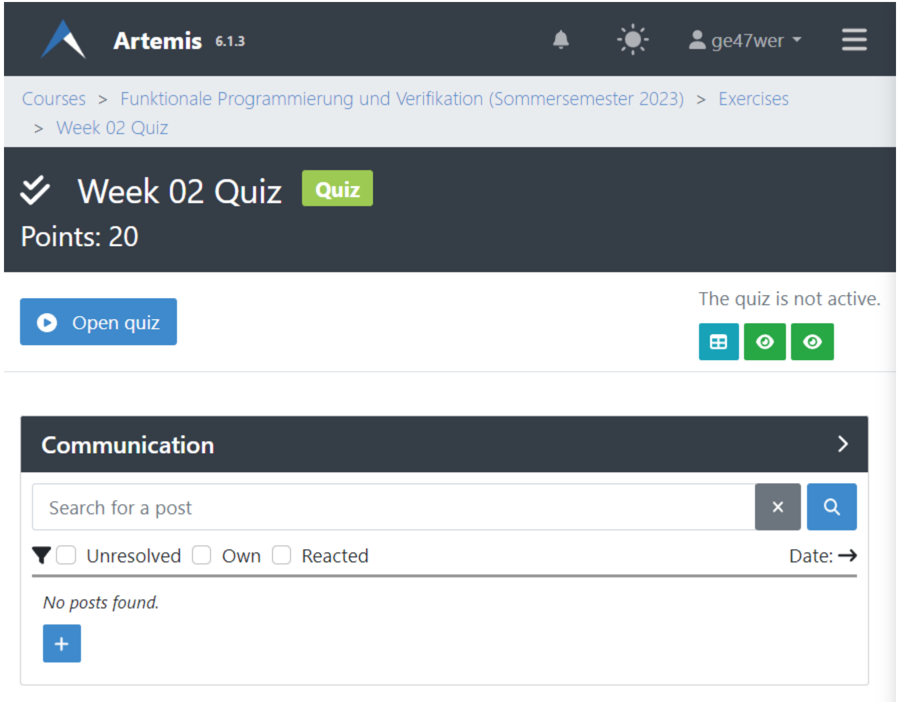
Woche 2

Preconditions, Postconditions and Local Consistency

Manuel Lerchner

03.05.2023

Quiz

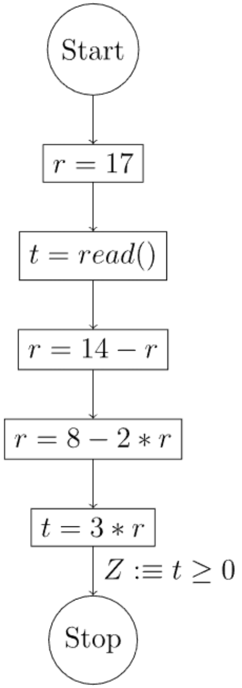


Passwort:

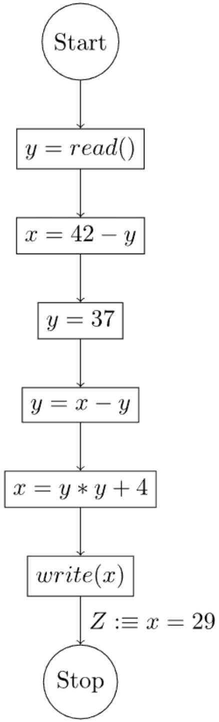
____

T01: From Post- to Preconditions

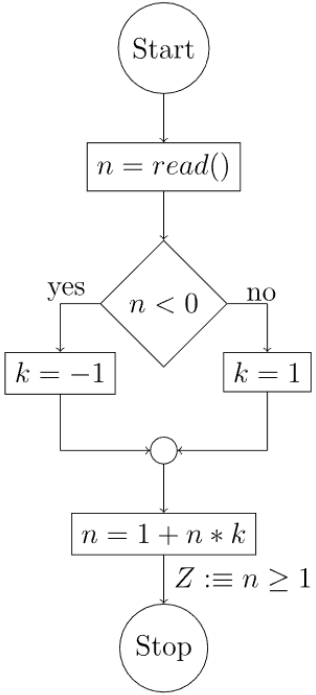
1.



2.



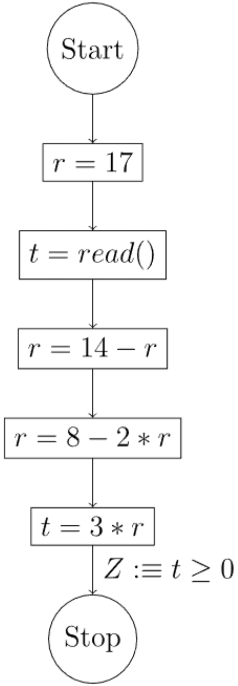
3.



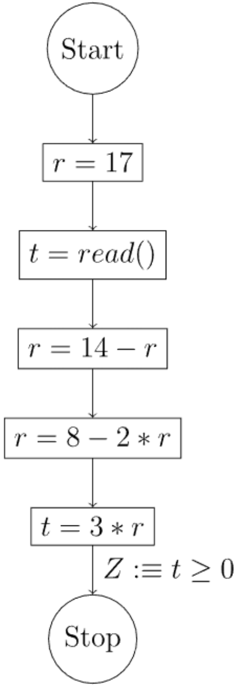
1. For each of these graphs show whether the assertion Z holds…
   (a) …using strongest postconditions and
   (b) …using weakest preconditions.
2. Discuss advantages and disadvantages of either approach.

T01: From Post- to Preconditions 1

Post-Condition:

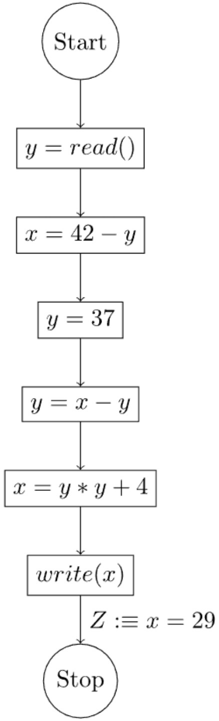


Pre-Condition:

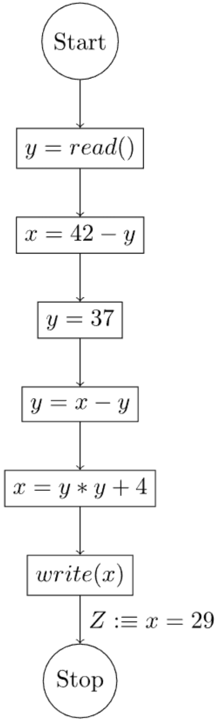


T01: From Post- to Preconditions 2

Post-Condition:

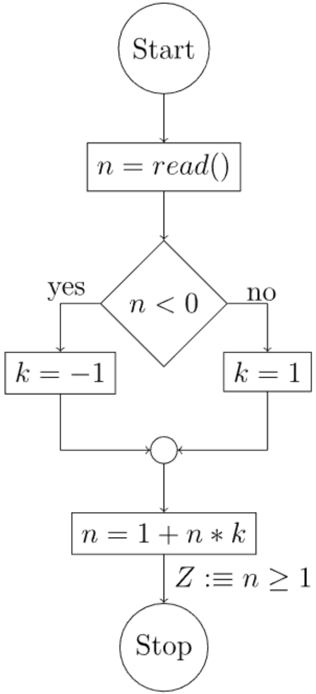


Pre-Condition:

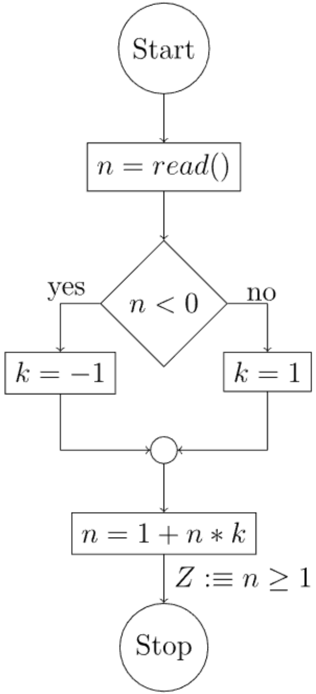


T01: From Post- to Preconditions 3

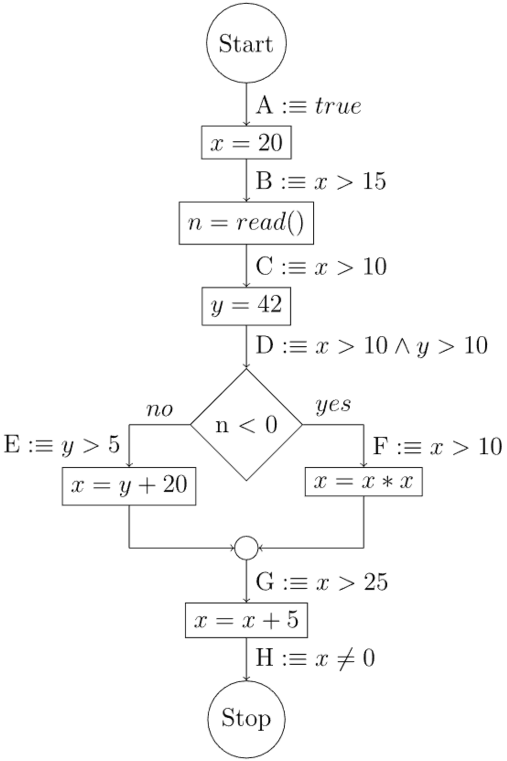
Post-Condition:



Pre-Condition:

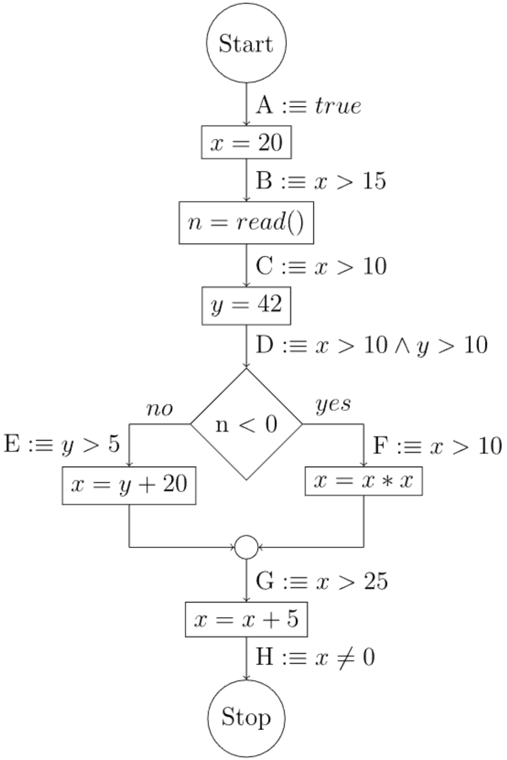


T02: Local Consistency

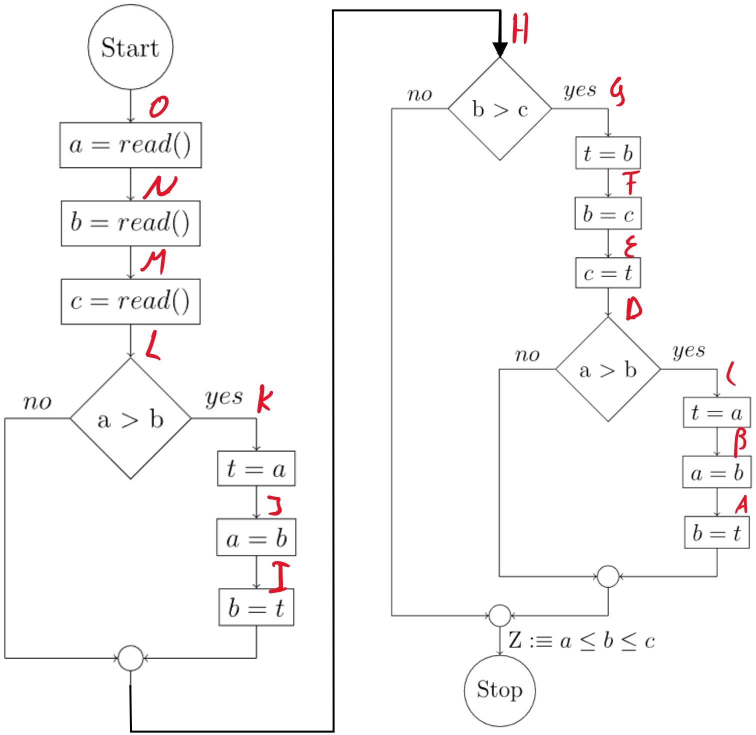


Check whether the annotated assertions prove that the program computes an $x \neq 0$ and discuss why this is the case.

T02: Local Consistency (Extra Space)

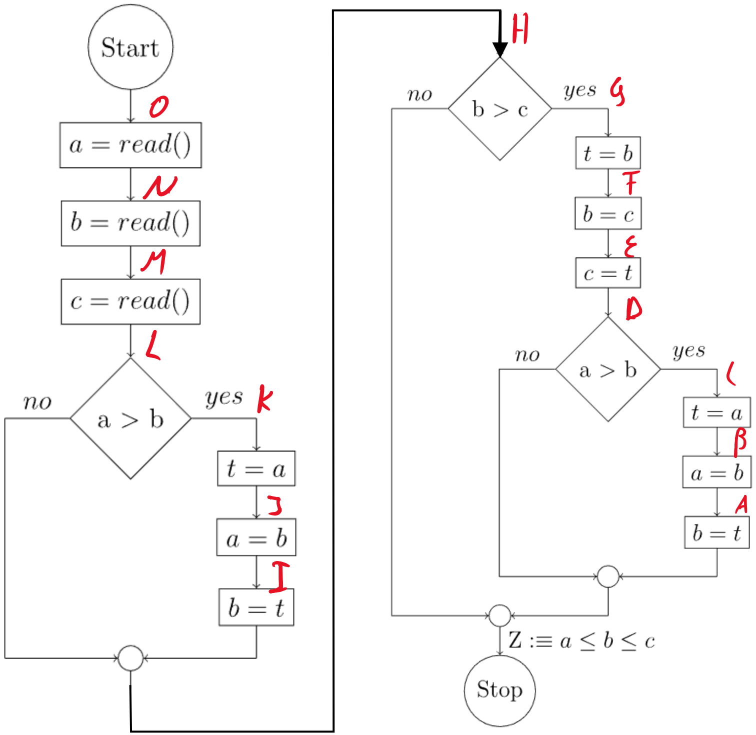


T03: Trouble Sort



1. Annotate each program point in the following control flow diagram with a suitable assertion, then show that your annotations are locally consistent and prove that Z holds at the given program point.
2. Discuss the drawbacks of annotating each program point with an assertion before applying weakest preconditions, and discuss how you could optimize the approach to proving that Z holds.

T03: Trouble Sort (Extra Space)



FPV Tutorübung

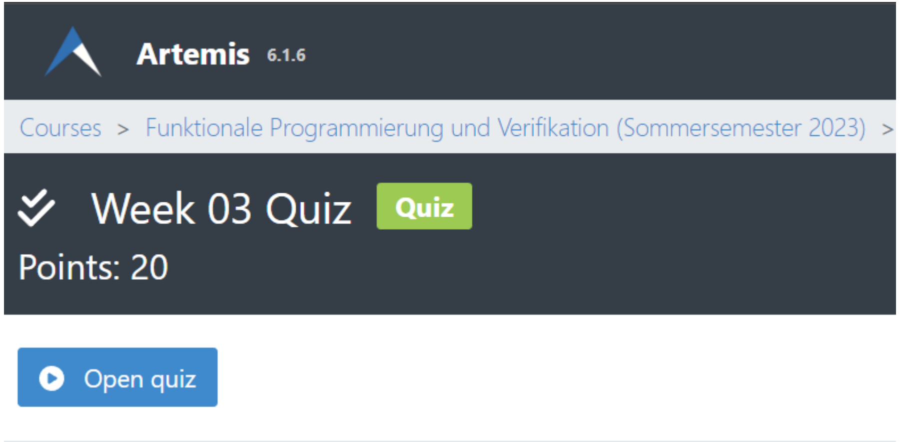
Woche 3

MiniJava 2.0, Loop Invariants

Manuel Lerchner

09.05.2023

Quiz



Passwort:

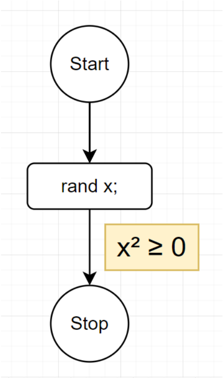
T01: MiniJava 2.0

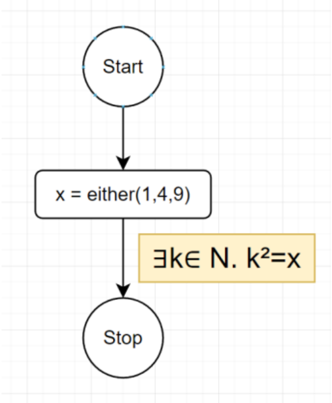
In the lecture, the weakest precondition operator has been defined for all statements of MiniJava. In this assignment, we consider an extension of the MiniJava language, which provides four new statements:

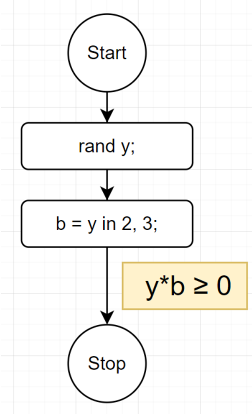
1. `rand x`:
   Assigns a random value to variable x,
2. `x = either` $e_0, \ldots, e_k$:
   Assigns one of the values of the expressions $e_0, \ldots, e_k$ to variable `x` non-deterministically,
3. `x = e in a, b`:
   Assigns the value 1 to variable `x`, if the value of expression `e` is in the range $[a, b]$ and 0 if `e` is not in the range or the range is empty $(a > b)$,
4. `stop`:
   Immediately stops the program.

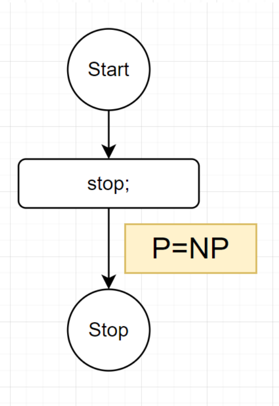
Define the weakest precondition operator $\mathbf{WP}[\![\ldots]\!](B)$ for each of these statements. (In terms of B)

Beispiele zum Testen:



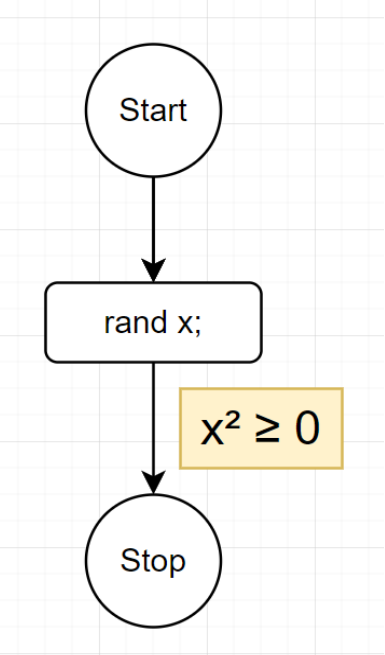






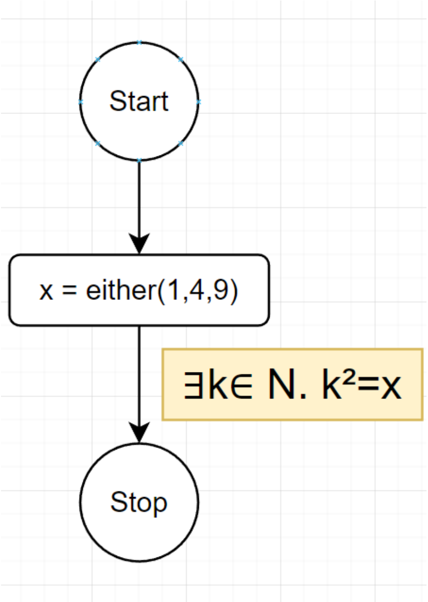
T01: MiniJava 2.0

WP[rand x;](B) =



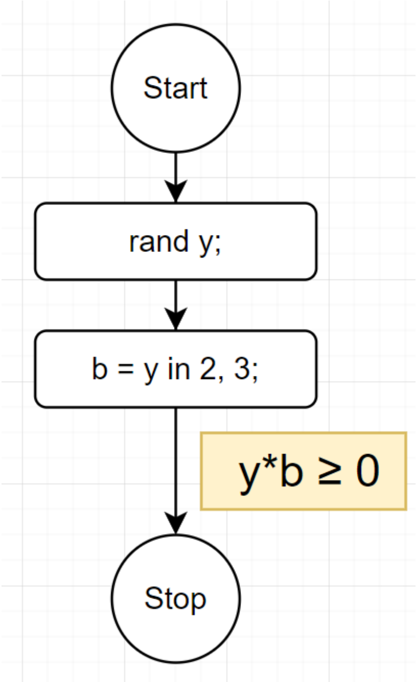
T01: MiniJava 2.0

$\text{WP}[x = \text{either } e_0, e_1 \ldots e_k](B) =$



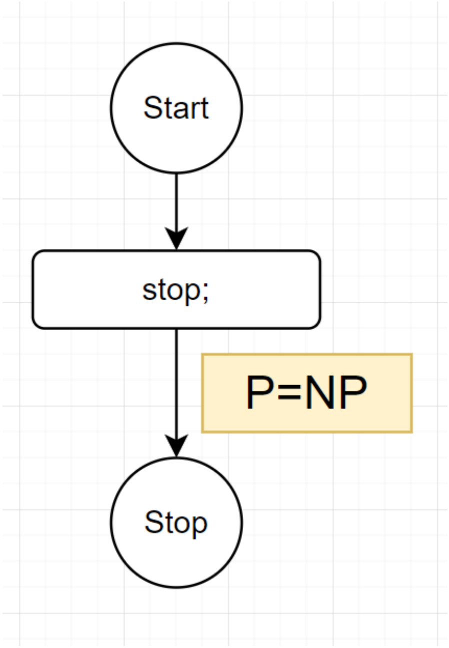
T01: MiniJava 2.0

WP[x e in a, b](B) =



T01: MiniJava 2.0

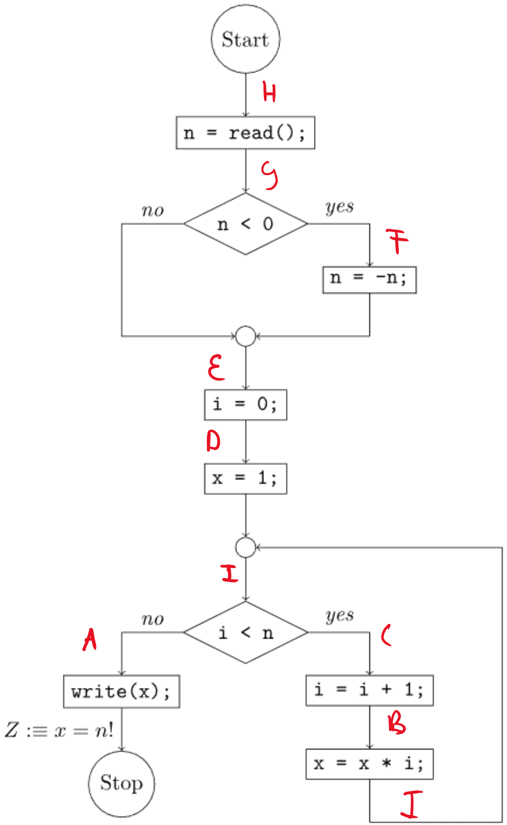
WP[stop](B) =



T02: Loop Invariants

1. Discuss the problem that arises when computing weakest preconditions to prove Z.
2. How can you use weakest preconditions to prove Z anyway?
3. Try proving Z using the the loop invariants $x \geq 0$ and $i = 0 \land x = 1 \land n = 0$ **at the end of the loop body** and in particular discuss these questions:
   - a) How has a useful loop invariant be related to Z?
   - b) What happens if the loop invariant is chosen too strong?
   - c) What happens if the loop invariant is chosen too weak?
   - d) Can you give a meaningful lower and upper bound for useful loop invariants?
4. Retry proving Z using the loop invariant $x = i!$ (again at the end of the loop body) and improve this invariant until the proof succeeds.

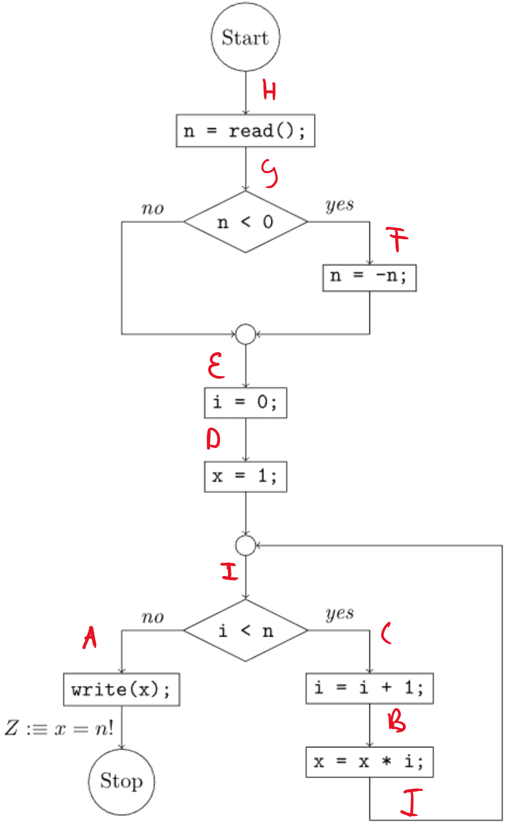
A program computes the factorial of its input:



T02: Loop Invariants 1

3. Try proving Z using the the loop invariants $x \geq 0$ and $i = 0 \wedge x = 1 \wedge n = 0$ **at the end of the loop body** and in particular discuss these questions:

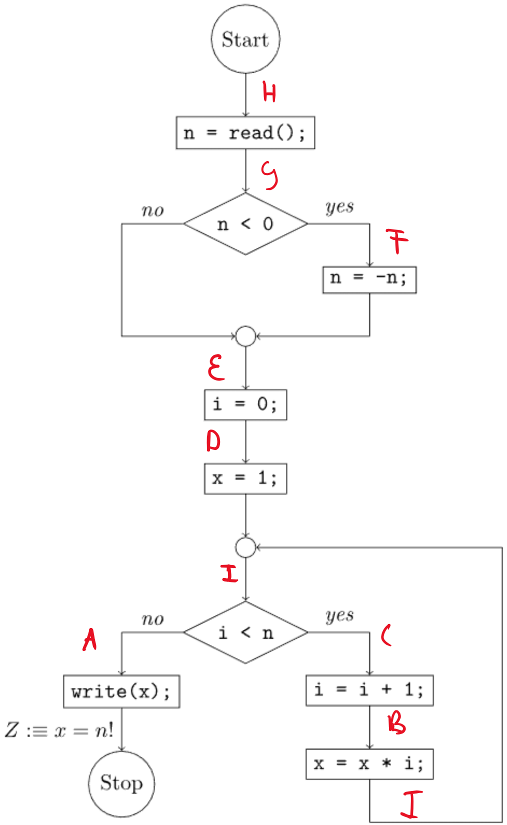
A program computes the factorial of its input:



T02: Loop Invariants 2

3. Try proving Z using the the loop invariants $x \geq 0$ and $i = 0 \land x = 1 \land n = 0$ **at the end of the loop body** and in particular discuss these questions:

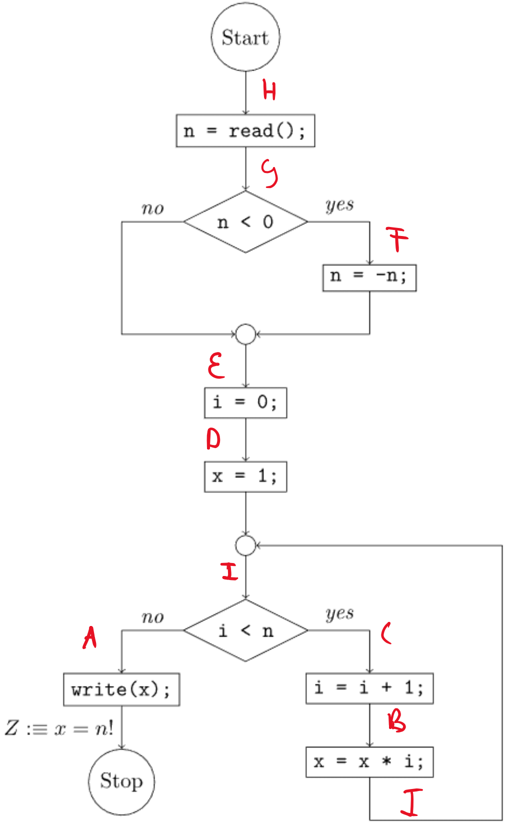
A program computes the factorial of its input:



T02: Loop Invariants 3

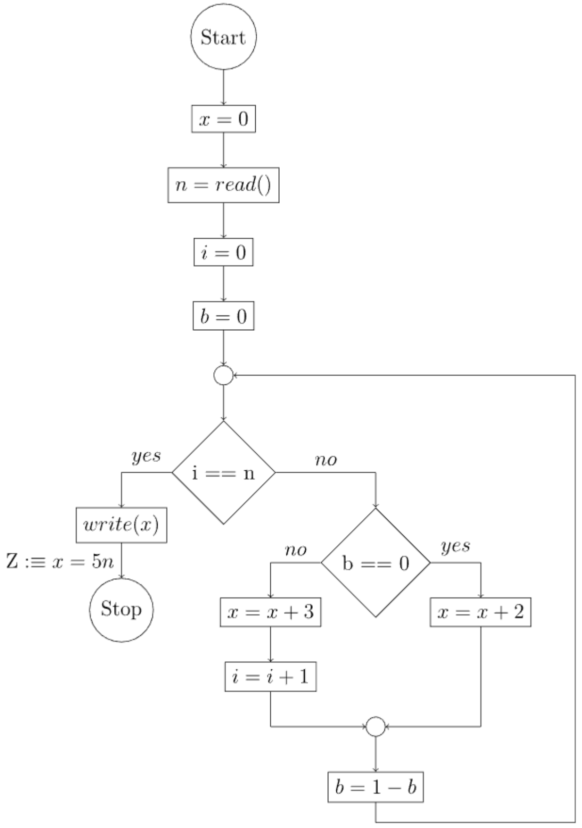
4. Retry proving Z using the loop invariant $x = i!$ (again at the end of the loop body) and improve this invariant until the proof succeeds.

A program computes the factorial of its input:



T03: Two b, or Not Two b

Prove Z using weakest preconditions.



T03: Two b, or Not Two b

Tipps zum finden von Loop Invarianten:
https://ttt.in.tum.de/recordings/Info2_2017_11_24-1/Info2_2017_11_24-1.mp4

Beispieltrace: n=3

Variable \ Schleifendurchgang	0	1	2	3	4	5	6
x	**0**	**2**	**5**	**7**	**10**	**12**	**15**
i	**0**	**0**	**1**	**1**	**2**	**2**	**3**
b	**0**	**1**	**0**	**1**	**0**	**1**	**0**