$$g_1(n) = \cos(\omega_0 n) \qquad n = 0, \ldots, N-1$$

$$g_2(n) = \cos(\omega_0 n + \theta) \qquad n = 0, \ldots, N-1$$

$$g_2(0) = g_1(N)$$

$$\cos(\theta) = \cos(\omega_0 N) =$$

$$= \cos(\omega_0 N + 2\pi K) \qquad K \in \mathbb{Z}$$

$$\theta = \omega_0 N + 2\pi K \qquad K \in \mathbb{Z}$$

$$\frac{\theta}{2\pi} = \frac{\omega_0 N}{2\pi} + K$$

$$\frac{\omega_0 N}{2\pi} - \left\lfloor \frac{\omega_0 N}{2\pi} \right\rfloor$$

So we can use $\theta = 2\pi\left(\frac{\omega_0 N}{2\pi} - \left\lfloor \frac{\omega_0 N}{2\pi} \right\rfloor \right)$

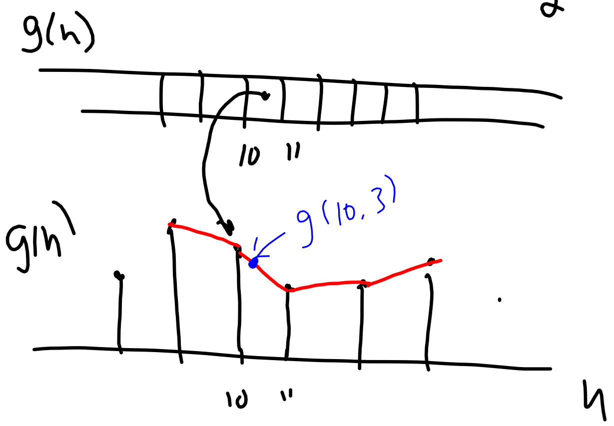
OR use "remainder" or "mod" function which is % in Python.

Vibrato Effect

$$x(t) \longrightarrow \boxed{\text{delay } \tau} \longrightarrow y(t) = x(t-\tau)$$

$$\tau = T_0 + W \sin(2\pi f_0 t)$$

Fraction index into signal (or buffer)



$g(10.3) = ?$

One simple method:

use $g(\text{round}(10.3))$

i.e., $g(10)$.

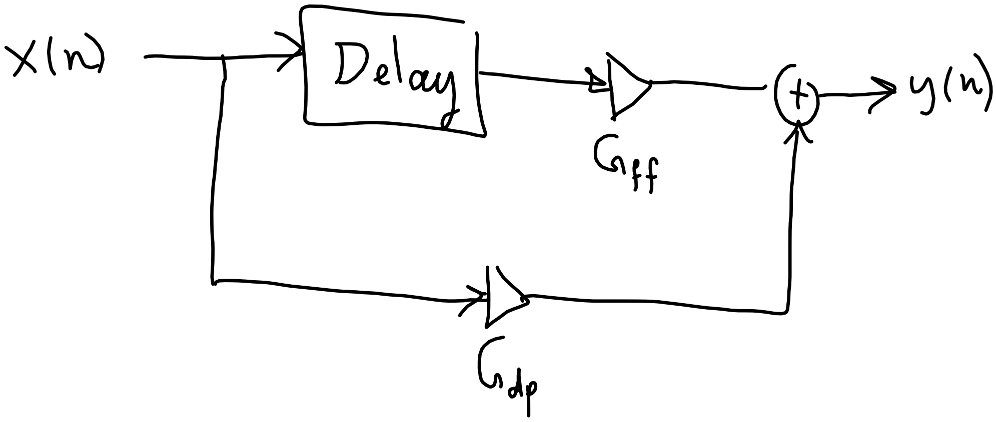
(closest value)

Linear interpolation

$$g(10.3) = 0.7\, g(10) + 0.3\, g(11).$$

$0 \le \alpha < 1$

$$g(n+\alpha) = (1-\alpha)\, g(n) + \alpha\, g(n+1)$$


x(n)
Delay
+
y(n)
Gff
Gdp

$$x(n) \to \boxed{\text{delay } \tau(n)} \to y(n)$$

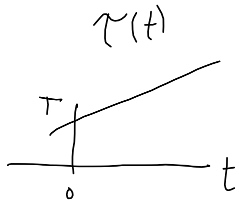
$$y(n) = x(n - \tau(n))$$

for any n.

(1) If $\tau(n) < 0$, then the system is noncausal! → leads to artifacts in output signal.

(2) ALSO, if delay $\tau(n) >$ buffer length, the output signal will have artifacts.

$$x(t) \longrightarrow \boxed{\text{delay}} \longrightarrow y(t) = x(t - \tau(t))$$



Say $\tau(t) = \alpha t + T$

Then $y(t) = x(t - \alpha t - T)$

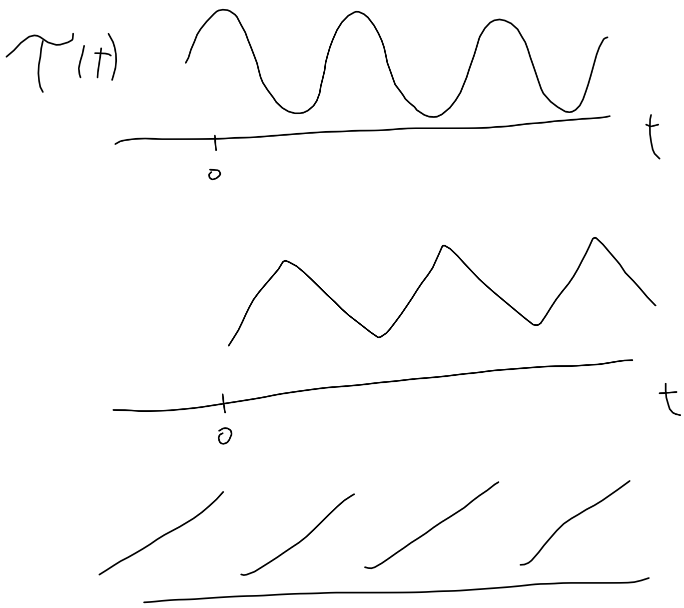
$= x((1-\alpha)t - T)$

- frequency of an input sinusoid will change.
- sinusoid input produces sinusoid output. but with different frequency.

$\tau(t) = \alpha t + T$ is not realizable.

but a delay $\tau(t) = T + W \sin(2\pi f_0 t)$ is realizable.

Time-Varying delay



Say $x(t) = \sin(\phi(t))$

fixed frequency. $\phi(t) = 2\pi f_0 t + \theta$

$$\phi'(t) = 2\pi f_0$$

$$\frac{1}{2\pi}\phi'(t) = f_0$$

$\frac{1}{2\pi}\phi'(t)$ is the "instantaneouse frequency"

Say $\phi(t) = 2\pi f_0 (t - \tau(t))$

$\sin(\phi(t)) = \sin(2\pi f_0 (t - \tau(t)))$ output of a vibrato effect with sin input.

inst. freq. is

$$\frac{1}{2\pi}\phi'(t) = \cancel{2\pi} f_0 (1 - \tau'(t))$$

$$= f_0 - \tau'(t)$$

Time-derivative of time-varying delay gives the inst. freq.