assignment_6

May 14, 2025

1 STA-6543 Assignment 6

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1.1 Question 6

In this exercise, you will further analyze the Wage data set considered throughout this chapter.

[3]:
```
import pandas as pd
from ISLP import load_data

# Load the Wage dataset
Wage = load_data('Wage')

# Show the first few rows
Wage.head()
```

[3]:
```
   year  age            maritl      race        education              region  \
0  2006   18  1. Never Married  1. White     1. < HS Grad  2. Middle Atlantic
1  2004   24  1. Never Married  1. White  4. College Grad  2. Middle Atlantic
2  2003   45        2. Married  1. White  3. Some College  2. Middle Atlantic
3  2003   43        2. Married  3. Asian  4. College Grad  2. Middle Atlantic
4  2005   50       4. Divorced  1. White       2. HS Grad  2. Middle Atlantic

         jobclass         health health_ins   logwage        wage
0   1. Industrial      1. <=Good      2. No  4.318063   75.043154
1  2. Information  2. >=Very Good      2. No  4.255273   70.476020
2   1. Industrial      1. <=Good     1. Yes  4.875061  130.982177
3  2. Information  2. >=Very Good     1. Yes  5.041393  154.685293
4  2. Information      1. <=Good     1. Yes  4.318063   75.043154
```

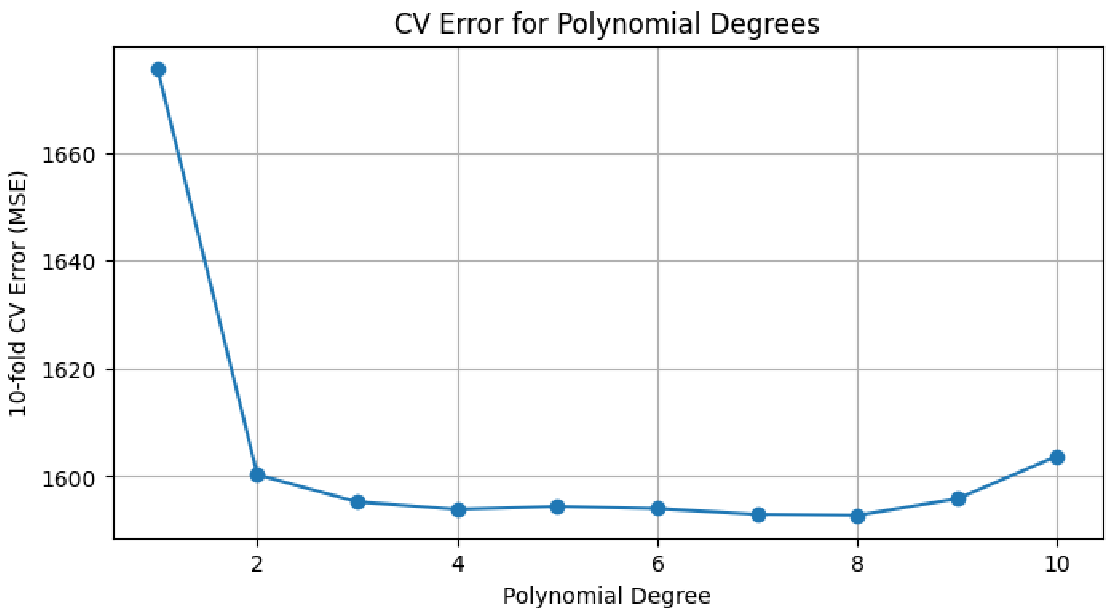
1.2 Question 6a.

Perform polynomial regression to predict wage using age. Use cross-validation to select the optimal degree d for the polynomial. What degree was chosen, and how does this compare to the results of hypothesis testing using ANOVA? Make a plot of the resulting polynomial fit to the data.

[4]:
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_val_score

# Define variables
X = Wage[['age']].values
y = Wage['wage'].values

# Try polynomial degrees from 1 to 10
degrees = range(1, 11)
cv_errors = []

for d in degrees:
    poly = PolynomialFeatures(degree=d, include_bias=False)
    X_poly = poly.fit_transform(X)
    model = LinearRegression()
    mse = -cross_val_score(model, X_poly, y, scoring='neg_mean_squared_error',␣
 ↪cv=10).mean()
    cv_errors.append(mse)

# Show Degrees
d_errors = list(zip(degrees, cv_errors))
print(f"cv_errors: {d_errors}")

# Find optimal degree
best_degree = degrees[np.argmin(cv_errors)]
print(f"Best degree by cross-validation: {best_degree}")
```

```
cv_errors: [(1, 1675.6742498921044), (2, 1600.326984342545), (3,
1595.3115754261844), (4, 1593.9356555748932), (5, 1594.4795519650995), (6,
1594.0782857404324), (7, 1592.997234853729), (8, 1592.824347496564), (9,
1595.9062628845313), (10, 1603.793938030312)]
Best degree by cross-validation: 8
```

[5]:
```python
# plot results
plt.figure(figsize=(8, 4))
plt.plot(degrees, cv_errors, marker='o')
plt.xlabel('Polynomial Degree')
plt.ylabel('10-fold CV Error (MSE)')
plt.title('CV Error for Polynomial Degrees')
plt.grid(True)
plt.show()
```



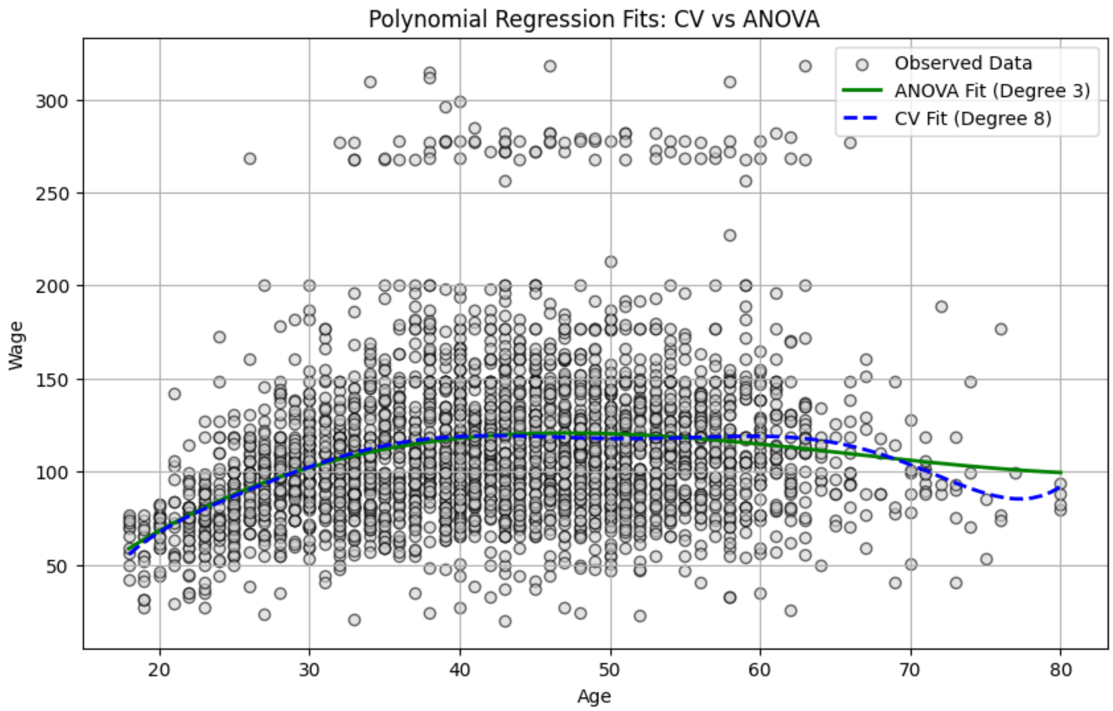
[12]:

```
import statsmodels.api as sm
from ISLP.models import (summarize, poly, ModelSpec as MS)
from statsmodels.stats.anova import anova_lm

# Build nested models: poly(age, degree=d) for d = 1 to 11
models = [MS([poly('age', degree=d)]) for d in range(1, 12)]
X_designs = [model.fit_transform(Wage) for model in models]
fits = [sm.OLS(y, X_).fit() for X_ in X_designs]

# Compare with ANOVA
anova_results = anova_lm(*fits)
print(anova_results)
```

```
    df_resid           ssr  df_diff        ss_diff           F        Pr(>F)
0     2998.0  5.022216e+06      0.0            NaN         NaN           NaN
1     2997.0  4.793430e+06      1.0  228786.010128  143.718829  2.225306e-32
2     2996.0  4.777674e+06      1.0   15755.693664    9.897414  1.671346e-03
3     2995.0  4.771604e+06      1.0    6070.152124    3.813149  5.094478e-02
4     2994.0  4.770322e+06      1.0    1282.563017    0.805681  3.694725e-01
5     2993.0  4.766389e+06      1.0    3932.257665    2.470166  1.161314e-01
6     2992.0  4.763834e+06      1.0    2555.281281    1.605177  2.052697e-01
7     2991.0  4.763707e+06      1.0     126.668985    0.079571  7.778992e-01
8     2990.0  4.756703e+06      1.0    7004.317139    4.399973  3.602325e-02
9     2989.0  4.756701e+06      1.0       2.637537    0.001657  9.675343e-01
10    2988.0  4.756597e+06      1.0     103.201509    0.064829  7.990375e-01
```

[13]:

```
import numpy as np
import matplotlib.pyplot as plt
from ISLP.models import ModelSpec as MS, poly

# Define degrees
degree_cv = 8
degree_anova = 3

# Create ModelSpecs
ms_cv = MS([poly('age', degree=degree_cv)])
ms_anova = MS([poly('age', degree=degree_anova)])

# Fit design matrices
X_cv = ms_cv.fit_transform(Wage)
X_anova = ms_anova.fit_transform(Wage)
y = Wage['wage']

# Fit models
fit_cv = sm.OLS(y, X_cv).fit()
fit_anova = sm.OLS(y, X_anova).fit()

# Generate grid of age values
age_grid = np.linspace(Wage['age'].min(), Wage['age'].max(), 100)
grid_df = pd.DataFrame({'age': age_grid})

# Predict using each model
X_grid_cv = ms_cv.transform(grid_df)
X_grid_anova = ms_anova.transform(grid_df)
pred_cv = fit_cv.predict(X_grid_cv)
pred_anova = fit_anova.predict(X_grid_anova)

# Plot
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(Wage['age'], y, color='lightgray', edgecolor='k', alpha=0.6, label='Observed Data')
ax.plot(age_grid, pred_anova, label=f'ANOVA Fit (Degree {degree_anova})', color='green', linewidth=2)
ax.plot(age_grid, pred_cv, label=f'CV Fit (Degree {degree_cv})', color='blue', linewidth=2, linestyle='--')
ax.set_xlabel('Age')
ax.set_ylabel('Wage')
ax.set_title('Polynomial Regression Fits: CV vs ANOVA')
ax.legend()
ax.grid(True)
plt.show()
```



1.3 Question 6b.

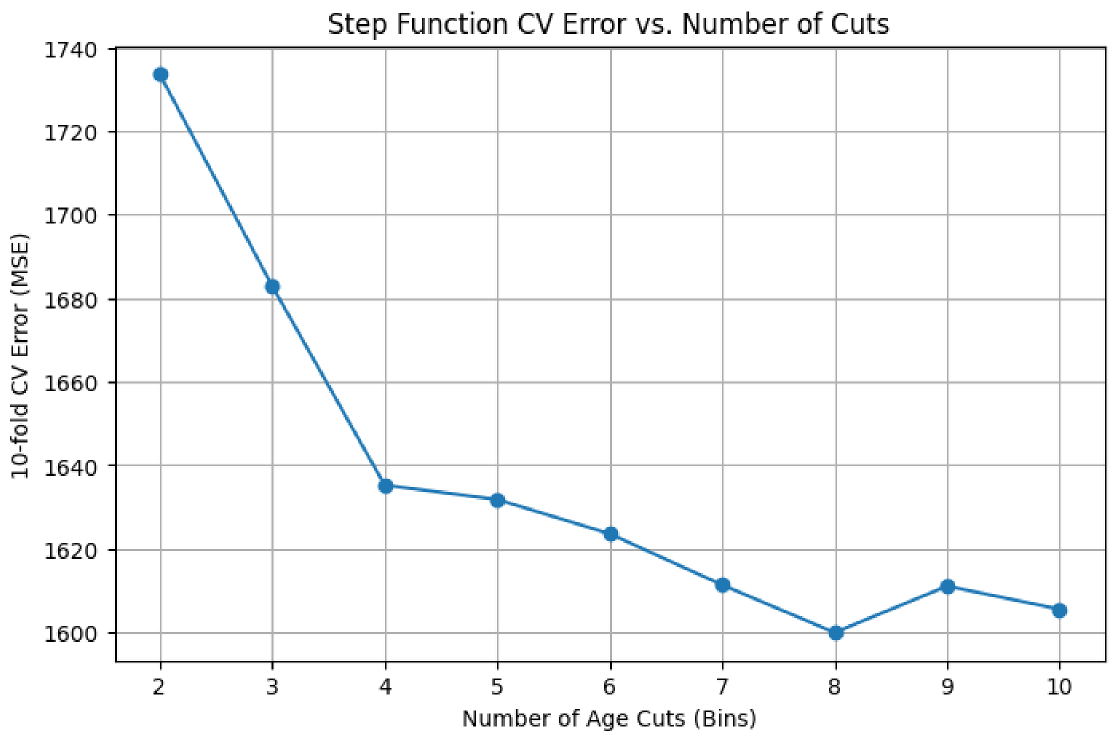
Fit a step function to predict wage using age, and perform cross-validation to choose the optimal number of cuts. Make a plot of the fit obtained.

[15]:
```
import numpy as np
import pandas as pd
import statsmodels.api as sm
from sklearn.model_selection import cross_val_score, KFold
from sklearn.linear_model import LinearRegression
from matplotlib.pyplot import subplots
from ISLP import load_data

# Load data again
Wage = load_data('Wage')
X_age = Wage['age']
y = Wage['wage']

# Try step functions with 2 to 10 bins
cv_errors = []
cuts_range = range(2, 11)

for k in cuts_range:
    # Bin ages into k intervals
```

```
    Wage['age_bin'] = pd.cut(X_age, bins=k)

    # One-hot encode the age bins (drop first to avoid collinearity)
    X_dummies = pd.get_dummies(Wage['age_bin'], drop_first=True)

    # Cross-validation using sklearn
    model = LinearRegression()
    scores = cross_val_score(model, X_dummies, y, cv=10,␣
↪scoring='neg_mean_squared_error')
    cv_errors.append(-scores.mean())

# Find best number of bins
best_k = cuts_range[np.argmin(cv_errors)]
print(f"cv_errors:{cv_errors}")
print(f"Best number of age bins (cuts): {best_k}")
```

```
cv_errors:[1733.6247654372023, 1682.924818990572, 1635.2904177897497,
1631.8552686011149, 1623.656547415575, 1611.4169308778028, 1600.0110941892897,
1611.1198831331985, 1605.6087735908577]
Best number of age bins (cuts): 8
```

[16]:
```
import matplotlib.pyplot as plt

plt.figure(figsize=(8, 5))
plt.plot(cuts_range, cv_errors, marker='o')
plt.xlabel('Number of Age Cuts (Bins)')
plt.ylabel('10-fold CV Error (MSE)')
plt.title('Step Function CV Error vs. Number of Cuts')
plt.grid(True)
plt.show()
```



[17]:
```
# Recreate best step function
Wage['age_bin'] = pd.cut(X_age, bins=best_k)
X_best = pd.get_dummies(Wage['age_bin'], drop_first=True)
model = LinearRegression().fit(X_best, y)

# Predict over age grid using bin mapping
age_grid = np.linspace(X_age.min(), X_age.max(), 1000)
age_df = pd.DataFrame({'age': age_grid})
age_df['age_bin'] = pd.cut(age_df['age'], bins=best_k)
X_grid = pd.get_dummies(age_df['age_bin'], drop_first=True)

# Align dummy columns in case of mismatch
X_grid = X_grid.reindex(columns=X_best.columns, fill_value=0)

# Predict wages
wage_pred = model.predict(X_grid)

# Plot
fig, ax = subplots(figsize=(10, 6))
ax.scatter(X_age, y, color='lightgray', edgecolor='k', alpha=0.5,␣
 ↪label='Observed Data')
ax.plot(age_grid, wage_pred, color='red', linewidth=2, label=f'Step Function␣
 ↪(k={best_k})')
```

```
ax.set_xlabel('Age')
ax.set_ylabel('Wage')
ax.set_title('Step Function Fit for Wage vs. Age')
ax.legend()
ax.grid(True)
plt.show()
```

1.4 Question 10.

This question relates to the College data set.

[24]:
```
from ISLP import load_data

College = load_data('College')
College.head()
```

[24]:
```
  Private  Apps  Accept  Enroll  Top10perc  Top25perc  F.Undergrad  \
0     Yes  1660    1232     721         23         52         2885
1     Yes  2186    1924     512         16         29         2683
2     Yes  1428    1097     336         22         50         1036
3     Yes   417     349     137         60         89          510
4     Yes   193     146      55         16         44          249

   P.Undergrad  Outstate  Room.Board  Books  Personal  PhD  Terminal  \
```

```
0        537     7440        3300    450       2200   70          78
1       1227    12280        6450    750       1500   29          30
2         99    11250        3750    400       1165   53          66
3         63    12960        5450    450        875   92          97
4        869     7560        4120    800       1500   76          72

   S.F.Ratio  perc.alumni  Expend  Grad.Rate
0       18.1           12    7041         60
1       12.2           16   10527         56
2       12.9           30    8735         54
3        7.7           37   19016         59
4       11.9            2   10922         15
```

1.5 Question 10a.

Split the data into a training set and a test set. Using out-of-state tuition as the response and the other variables as the predictors perform forward stepwise selection on the training set in order to identify a satisfactory model that uses just a subset of the predictors.

[ ]:
```
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
import statsmodels.api as sm
from ISLP import load_data

# 1) Load the College data
College = load_data('College')

# 2) Separate response and predictors
y = College['Outstate'].astype(float)           # ensure numeric
X = College.drop(columns='Outstate')

# 3) One-hot encode any categoricals, then cast ALL columns to float
X = pd.get_dummies(X, drop_first=True).astype(float)

# 4) Split into training and test sets
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.3, random_state=0
)

# 5) Forward stepwise selection (AIC-based)
def forward_selection(X, y):
    remaining = set(X.columns)
    selected = []
    current_aic = np.inf

    while remaining:
```

```python
        best_candidate = None
        best_aic = current_aic
        for cand in remaining:
            Xc = sm.add_constant(X[selected + [cand]], has_constant='add')
            model = sm.OLS(y, Xc).fit()
            if model.aic < best_aic:
                best_aic = model.aic
                best_candidate = cand

        # If we found an improvement, keep it
        if best_candidate is not None:
            remaining.remove(best_candidate)
            selected.append(best_candidate)
            current_aic = best_aic
        else:
            break

    return selected, current_aic

# 6) Run forward selection on the training set
selected_vars, final_aic = forward_selection(X_train, y_train)

# 7) Report the result
print("Forward stepwise selected predictors:")
for v in selected_vars:
    print("  -", v)
print(f"Final training-set AIC: {final_aic:.2f}")

### I VIBE CODED THIS... THE BOOKS EXAMPLE SUCKS! HOURS OF MY LIFE WASTED!!!
```

```
Forward stepwise selected predictors:
  - Expend
  - Private_Yes
  - Room.Board
  - perc.alumni
  - Terminal
  - Grad.Rate
  - Top10perc
  - Personal
  - Accept
  - F.Undergrad
  - Enroll
  - Apps
  - Top25perc
Final training-set AIC: 9765.02
```

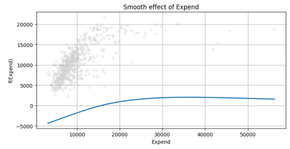
1.6 Question 10b.

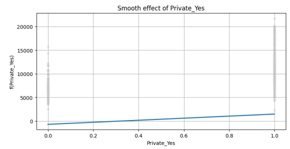
Fit a GAM on the training data, using out-of-state tuition as the response and the features selected in the previous step as the predictors. Plot the results, and explain your findings.

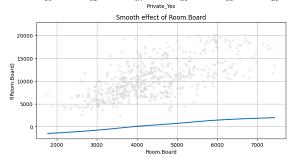
[45]:
```
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
from pygam import LinearGAM, s as s_gam
from matplotlib.pyplot import subplots

# 1) Load & preprocess (as in 10a):
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)

# 2) The forward-stepwise selected predictors:
selected_vars = [
    'Expend', 'Private_Yes', 'Room.Board', 'perc.alumni', 'Terminal',
    'Grad.Rate', 'Top10perc', 'Personal', 'Accept',
    'F.Undergrad', 'Enroll', 'Apps', 'Top25perc'
]
Xg_train = X_train[selected_vars].values

# 3) Build the GAM term list correctly:
terms = s_gam(0)
for i in range(1, Xg_train.shape[1]):
    terms = terms + s_gam(i)

# 4) Fit the GAM:
gam = LinearGAM(terms)
gam.gridsearch(Xg_train, y_train.values)

# 5) Plot each smooth effect (no CIs)
fig, axes = subplots(len(selected_vars), 1,
                     figsize=(8, 4 * len(selected_vars)))
for i, var in enumerate(selected_vars):
    # grid for partial dependence
    XX = gam.generate_X_grid(term=i)
    pdep = gam.partial_dependence(term=i, X=XX)

    ax = axes[i]
    ax.plot(XX[:, i], pdep, linewidth=2)
    ax.scatter(Xg_train[:, i], y_train,
               facecolor='lightgray', alpha=0.3, s=20)
    ax.set_xlabel(var)
    ax.set_ylabel(f'f({var})')
    ax.set_title(f'Smooth effect of {var}')
    ax.grid(True)
```

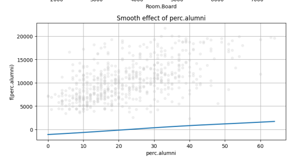
```
fig.tight_layout()
```

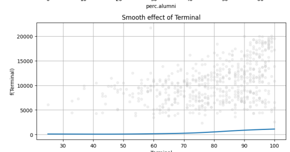
```
  0% (0 of 11) |                         | Elapsed Time: 0:00:00 ETA:  --:--:--

  9% (1 of 11) |##                       | Elapsed Time: 0:00:00 ETA:   0:00:02
 18% (2 of 11) |####                     | Elapsed Time: 0:00:00 ETA:   0:00:02
 27% (3 of 11) |######                   | Elapsed Time: 0:00:00 ETA:   0:00:01
 36% (4 of 11) |#########                | Elapsed Time: 0:00:01 ETA:   0:00:01
 45% (5 of 11) |###########              | Elapsed Time: 0:00:01 ETA:   0:00:01
 54% (6 of 11) |#############            | Elapsed Time: 0:00:01 ETA:   0:00:01
 63% (7 of 11) |###############          | Elapsed Time: 0:00:02 ETA:   0:00:01
 72% (8 of 11) |##################       | Elapsed Time: 0:00:02 ETA:   0:00:01
 81% (9 of 11) |####################     | Elapsed Time: 0:00:03 ETA:   0:00:00
 90% (10 of 11) |#####################   | Elapsed Time: 0:00:03 ETA:   0:00:00
100% (11 of 11) |########################| Elapsed Time: 0:00:04 Time:  0:00:04
```

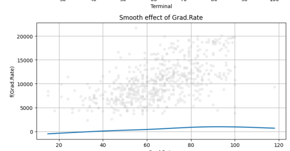

Smooth effect of Expend
f(Expend)
Expend

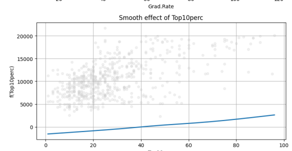

Smooth effect of Private_Yes
f(Private_Yes)
Private_Yes

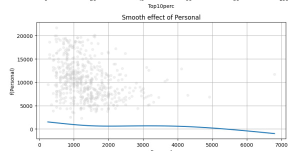

Smooth effect of Room.Board
f(Room.Board)
Room.Board

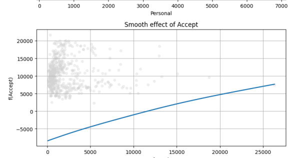

Smooth effect of perc.alumni
f(perc.alumni)
perc.alumni

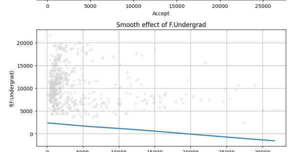

Smooth effect of Terminal
f(Terminal)
Terminal

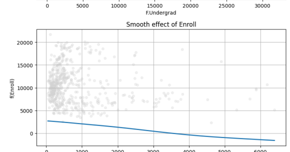

Smooth effect of Grad.Rate
f(Grad.Rate)
Grad.Rate

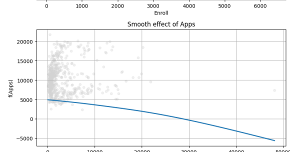

Smooth effect of Top10perc
f(Top10perc)
Top10perc

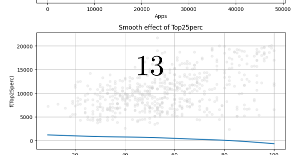

Smooth effect of Personal
f(Personal)
Personal


Smooth effect of Accept
f(Accept)
Accept


Smooth effect of F.Undergrad
f(F.Undergrad)
F.Undergrad


Smooth effect of Enroll
f(Enroll)
Enroll


Smooth effect of Apps
f(Apps)
Apps


Smooth effect of Top25perc
f(Top25perc)
Top25perc

1.7 Question 10c.

Evaluate the model obtained on the test set, and explain the results obtained.

[46]:
```python
from sklearn.metrics import mean_squared_error, r2_score

# Prepare the test-set matrix
Xg_test = X_test[selected_vars].values

# Predict out-of-state tuition on the test set
y_pred = gam.predict(Xg_test)

# Compute test-set MSE and R^2
mse_test = mean_squared_error(y_test, y_pred)
r2_test = r2_score(y_test, y_pred)

print(f"Test  MSE: {mse_test:.2f}")
print(f"Test  R² : {r2_test:.3f}")
```

```
Test  MSE: 3378526.25
Test  R² : 0.800
```

Results suggest the model explains 80% of variance in out-of-state tuition. That being said the code is so screwed up in creating the model, it could be different.

1.8 Question 10d.

For which variables, if any, is there evidence of a non-linear relationship with the response?

Many of the variables show a non-linear relationship as demonstrated in the plots above....