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DEPARTMENT OF MECHANICAL ENGINEERING
RICE UNIVERSITY

MECH 498/598 ELEC 498/598 COMP 498/598
INTRODUCTION TO ROBOTICS

EXAM Spring 2022

Due: ***Tuesday, April 12, 2022*** *at* ***8 PM*** ***(turn in to MECH drop box only)***

This exam is **open notes**, but **closed computer**. You may use the course textbook, notes, homework, and your own notes. You may not use MATLAB or Google, or any other outside resource. There is an additional problem for 598 students. You have eight (8) hours to complete the exam. It must be completed in one sitting.

Disassemble the exam and re-assemble with additional sheets if necessary. For partial credit, your work must be clear and easy to read. Final answers must be clearly indicated with a box, arrow, or similar.

Write your name on this cover sheet, and write and sign the pledge in the space provided below. **Exams without pledges will not be graded.**

Good luck!

Start time: Stop time:

Pledge:

Signed: ____

	498 students	**598 students**
Problem 1 (10 points)		
Problem 2 (15 points)		
Problem 3 (26 points)		
Problem 4 (15 points)		
Problem 5 (14 points)		
Problem 5 (10 points)		
Problem 6 (10 points)	----------------	
Total	**________ / 90**	**________ / 100**

Name: ____________________________

Problem 1 (10 points)

Consider the following sequence of rotations:

1. Rotate by ϕ about the world x-axis.
2. Rotate by θ about the current z-axis
3. Rotate by ψ about the world y-axis

Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication).

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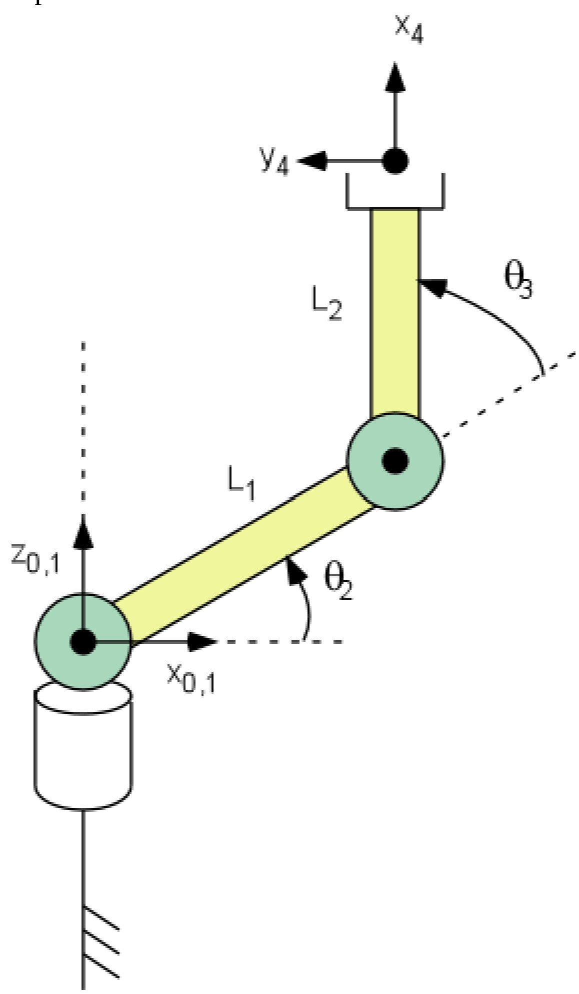
Problem 2 (15 points)

Consider the spatial RRR manipulator shown below.



a. (5pt) Assign and label frames {2} and {3} on the diagram above, according to the convention used in the Craig textbook.

b. (5pt) Create a table of the D-H parameters for all four frames of this manipulator.

c. (5pt) Derive the forward kinematics of this manipulator (${}^{0}_{4}T$)
You do NOT need to multiply all of the matrices together, the last step before matrix multiplication is acceptable.

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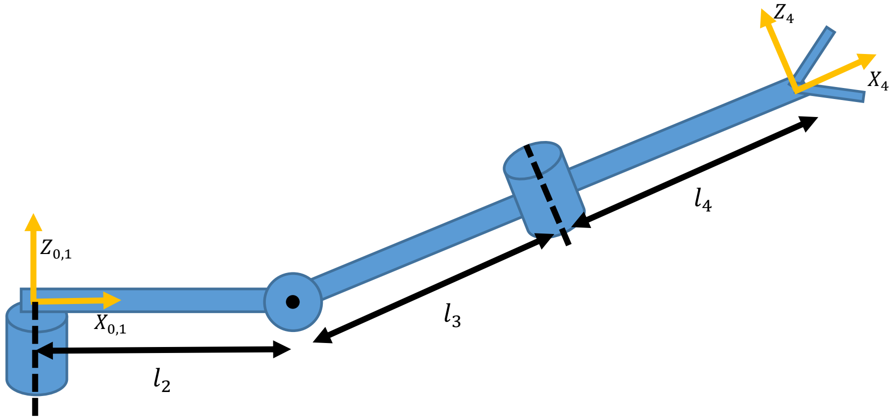
Problem 3 (26 points)

Consider the spatial RRR Manipulator shown below
(Joints 1 and 3 are parallel and 90deg offset from joint 2)



Given that the Transformation matrices are as follows:

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ -s_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^3_4T = \begin{bmatrix} 1 & 0 & 0 & l_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_4T = \begin{bmatrix} c_1c_2c_3 - s_1s_3 & -c_3s_1 - c_1c_2s_3 & -c_1s_2 & l_2c_1 - l_4(s_1s_3 - c_1c_2c_3) + l_3c_1c_2 \\ c_1s_3 + c_2c_3s_1 & c_1c_3 - c_2s_1s_3 & -s_1s_2 & l_4(c_1s_3 + c_2c_3s_1) + l_2s_1 + l_3c_2s_1 \\ c_3s_2 & -s_2s_3 & c_2 & s_2(l_3 + l_4c_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

a. (10pt) Find the Jacobian matrix ${}^0J_v$ for this manipulator, expressed in frame $\{0\}$ that transforms the joint velocities into end-effector linear velocities.

b. (6pt) Given that $\det(J(\theta)) = -l_4 s_3 * (l_2 + l_3 c_2) * (l_3 + l_4 c_3)$

I. (2pt) Define the orientations where the manipulator is singular mathematically

II. (3pt) Describe and/or draw each singular condition

III. (1pt) How could you manipulate the joint lengths of this manipulator to decrease the number of possible singularities?

c. (10pt) Consider a configuration of the robot where $\theta_1 = 0$, $\theta_2 = \pi/2$, and $\theta_3 = 0$. A force-torque vector ${}^0F = [f_x \quad 0 \quad f_z]^T$ is applied to the end effector by a visiting Cal Tech robotics student. Determine the resulting joint torques required to balance this load. You must justify your answer mathematically. Provide a description in words of why your result makes sense.

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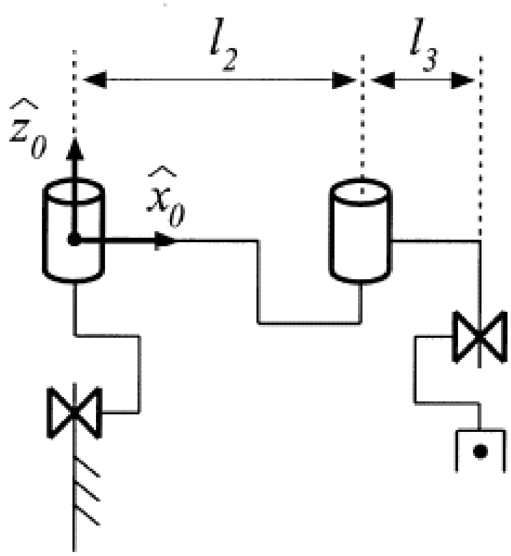
Problem 4 (15 points)

Consider the spatial PRRP manipulator shown below.



a. (3pt) Assuming no joint limits, sketch the workspace of this manipulator. Be sure to include dimensions in your drawing. Assume $l_2 > l_3$.

b. (3pt) Describe the (3D) dexterous workspace of this manipulator.

c. (3pt) With no joint limits, and considering only the position of the end-effector, how many inverse kinematic solutions are there (in general)? Explain briefly.

d. (3pt) Imagine that we remove the first prismatic joint, so that the first revolute joint now rotates around the base. Repeat part (c) for such an RRP manipulator.

e. (3pt) Imagine that we further modify the manipulator from part (d), by inserting another revolute joint between the two existing revolute joints, whose axis is oriented in the same directions as the other two. Repeat part (c) for such an RRRP manipulator.

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Problem 5: General Understanding Short Answer (14 points)
These questions are meant to allow you to demonstrate your practical and applied understanding of the material and mathematics. Please provide short answers for each. No need for a novel.

a. (2pt) When calculating torque on a joint using the equation: $\tau_i = {}^i n_i^T \, {}^i\hat{Z}_i$.

   Explain what the mathematical purpose of ${}^i\hat{Z}_i$ is and why we might choose <u>not</u> to apply it?

b. (2pt) What things must you consider when selecting a transmission type for a joint?
   I. What should you consider when deciding direct drive or transmission?
   II. Please explain considerations for at least 2 types of transmissions.

c. (2pt) In practice, what do you need to consider when setting the zero position of a robot joint?

d. (2pt) Please give one example of a robot where you could ignore the effects of the link inertia and one example of a robot you should <u>not</u> ignore the effects of link inertia.

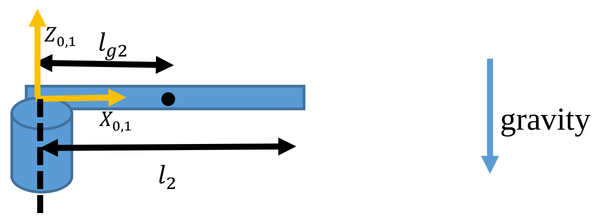
e. (2pt) In your own words, describe the two methods discussed in class for finding the solution to the inverse kinematics equation of a manipulator.

f. (2pt) Explain in your own words why the Jacobian is non-invertible at singularity.

g. (2pt) Describe the concept of “test-driven development” in software and give 3 considerations of what to test for when writing unit-tests for code.

Problem 6 (10 points)

Consider only the first joint from problem 3. Please calculate the torque from the joint dynamics using the Lagrangian method. Please show the process of calculation in your work. Show all equations and components even if their result is 0 (if the result is 0, explain why).
The CG of the link is located at l_{g2} as shown in the diagram



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Problem 7 (10 points)

[only for students taking the course for graduate credit e.g. registered for “598”]

In lecture, we discussed the Z-Y-X Euler angles. List all possible sets of Euler angles. Is it possible to have Z-Z-Y Euler angles? Why or why not?

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Bonus Question: 0 points

Circle the picture that best describes how you felt about the exam