

Aufgabe 3

$$\begin{aligned} \text{a) } f(x) &= (x+1)^{x^2+5} \\ &= e^{\ln((x+1)^{x^2+5})} \\ &= e^{(x^2+5) \cdot \ln(x+1)} \end{aligned}$$

$$\text{NR: } u = x^2+5 \quad v = \ln(x+1) \\ u' = 2x \quad v' = \frac{1}{x+1}$$

$$\begin{aligned} f'(x) &= e^{(x^2+5) \cdot \ln(x+1)} \cdot \left((x^2+5) \cdot \frac{1}{x+1} + (2x \cdot \ln(x+1)) \right) \\ &= (x+1)^{x^2+5} \cdot \left(\frac{x^2+5}{x+1} + 2x \ln(x+1) \right) \end{aligned}$$

$$\begin{aligned} \text{b) } f(x) &= 2^x \\ &= e^{\ln(2^x)} \\ &= e^{x \cdot \ln(2)} \end{aligned}$$

$$\text{NR: } u = x \quad v = \ln(2) \\ u' = 1 \quad v' = 0$$

$$\begin{aligned} f'(x) &= 2^x \cdot (1 \cdot \ln(2)) + (x \cdot 0) \\ &= 2^x \cdot \ln(2) \end{aligned}$$

$$\begin{aligned} g(x) &= 3^x \\ &= e^{\ln(3^x)} \\ &= e^{x \cdot \ln(3)} \end{aligned}$$

$$\text{NR: } u = x \quad v = \ln(3) \\ u' = 1 \quad v' = 0$$

$$\begin{aligned} g'(x) &= 3^x \cdot (1 \cdot \ln(3)) + (x \cdot 0) \\ &= 3^x \cdot \ln(3) \end{aligned}$$

→ Allgemeine Formel: $f(x) = a^x$

$$\begin{aligned} &= e^{\ln(a^x)} \\ &= e^{x \cdot \ln(a)} \end{aligned}$$

$$\text{NR: } u = x \quad v = \ln(a) \\ u' = 1 \quad v' = 0$$

$$\begin{aligned} f'(x) &= a^x \cdot (1 \cdot \ln(a)) + (0 \cdot x) \\ &= a^x \cdot \ln(a) \end{aligned}$$