

Aufgabe 1

a) (i) $f(x) = 7x^5 + 3x^2 + x + 1$

$$f'(x) = 35x^4 + 6x + 1$$

(ii) $f(x) = (3x^7 - 4x^3 + x^2 - 3x + 1)^8$

$$f'(x) = (21x^6 - 12x^2 + 2x - 3) \cdot 8(3x^7 - 4x^3 + x^2 - 3x + 1)^7$$

(iii) $f(x) = (3x^4 + 2x) \cdot \sqrt{x^2 + 1}$

$$u = 3x^4 + 2x \quad v = (x^2 + 1)^{1/2}$$

$$u' = 12x^3 + 2 \quad v' = 2x \cdot \frac{1}{2} \cdot (x^2 + 1)^{-1/2}$$

$$f'(x) = (3x^4 + 2x) \cdot (x \cdot (x^2 + 1)^{-1/2}) + (12x^3 + 2) \cdot (x^2 + 1)^{1/2}$$

(iv) $f(x) = (x^3 + 1) \cdot \ln(x^4 + 3x^2 + 1)$

$$u = x^3 + 1 \quad v = \ln(x^4 + 3x^2 + 1)$$

$$u' = 3x^2 \quad v' = \frac{4x^3 + 6x}{x^4 + 3x^2 + 1}$$

$$\begin{aligned} f'(x) &= (x^3 + 1) \cdot \frac{4x^3 + 6x}{x^4 + 3x^2 + 1} + 3x^2 \cdot \ln(x^4 + 3x^2 + 1) \\ &= \frac{(x^3 + 1) \cdot (4x^3 + 6x)}{x^4 + 3x^2 + 1} + 3x^2 \cdot \ln(x^4 + 3x^2 + 1) \end{aligned}$$

(v) $f(x) = e^{x^3 + x^2 + 1} \cdot \sqrt{x}$

$$u = e^{x^3 + x^2 + 1} \quad v = \sqrt{x} = x^{1/2}$$

$$u' = (3x^2 + 2x) \cdot e^{x^3 + x^2 + 1} \quad v' = \frac{1}{2} x^{-1/2}$$

$$f'(x) = (e^{x^3 + x^2 + 1}) \cdot \left(\frac{1}{2} x^{-1/2}\right) + ((3x^2 + 2x) \cdot e^{x^3 + x^2 + 1}) \cdot \sqrt{x}$$

(vi) $f(x) = \sqrt{x^4 + 1} \cdot \ln(x)$

$$u = \sqrt{x^4 + 1} = (x^4 + 1)^{1/2} \quad v = \ln(x)$$

$$u' = 4x^3 \cdot \frac{1}{2} \cdot (x^4 + 1)^{-1/2} \quad v' = \frac{1}{x}$$

$$f'(x) = \frac{\sqrt{x^4 + 1}}{x} + 2x^3 \cdot (x^4 + 1)^{-1/2} \cdot \ln(x)$$

$$b) \quad q(x) = \frac{5x^2 + 1}{x - 3}$$

$$u = 5x^2 + 1$$

$$v = x - 3$$

$$u' = 10x$$

$$v' = 1$$

$$v^2 = (x - 3)^2$$

$$q'(x) = \frac{10x \cdot (x - 3) - (5x^2 + 1)}{(x - 3)^2}$$

$$= \frac{5x^2 - 30x - 1}{(x - 3)^2}$$

$$u = 5x^2 - 30x - 1$$

$$v = (x - 3)^2$$

$$u' = 10x - 30$$

$$v' = 1 \cdot 2(x - 3) = 2(x - 3)$$

$$v^2 = (x - 3)^4$$

$$q''(x) = \frac{(10x - 30) \cdot (x - 3)^2 - ((5x^2 - 30x - 1) \cdot (2 \cdot (x - 3)))}{(x - 3)^4}$$

$$= \frac{(x - 3) \cdot (10x^2 - 30x - 30x + 90) - (10x^2 - 60x - 2)}{(x - 3)^4}$$

$$= \frac{10x^2 - 60x + 90 - 10x^2 + 60x + 2}{(x - 3)^3}$$

$$= \frac{92}{(x - 3)^3}$$

$$u = 92$$

$$v = (x - 3)^3$$

$$u' = 0$$

$$v' = 1 \cdot 3 \cdot (x - 3)^2 = 3(x - 3)^2$$

$$v^2 = (x - 3)^6$$

$$q'''(x) = \frac{92 \cdot (3 \cdot (x - 3)^2)}{(x - 3)^6}$$

$$= \frac{276 \cdot (x - 3)^2}{(x - 3)^6}$$

$$= \frac{276}{(x - 3)^4}$$