User's Guide to Running the Draft NIST SP 800-90B Section 9 Entropy Estimation Tests

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This is a brief introduction on how to run the Python command-line programs that implement the statistical entropy estimation tests found in Section 9 of the Draft NIST SP 800-90B (August 2012). It is not a description or explanation of the tests themselves. Please refer to the standard itself for definitions and descriptions of the tests and their rationales.

Python files to implement the tests:

iidmain.py

- Contains main routine to give the IID entropy estimate, if IID assumption holds
- Run six shuffle tests to determine if IID
- Run chi-square independence and goodness of fit tests to determine if IID
- Estimate min entropy if passes above tests
- Run sanity check tests

noniid_main.py

- Contains main routine to give the non-IID entropy estimate
- Run five tests to estimate min-entropy
- · min-entropy as lowest of the five
- Run sanity check tests

shuffle_tests.py

• Contains six shuffle tests to determine if dataset is IID from Section 9.1.2

chi_square_tests.py

• Contains the chi square independence and goodness of fit for binary and non-binary data from Section 9.1.3

iid_tests.py

Contains the min entropy calculation for IID data from Section 9.2

sanity_checks.py

Contains the two sanity check tests from Section 9.4

noniid_collision.py

- the non-IID collision test from Section 9.3.3
- Test may not be valid for all datasets.

partial_collection.py

- The non-IID partial collection test from Section 9.3.4
- Test may not be valid for all datasets.

markov.py

- Contains the non-IID Markov test from Section 9.3.5
- Per SP 800-90B, only up to 6 bits per symbol used for Markov test

maurer.py

- Contains the non-IID compression test (Maurer Universal Statistic) from Section 9.3.6
- Test may not be valid for all datasets

frequency.py

Contains the non-IID frequency test from Section 9.3.7

util90b.py

• Contains utility functions

Sample dataset files:

Three files generated with TrueRand that should pass determine iid tests:

- 1 000 000 data samples
 - 1 bit per sample (*truerand_1bit.bin*)
 - 4 bits per sample (truerand_4bit.bin)
 - 8 bits per sample (truerand_8bit.bin)

One file generated with TrueRand that should pass shuffle tests but fail chi square tests:

• 1 000 000 data samples, 9 bits per sample (truerand_9bit.bin)

One file containing binary digits of pi that fails iid tests:

• data.pi.bin

This User Guide

user_guide.pdf

The code has been tested and run successfully on the following:

- Python 2.6 on Linux
- Python 2.7 on Mac OS X
- Python 3.3 on Windows 7

It should run on any OS with Python 2.6+ or Python 3.

Note that this tool does not come with a Python installation. If you do not already have Python installed on your system, go to https://www.python.org and select "Download." No additional modules or packages are required to run the code. However, some routines will run faster if you have the **numpy** package installed. You can get **numpy** at http://www.scipy.org. If you are running a Windows OS, you can also find it here: http://www.lfd.uci.edu/~gohlke/pythonlibs. Alternatively, you can download the entire **scipy-stack**, which includes **numpy**.

Running the tests

The help message for the IID tests is:

Examples of running the IID tests follow.

Run the IID tests on the included truerand_8bit.bin dataset, which contains 8 bits per sample. Use 1,000 shuffles of the data subsets and append the verbose flag for detailed output:

```
C:\est>python iid main.py truerand 8bit.bin 8 1000 -v
Read in file truerand 8bit.bin, 1000000 bytes long.
Dataset: 1000000 8-bit symbols.
Output symbol values: min = 0, max = 255
Compression Test:
      Scores
                                Ranks
      106842
                                 670
      106886
                                 856
      106858
                                 673
      106718
                                106
      106845
                                 564
      106899
                                867
      106752
                                224
      106867
                                741
      106936
                                981*
      106849
                                 695
                                   1
Passed Compression Test
. . .
```

The full program output is not listed for space considerations. The first three lines of output are information about the dataset: its name, total size in bytes, how the raw bytes are interpreted (1,000,000 8-bit symbols as opposed to 500,000 16-bit symbols, for example) and the range of sample values in the dataset.

Following this is detailed information about the individual shuffle tests. The test name is followed by the scores and ranks of the 10 original (unshuffled) data subsets. If the rank of the score an original (unshuffled) data subset in a ranked ordering of the scores of all 1,000 shuffled data subsets is in the top or bottom 5%, then the rank is marked with an asterisk. For 1,000 shuffles, this works out to ranks of greater than or equal 950 or less than or equal to 50. If eight or more of the data subsets fall in this range, then the test fails, indicating that the IID assumption does not hold. Please see the Draft NIST SP 800-90B (August 2012) for an explanation and more details on this. A similar display of scores, ranks and Passed/Failed verdict is output for the other five shuffle tests.

If the dataset passes all of the shuffle tests, as is the case for truerand_8bit.bin, then the program output indicates this and prints out details and results of the Chi-square tests and the overall determination of the IID assumption. If the determination is that the IID assumption holds, as is true for our example, the min-entropy estimate is output, followed by the details and results of the two sanity checks.

```
** Passed iid shuffle tests
Chi square independence
        score = 65212.5, degrees of freedom = 65280, cut-off = 66402.2
** Passed chi-square independence test
Chi square stability
        score = 2449.48, degrees of freedom = 2313 cut-off = 2528.88
** Passed chi-square stability test
IID = True
min-entropy = 7.87108
Compression sanity check...
        dataset 1 compressed length = 854736, cutoff = 787108...Pass
        dataset 2 compressed length = 855088, cutoff = 787108...Pass
        dataset 3 compressed length = 854864, cutoff = 787108...Pass
        dataset 4 compressed length = 853744, cutoff = 787108...Pass
        dataset 5 compressed length = 854760, cutoff = 787108...Pass
        dataset 6 compressed length = 855192, cutoff = 787108...Pass
        dataset 7 compressed length = 854016, cutoff = 787108...Pass
        dataset 8 compressed length = 854936, cutoff = 787108...Pass
        dataset 9 compressed length = 855488, cutoff = 787108...Pass
        dataset 10 compressed length = 854792, cutoff = 787108...Pass
Collision sanity check...
        Dividing dataset into 4-tuples
        Check rule 1 - do three or more 4-tuples have the same value?...Pass
        Check rule 2 - probability of number of collisions below cutoff
                number of collisions = 6, cutoff = 10.4023...Pass
sanity check = PASS
```

If the same dataset and test parameters are run without the verbose flag, only the results of the IID determination, min-entropy estimate and sanity check are output:

```
C:\est>python iid_main.py truerand_8bit.bin 8 1000
IID = True
min-entropy = 7.87108
sanity check = PASS
```

Note that 1,000 is used as the number of shuffles for the examples above. 1,000 shuffles is specified in Draft NIST SP 800-90B (August 2012) and thus what should be used in order to run the tests in conformance with the standard. However, you may use a different number if you choose.

The help message for the non-IID tests is:

Next are some examples of running the non-IID tests.

First example: run the non-IID tests on the included truerand_4bit.bin dataset, which has 4 bits per sample. The verbose flag is set in order to obtain detailed test results. Note that the results of each individual non-IID test are shown. The output is below:

```
C:\est>python noniid main.py truerand 4bit.bin 4 -v
Read in file truerand 4bit.bin, 1000000 bytes long.
Dataset: 1000000 4-bit symbols.
Output symbol values: min = 0, max = 15
- Collision test
                          : p(max) = 0.0715332, min-entropy = 3.80524
- Partial collection test : p(max) = 0.074295, min-entropy = 3.75059
- Markov test
                          : p(max) = 3.02369e-153, min-entropy = 3.95827
- Compression test
                          : p(max) = 0.0789795, min-entropy = 3.66238
- Frequency test
                          : p(max) = 0.063134, min-entropy = 3.95774
min-entropy = 3.66238
Compression sanity check...
      dataset 1 compressed length = 435232, cutoff = 366238...Pass
      dataset 2 compressed length = 435720, cutoff = 366238...Pass
      dataset 3 compressed length = 435904, cutoff = 366238...Pass
      dataset 4 compressed length = 435336, cutoff = 366238...Pass
      dataset 5 compressed length = 435768, cutoff = 366238...Pass
      dataset 6 compressed length = 435480, cutoff = 366238...Pass
      dataset 7 compressed length = 435288, cutoff = 366238...Pass
      dataset 8 compressed length = 435632, cutoff = 366238...Pass
      dataset 9 compressed length = 435648, cutoff = 366238...Pass
      dataset 10 compressed length = 435936, cutoff = 366238...Pass
Collision sanity check...
      Dividing dataset into 9-tuples
      Check rule 1 - do three or more 9-tuples have the same value?...Pass
      Check rule 2 - probability of number of collisions below cutoff
            number of collisions = 0, cutoff = 0.738097...Pass
sanity check = PASS
```

As with the IID tests, not setting the verbose flag produces compact results output:

```
C:\est>python noniid_main.py truerand_4bit.bin 4
min-entropy = 3.66238
sanity check = PASS
```

The usebits option allows you to instruct the program to consider only a lower order subset of the bits per sample. This is useful when almost all of the entropy is in these low order bits. Below is the case where only the lowest order two bits of the four bit samples are used.

```
C:\est>python noniid main.py truerand 4bit.bin 4 --usebits 2 -v
Read in file truerand_4bit.bin, 1000000 bytes long.
Dataset: 1000000 4-bit symbols.
Output symbol values: min = 0, max = 15
* Using only low 2 bits out of 4.
* Using output symbol values: min = 0, max = 3
- Collision test
                          : p(max) = 0.277344, min-entropy = 1.85025
- Partial collection test : p(max) = 0.276367, min-entropy = 1.85534
                          : p(max) = 1.48356e-77, min-entropy = 1.9939
- Markov test
- Compression test
                          : p(max) = 0.276001, min-entropy = 1.85725
                          : p(max) = 0.250906, min-entropy = 1.98776
- Frequency test
min-entropy = 1.85025
Compression sanity check...
      dataset 1 compressed length = 215816, cutoff = 185025...Pass
      ...<output deleted to save space>...
      dataset 10 compressed length = 215928, cutoff = 185025...Pass
Collision sanity check...
      Dividing dataset into 17-tuples
      Check rule 1 - do three or more 17-tuples have the same value?...Pass
      Check rule 2 - probability of number of collisions below cutoff
            number of collisions = 0, cutoff = 0.587999...Pass
sanity check = PASS
```

Disclaimers

 This code is made available without any assertion or guarantee, implied or otherwise, of correctness or completeness.

- No support is provided for this code.
- The identification of any commercial product or trade name does not imply endorsement or recommendation by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

Appendix: Selected Runs of IID and Non-IID Tests

Non-IID Tests on 1 bit TrueRand data:

```
C:\est>python noniid main.py truerand 1bit.bin 1 -v
Read in file truerand_1bit.bin, 1000000 bytes long.
Dataset: 1000000 1-bit symbols.
Output symbol values: min = 0, max = 1
                          : p(max) = 0.53125, min-entropy = 0.912537
- Collision test
- Partial collection test : p(max) = 0.526367, min-entropy = 0.925859
                          : p(max) = 3.39255e-39, min-entropy = 0.998381

    Markov test

- Compression test
                          : p(max) = 0.529541, min-entropy = 0.917186
                          : p(max) = 0.500433, min-entropy = 0.995227
- Frequency test
min-entropy = 0.912537
Compression sanity check...
      dataset 1 compressed length = 125008, cutoff = 91253.7...Pass
      dataset 2 compressed length = 124792, cutoff = 91253.7...Pass
      dataset 3 compressed length = 124920, cutoff = 91253.7...Pass
      dataset 4 compressed length = 124896, cutoff = 91253.7...Pass
      dataset 5 compressed length = 124904, cutoff = 91253.7...Pass
      dataset 6 compressed length = 124824, cutoff = 91253.7...Pass
      dataset 7 compressed length = 124912, cutoff = 91253.7...Pass
      dataset 8 compressed length = 124920, cutoff = 91253.7...Pass
      dataset 9 compressed length = 125032, cutoff = 91253.7...Pass
      dataset 10 compressed length = 125240, cutoff = 91253.7...Pass
Collision sanity check...
      Dividing dataset into 32-tuples
      Check rule 1 - do three or more 32-tuples have the same value?...Pass
      Check rule 2 - probability of number of collisions below cutoff
            number of collisions = 0, cutoff = 0.791109...Pass
sanity check = PASS
```

Non-IID Tests on 9-bit TrueRand data:

```
C:\est>python noniid main.py truerand 9bit.bin 9 -v
Read in file truerand 9bit.bin, 2000000 bytes long.
Dataset: 1000000 9-bit symbols.
Output symbol values: min = 0, max = 511
- Collision test
                          : p(max) = 0.00601578, min-entropy = 7.37703
- Partial collection test : p(max) = 0.00437081, min-entropy = 7.83788
- Markov test (map 6 bits): p(max) = 5.70333e-223, min-entropy = 5.7678
- Compression test
                          : p(max) = 0.00553894, min-entropy = 7.49617
- Frequency test
                          : p(max) = 0.002091, min-entropy = 8.23683
min-entropy = 5.7678
Compression sanity check...
      dataset 1 compressed length = 948384, cutoff = 576780...Pass
      dataset 2 compressed length = 947576, cutoff = 576780...Pass
      dataset 3 compressed length = 946792, cutoff = 576780...Pass
      dataset 4 compressed length = 949304, cutoff = 576780...Pass
      dataset 5 compressed length = 947480, cutoff = 576780...Pass
      dataset 6 compressed length = 947896, cutoff = 576780...Pass
      dataset 7 compressed length = 950720, cutoff = 576780...Pass
      dataset 8 compressed length = 947952, cutoff = 576780...Pass
      dataset 9 compressed length = 946872, cutoff = 576780...Pass
      dataset 10 compressed length = 947384, cutoff = 576780...Pass
Collision sanity check...
      Dividing dataset into 6-tuples
      Check rule 1 - do three or more 6-tuples have the same value?...Pass
      Check rule 2 - probability of number of collisions below cutoff
            number of collisions = 0, cutoff = 0.530863...Pass
sanity check = PASS
```

IID Tests run on binary digits of pi

Note that since the reordering of samples in the shuffle tests is random, the ranks may change from run to run of the IID tests for the same dataset.

```
C:\est>python iid main.py data.pi.bin 1 1000 -v
Read in file data.pi.bin, 1165666 bytes long.
Dataset: 1165666 1-bit symbols.
Output symbol values: min = 0, max = 1
Compression Test:
     Scores
                              Ranks
     17867
                                1*
     17865
                                1*
     17874
                                1*
     17904
                                1*
     17851
                                1*
     17846
                                1*
     17894
                                1*
     17888
                                1*
                                1*
     17877
     17910
                                1*
                               10
Failed Compression Test
Over/under Test:
     Scores
                              Ranks
     22 52034
                              826
                                      1*
     19 52410
                              501
                                      1*
                              501
     19 52318
                                      1*
     20 52577
                              549
                                     1*
     20 52347
                              501
                                      1*
     18 52090
                              328
                                      1*
     20 52132
                              547
                                     1*
     19 52169
                               501
                                     1*
     19 52136
                               501
                                     1*
     19 52131
                               501
                                      1*
                                0
                                     10
Failed Over/under Test
```

Excursion Test:	
Scores	Ranks
156.625	642
125.619	364
160.774	699
138.821	507
84.1666	42*
85.0692	54
114.445	229
93.5417	80
102.259	123
109.691	238
	1

Passed Excursion Test

Directional runs Test:

Scores			Ranks		
8735	8	6102	1000*	187	1000*
8721	8	6043	1000*	202	1000*
8710	9	6090	1000*	501	1000*
8734	9	6072	1000*	501	1000*
8779	8	6096	1000*	182	1000*
8810	9	6088	1000*	501	1000*
8711	8	6046	1000*	162	1000*
8736	8	6064	1000*	190	1000*
8842	9	6158	1000*	501	1000*
8636	9	6086	979*	501	1000*
			10	0	10

Failed Directional runs Test

Covariance Test:

Scores	Ranks
0.0219853	1000*
0.0203951	1000*
0.0210892	1000*
0.0200727	1000*
0.0205968	1000*
0.021612	1000*
0.0218409	1000*
0.0212882	1000*
0.0217061	1000*
0.0216055	1000*

10

Failed Covariance Test

Collision Test: Scores Ranks 1 18.4866 56 501 626 635 1 18.9863 49 501 964* 124 1 18.051 46 13* 501 146 1 17.9182 56 501 59 622 1 18.2407 49 501 390 132 1 17.9208 47 501 116 38* 1 18.5787 52 501 662 370 1 17.9169 51 501 124 276 2 17.9608 56 971* 102 634 1 18.0984 49 501 235 135 1 1 2

Passed Collision Test

** Failed iid shuffle tests

IID = False