COMP550 Natural Language Processing
Assignment 3

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Question 1

(a)

$$
\begin{aligned}
(\lambda x.xx)(\lambda y.yx)z =& (\lambda y.yx)(\lambda y.yx)z \\
=& (\lambda y.yx)xz \\
=& xxz
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
(\lambda uvw.wvu)aa(\lambda pq.q)z =& (\lambda pq.q)aa \\
=& a
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
[(\lambda v.vv)(\lambda u.u)][(\lambda v.v)(\lambda v.w)] =& [(\lambda u.u)(\lambda u.u)](\lambda v.w) \\
=& (\lambda u.u)(\lambda v.w) \\
=& (\lambda v.w)
\end{aligned} \tag{3}
$$

(b)

```
DET -> no
V -> hates
```

The word *no* is used to say none of the object with characteristic P have the attribute Q (e.g. no tomatoes are pink, there is no human on mars,...). So it can be read has *for all x such that x is P, x is not Q*.

$$\lambda P.\lambda Q.\forall x.P(x) \rightarrow \neg Q(x)$$

When the verb *hates* is used, there is an event e:*something is hated*, the event need a *hater* z, and a *hatee* x

$$\lambda w.\lambda z.w[\forall x.\exists e\ hates(e) \wedge hater(e, z) \wedge hatee(e, x)]$$

Parsing tree.
Let `A`$\equiv \lambda z.\exists e.\ hates(e) \wedge hater(e,z) \wedge hatee(e, COMP550)$

The semantis of `S` is given by

$$\begin{aligned}
&[\lambda Q.\forall x.student(x) \rightarrow \neg Q(x)](A)\\
&\quad = \lambda Q.\forall x.student(x) \rightarrow \neg A(x)\\
&\quad = \lambda Q.\forall x.student(x) \rightarrow \neg[\lambda z.\exists e.\ hates(e) \wedge hater(e,z) \wedge hatee(e, COMP550)](x)\\
&\quad = \lambda Q.\forall x.student(x) \rightarrow \neg[\exists e.\ hates(e) \wedge hater(e,X) \wedge hatee(e, COMP550)]
\end{aligned} \tag{4}$$

`S` $< \lambda Q.\forall x.student(x) \rightarrow \neg\exists e.\ hates(e) \wedge hater(e,X) \wedge hatee(e, COMP550) >$
`S -> NP VP`

`NP` $< \lambda P.\lambda Q.\forall x.student(x) \rightarrow \neg Q(x) >$
`NP -> DET N`

`DET` $< \lambda P.\lambda Q.\forall x.P(x) \rightarrow \neg Q(x) >$
`DET -> no`

`N` $< \lambda x.student(x) >$
`N-> student`

`VP` $< \lambda z.\exists e.\ hates(e) \wedge hater(e,z) \wedge hatee(e, COMP550) >$
`VP -> V PN`

`V` $< \lambda w.\lambda z.w[\forall x.\exists e\ hates(e) \wedge hater(e,z) \wedge hatee(e,x)] >$
`V -> hates`

`PN` $< \lambda X.x(COMP550) >$
`PN -> COMP550`

(c) The representation of wants is

$$\exists e wants(e) \wedge wanter(e, s_1) \wedge wantee(e, s_2)$$

First interpretation: Start with the predicate *there is an exam* y *that is wanted by* s_1 and then *for each student* x *negate this predicate over* s_1

$$\begin{aligned}
&(\lambda Q.\exists y.exam(y) \wedge Q(y))(\lambda s_2.\exists e.wants(e) \wedge wanter(e,s_1) \wedge wantee(e,s_2))\\
&\quad = \exists y.exam(y) \wedge \exists e.want(e) \wedge wanter(e,s_1) \wedge wantee(e,y)
\end{aligned} \tag{5}$$

$$\begin{aligned}
&(\lambda Q.\forall x.student(x) \rightarrow \neg Q(x))(\lambda s_1.\exists y.exam(y) \wedge \exists e.want(e) \wedge wanter(e,s_1) \wedge wantee(e,y))\\
&\quad = \forall x.student(x) \rightarrow \neg[\lambda s_1.\exists y.exam(y) \wedge \exists e.want(e) \wedge wanter(e,s_1) \wedge wantee(e,y)](x)\\
&\quad = \forall x.student(x) \rightarrow \neg\exists y.exam(y) \wedge \exists e.want(e) \wedge wanter(e,x) \wedge wantee(e,y)
\end{aligned} \tag{6}$$

Interpretation 1: There is no exam anywhere that any students want.

Second interpretation: Start with *There is an object s_2 that is not wanted by any student* x and then replace this object by an exam y that can have the property Q (of not being wanted in that case).

$$\begin{aligned}&(\lambda Q.\forall x.student(x) \rightarrow \neg Q(x))(\lambda s_1.\exists e.want(e) \wedge wanter(e, s_1) \wedge wantee(e, s_2)) \\ &=\forall x.student(x) \rightarrow \neg(\exists e.want(e) \wedge wanter(e, x) \wedge wantee(e, s_2))\end{aligned} \tag{7}$$

$$\begin{aligned}&(\lambda Q.\exists y.exam(y) \wedge Q(y))(\lambda s_2.\forall x.student(x) \rightarrow \neg(\exists e.want(e) \wedge wanter(e, x) \wedge wantee(e, s_2))) \\ &=(\exists y.exam(y)) \wedge [\forall x.student(x) \rightarrow \neg(\exists e.want(e) \wedge wanter(e, x) \wedge wantee(e, y))]\end{aligned} \tag{8}$$

Interpretation 2: There is a particular exam that no student want.