Name :_____

Permanent code : _____

Place numer :_____

Directives :

- Write your name, first name, permanent code and place number.
- Read carefully all questions and **answer directly on the questionnaire**.
- You can only use a pen or pencil, **no documentation, calculator, phone, computer, or any other object**.
- This exam contains 10 questions for 150 points.
- **There are 15 bonus points.**
- This exam contains 21 pages, including 4 draft and detachable pages at the end.
- **Write lisibly and detail your answers**.
- You have 160 minutes to complete this exam.

GOOD LUCK !

1	/ 15
2	/ 15
3	/ 15
4	/ 10
5	/ 20
6	/ 10
7	/ 10
8	/ 20
9	/ 25
10	/ 25
Total	/150

1. (15) Show that $\sum_{i=1}^{i^2}$ is $O(n^3)$.

2. (15) Given a list A of size $n \geq 2$ that contains integers 1 to n-1, including one and only one repeated integer. Describe a fast algorithm, O(n), to find the repeated integer in A.

3. (15) Suppose we insert three values in a stack S in a random order. Give a code without loops nor recursive function, but that uses one and only one variable, x, such that at the end of the execution x contains the greatest value of the three with probability 2/3. Say why your method is correct.

4. (10) An airport wants to develop a simulation system to manage air traffic, i.e. departures and arrivals. Each event is labelled by a time stamp, that is when the event occurs. The program must perform the two fundamental following operations:

    1. Insert a futur event with given time
    2. Extract the next event to occur.

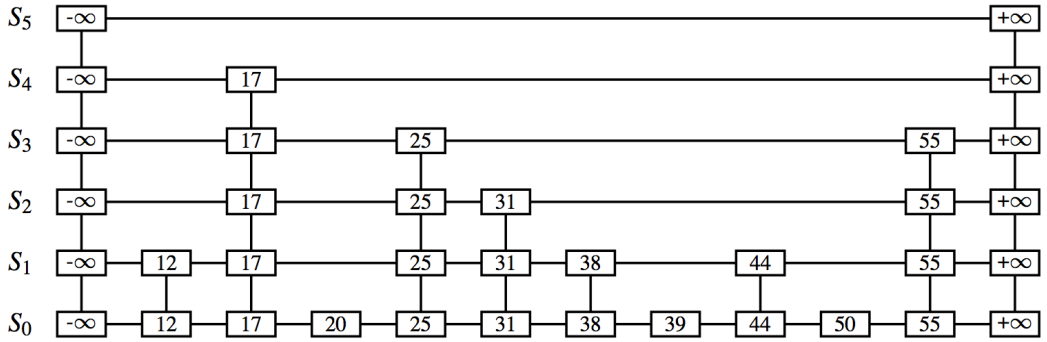
    What data structure should they use to implement this simulation system and why?

5. (20) Draw the hashing table resulting from using the hashing function h(i) = (3i+5) mod 11, to insert the following keys : 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, and 5.

   a) (10) If the collisions are solved by external chaining.

b) (10) If the collision are solved by linear probing.

6. (10) Consider the following « Skip List », S:

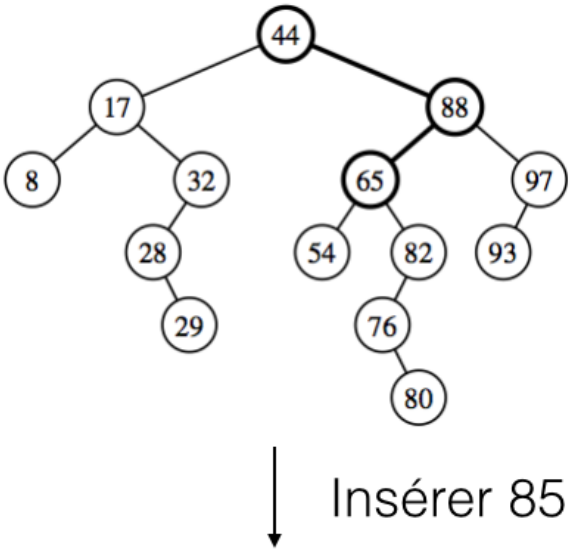


a) (5) Draw the resulting Skip List after executing the operation `del S[17]`.

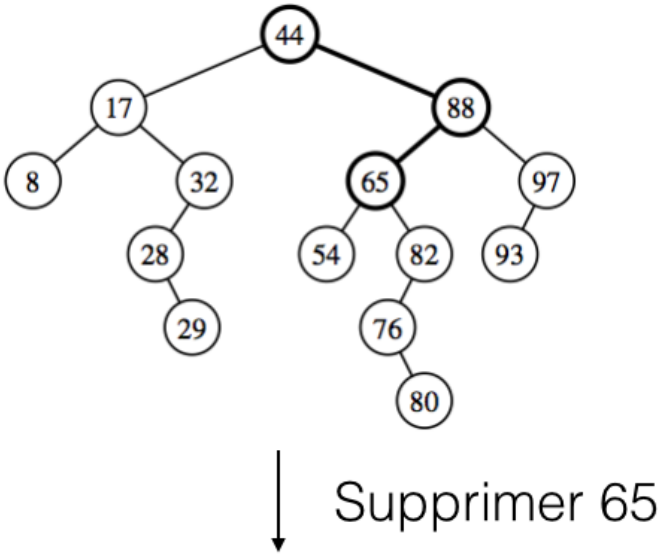
b) (5) From the initial Skip List (the starting one, not the one you obtained in (a)), draw the resulting « Skip List » after executing the operation `S[45] = 'x'`, if it is randomly determined that the height of the tower must be increased three times.

7. (10) Show the result of executing the following operations in a binary search tree.

a) (5) Insertion of a key.

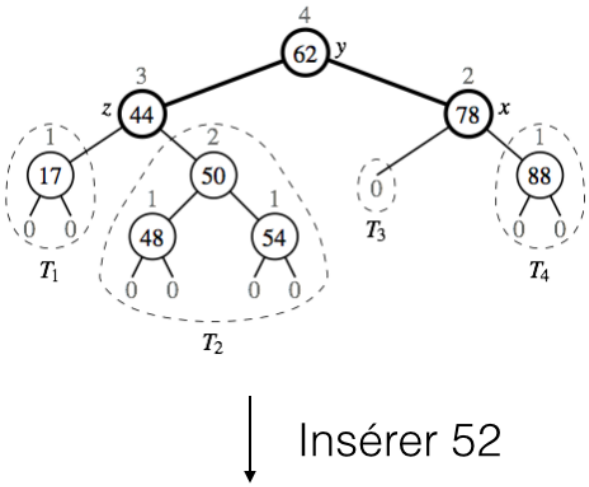


b) (5) Suppression of a key.

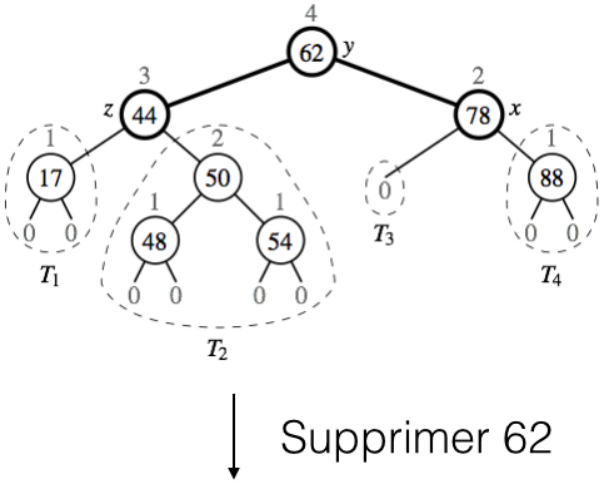


8. (20) Show the result of executing the following operations in a AVL tree. Draw the resulting AVL tree and indicate the height of each node.

a) (10) Insertion of a key.



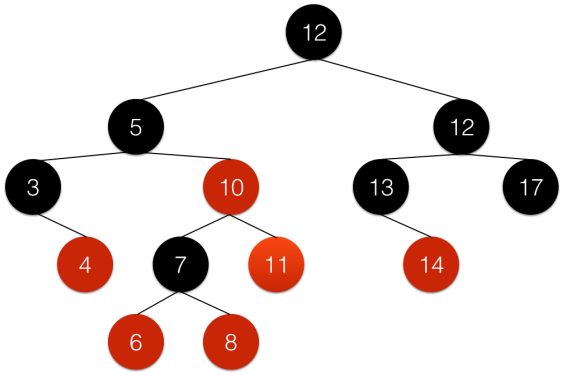
b) (10) Suppression of a key.



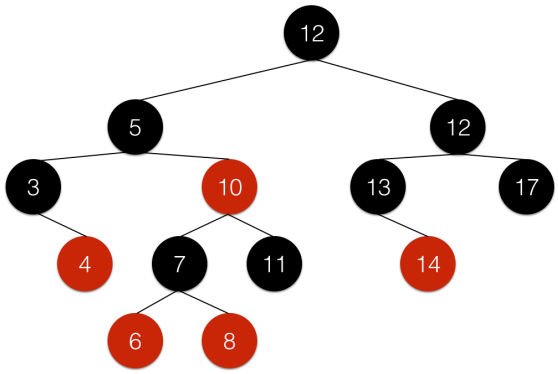
9. (25) Consider the insertion of the following keys: 5, 16, 22, 45, 2, 10, 18, 30, 50, 12, and 1. Draw the resulting trees you obtain after each insertion if the keys are inserted in this order in a (2,4) tree initially empty.

10. (25) Say if the following Red-Black trees are valid. In the case the Red-Black tree is not valid, say the property or properties that are not respected.

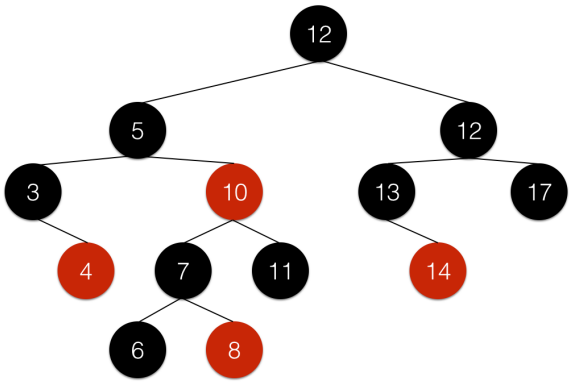
a) (5)



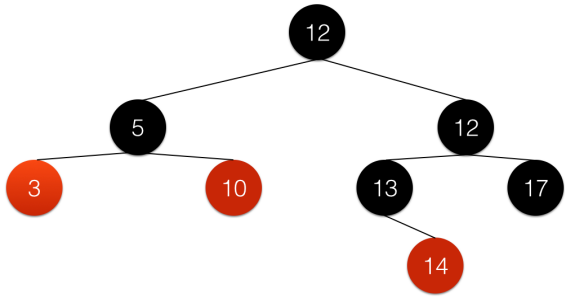
b) (5)



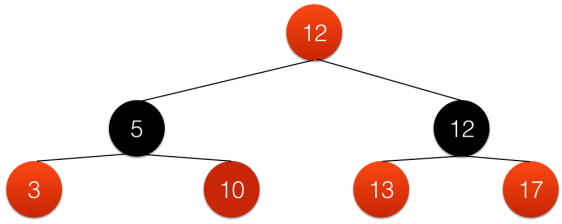
c) (5)



d) (5)



e) (5)



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