Name :_____

Permanent code :   _____

Place number :   _____

Directives:
- Write your name, permanent code and place number above.
- **Read all questions and answer directly on the sheets**
- Only a pen/pencil is permitted, **no documentation**, calculator, phone, computer, or any other object
- This exam contains 10 questions for a total of 160 points.
- Points over 150 are considered bonuses. However, pay attention at your time as 1 point = 1 minute. You can thus drop 10 points without penalty.
- This exam contains 19 pages, including 3 removable pages at the end for your drafts.
- For the developing questions, **write clearly** and **detail your answers**
- You have 150 minutes to complete this exam

GOOD LUCK!

1	/ 10
2	/ 10
3	/ 15
4	/ 15
5	/ 25
6	/ 15
7	/ 15
8	/ 20
9	/10
10	/25
Total	/150

1.   (10) Consider the following general ontology taxonomic tree:
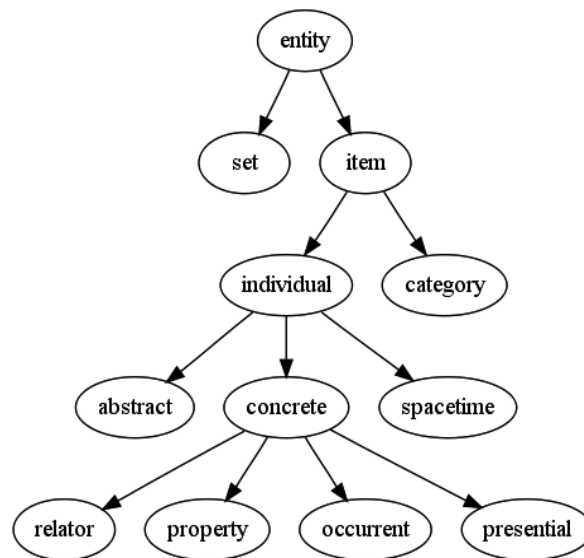
a)   (1) What node is the root?

b)   (2) What are the internal nodes?

c)   (1) How many descendants does node « individual » have?

d)   (1) How many ancestors node « spacetime » have?

e)   (1) How many siblings node « item » have?

f)   (1) Which nodes are in the subtree rooted by « individual »?

g)   (2) What is the depth of node « category »?

h)   (1) What is the height of the tree?

2.   (10) In which order the nodes of the tree in question (1) are visited by:

    a)   (5) a preorder traversal

    b)   (5) a postorder traversal

3. (15) Show how to implement the stack ADT (operations `push` and `pop`) using a priority queue (min queue and operations `enqueue` and `dequeue`) and one additional integer instance variable. (<u>hint</u>: the elements or a priority queue are composed of a key and a value).

4. (15) Consider a binary search tree (BST).

    a) (8) Draw the BST after each insertion of the keys 30, 40, 24, 58, 48, 26, 11, 13 (in this order).

b)   (7) Draw all possible BSTs containing the keys 1, 2 and 3.

5.   (20) Draw step by step the AVL trees resulting from the following operations:

    a)   (10) Insertion of the following keys in an empty AVL tree at the beginning: 14, 17, 11, 7, 53, 4, 13, 12 and 8 (in this order).
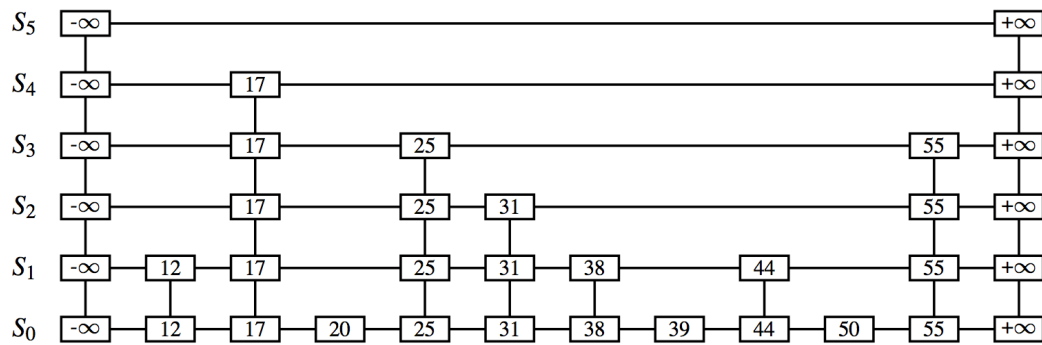
b)   (10) From the AVL tree you obtained in (a), deletion of the following keys: 53, 11 and 8 (in this order).

6. (15) Consider the following hash table with external chaining:
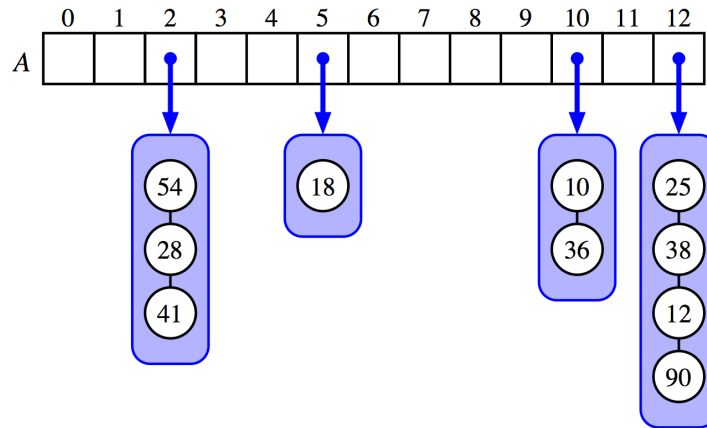
a) (5) What is the worst-case time for putting n entries in an initially empty hash table, with collisions resolved by chaining with a singly-linked list?

b) (10) Draw the new table after rehashing in a table of size 19 and with a new hashing function, $h(k) = k \bmod 17$.

7.   (15) Consider the following skip list:

	$-\infty$	12	17	20	25	31	38	39	44	50	55	$+\infty$
S_5	$-\infty$											$+\infty$
S_4	$-\infty$		17									$+\infty$
S_3	$-\infty$		17		25						55	$+\infty$
S_2	$-\infty$		17		25	31					55	$+\infty$
S_1	$-\infty$	12	17		25	31	38		44		55	$+\infty$
S_0	$-\infty$	12	17	20	25	31	38	39	44	50	55	$+\infty$

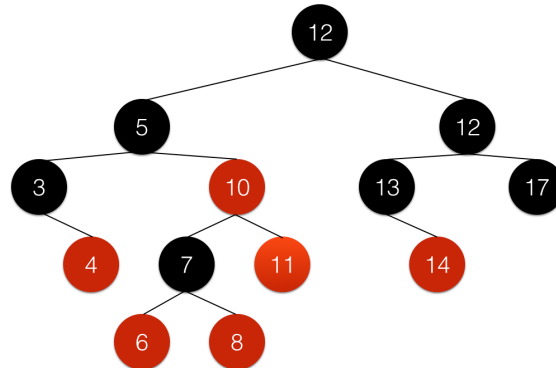a)   (5) Draw the resulting skip list after the operation `del S[38]`.

b)   (10) From the above skip list (the starting one, not the one you obtained in (a)), draw the resulting skip list after operation `S[48] = 'x'` if the results of the coin flipping are HEAD, HEAD, HEAD, TAIL, HEAD.

8. (20) Consider red-black trees (RBT). For each of the following statements, give a justification if true, or a counter example if false.

   a) (5) A subtree of a RBT is itself a RBT.

   b) (5) A RBT node that has no sibling is red.

   c) (5) Each RBT has a unique associated (2,4) tree.

   d) (5) Each (2,4) tree has a unique associated RBT.
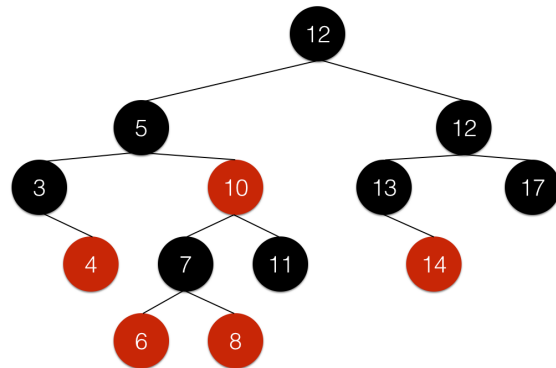
9.   (10) Explain why you would get the same output in an inorder listing of the entries in a BST, AVL, splay tree, or RBT.

10. (25) For each of the following RBT, indicate if it is valid, and if not which of the RBT properties is violated.
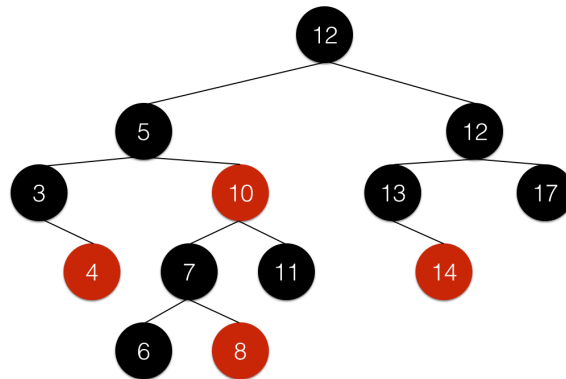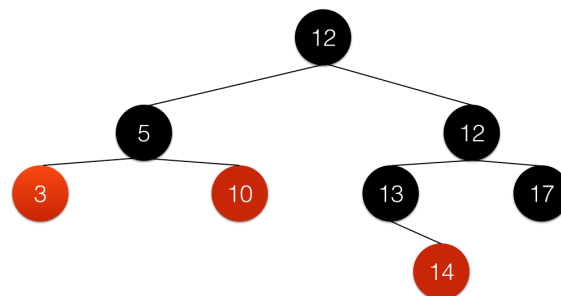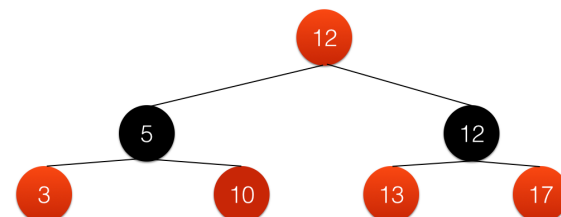
a) (5)

b) (5)

c)   (5)

d)   (5)

e)   (5)

Draft :

Draft :

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