1. grep and many other software implementations of regular expressions include the question mark , '?', as a special symbol which marks the preceding expression as optional. For example, the regular expression `dog(gy)?` matches the strings 'dog' and 'doggy'.

   Let $\mathcal{REQ}$ be an extension of our familiar language of regular expressions with the question mark operator added. We will formally define the set $\mathcal{REQ}$ by extending the definition of $\mathcal{RE}$ (definition 7.6 in the Vassos course notes) to add the following induction step: If $R \in \mathcal{REQ}$, then $(R)? \in \mathcal{REQ}$.

   (a) Definition 7.7 in the Vassos course notes is a recursive definition of the language denoted by a regular expression $R \in \mathcal{RE}$. Give an extended version of this definition for $\mathcal{REQ}$.

   (b) Show that $\mathcal{REQ}$ has no more expressive power than $\mathcal{RE}$, by proving the following statement: $\forall R_1 \in \mathcal{REQ}, \exists R_2 \in \mathcal{RE}, \mathcal{L}(R_2) = \mathcal{L}(R_1)$. Your proof should use structural induction.

2. Given a DFSA $M = (Q, \Sigma, \delta, s, F)$, we will say that M is **frumious** if the following is true:

   $\forall a \in \Sigma, \exists q_1 \in Q, \forall q_2 \in Q, \delta(q_2, a) = q_1$

   (a) Give a short English description of what it means for a DFSA to be frumious.

   (b) If M is frumious, what can we say about the language accepted by M, $\mathcal{L}(M)$?

   (c) How many distinct languages over the alphabet $\{0, 1\}$ can be recognized by frumious DFSAs? Briefly explain your answer.

3. Suppose L is an infinite regular language. Does it follow that there exists a finite language S such that $L = SS^*$? If yes, prove it. If no, find a counterexample language L and prove that it cannot be formed this way.