CSC236 Assignment 1

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1.

P(n): $f(n) = 2^{(n/2)} \cdot (\frac{n}{2})!$

WTS: $\forall\ n \in \mathbb{N}$, n is even $\Rightarrow$ P(n)

Base Case:

$f(0) = 1 = (2^{(0/2)}) \cdot (\frac{0}{2})!$

P(0) holds

Inductive Steps:

Let n be an arbitrary even natural number

Assume P(n) holds:

$f(n) = 2^{(n/2)} \cdot (\frac{n}{2})!$

WTS: P(n+2) i.e., $f(n+2) = 2^{((n+2)/2)} \cdot (\frac{n+2}{2})!$

$$\begin{aligned}
f(n+2) &= (n+2) \cdot f(n) \\
&= (n+2) \cdot 2^{(n/2)} \cdot (\tfrac{n}{2})! \\
&= (\tfrac{n}{2} + 1) \cdot 2 \cdot 2^{(n/2)} \cdot (\tfrac{n}{2})! \\
&= 2 \cdot 2^{(n/2)} \cdot (\tfrac{n}{2} + 1) \cdot (\tfrac{n}{2})! \\
&= 2 \cdot 2^{(n/2)} \cdot (\tfrac{n}{2} + 1)! \\
&= 2^{((n+2)/2)} \cdot (\tfrac{n+2}{2})!
\end{aligned}$$

So $P(n) \Rightarrow P(n+2)$ for all even natural number n

So, $\forall\ n \in \mathbb{N}$, n is even $\Rightarrow$ P(n) ■

2. (a)

WTS: $\forall$ n $\in \mathbb{N}$, P(n)

Let n $\in \mathbb{N}$

Inductive Hypothesis: If n > 1, P(k) holds for $\forall$ k $\in \mathbb{N}$, 0 < k < n

Case 1: n = 1

P(1) holds # (1)

Case 2: n > 1, n is even

$P(\frac{n}{2})$ holds # $\frac{n}{2} \in \mathbb{N} \wedge$ I.H

$P(\frac{n}{2}) \Rightarrow P(n)$ # (3)

P(n) holds

Case 3: n > 1, n is odd

$P(\frac{n+1}{2})$ holds # $\frac{n+1}{2} \in \mathbb{N} \wedge \frac{n+1}{2} < n$ (Since n < 1) $\wedge$ I.H

$P(\frac{n+1}{2}) \Rightarrow P(n+1)$ # (3)

$P(n+1) \Rightarrow P(n)$ # (2)

P(n) holds

Case 4: n = 0

P(1) holds # (1)

$P(1) \Rightarrow P(0)$ # (2)

P(0) holds

$\forall$ n $\in \mathbb{N}$, P(n) holds

■

2. (b)

We can only prove $P(1)$ and $P(0)$

$P(1)$ holds # (1)

$P(1) \Rightarrow P(0)$ # (2) $\wedge\ 1 \in \mathbb{N}^+$

We can't go any further

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3.

Let S be such a set defined in the problem.

P(n): n does not contain substring yh.

Let $n \in S$

WTS $\forall\ n \in S, P(n)$

Base Case: $n = u$

$u \in S$

u does not contain yh

P(u) holds

Inductive steps:

Let $s1, s2 \in S$

Inductive Hypothesis: Assume $P(s1) \wedge P(s2)$

Lemma 1: y can't be postfix of any element in S

Because y can only add to one string's leftmost digit to form a new element in S, and y is not in S

Lemma 2: h can't be prefix of any element in S

Because h can only add to one string's rightmost digit to

form a new element in S, and h is not in S

Case 1: n = ys1

The first digit in s1 is not h # Lemma 2

So, the first two digits in ys1 is not yh.

So, ys1 does not contain yh.

P(ys1) holds

Case 2: n = s1h

The last digit in s1 is not y # Lemma 1

So, the last two digits in s1h is not yh.

So, s1h does not contain yh.

P(s1h) holds

Case 3: n = s1s2

The last digit in s1 is not y # Lemma 1

The first digit in s2 is not h # Lemma 2

So, s1s2 does not contain yh.

P(s1s2) holds

So, $\forall$ n $\in$ S, P(n)

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4.

I will prove this problem by two steps.

First, I will prove that for any natural number n, there exists at least one natural number m, containing exactly n sub-strings in its decimal representation which are prime numbers.

Second, I will prove that for any natural number n, there exists a smallest natural number A(n), containing exactly n sub-strings in its decimal representation which are prime numbers.

Step 1:

P(n): $\exists$ m $\in \mathbb{N}$, m containing exactly n sub-strings in its decimal representation which are prime numbers.

WTS: $\forall$ n $\in \mathbb{N}$, P(n)

Let n $\in \mathbb{N}$

Case 1: $n = 0$

Let m be 8

8 contains exactly 0 sub-strings in its decimal representation which are prime numbers.

P(0) holds.

Case 2: $n > 0$

Let m be 555555……5555555 (n digit of 5)

5 is a prime number

55……55(k digit of 5, $k \in \mathbb{N}$, $k > 1$) is not a prime number, because it can be divided by 5. Proof below:

55……55 = 55……50 + 5 = 10t + 5(let t be such a natural number). WTS: $\exists\ r \in \mathbb{N}$, $10t + 5 = 5r$.

Let $r = 2t + 1$

$5r = 10t + 5 =$ 55……55

555555……5555555 (n digit of 5) contains exactly n sub-strings in its decimal representation which are prime numbers.

P(n) holds.

So, $\forall\ n \in \mathbb{N}$, P(n)

Step 2:

Q(n): $\exists\ A(n) \in \mathbb{N}$, A(n) is the smallest number containing exactly n sub-strings in its decimal representation which are prime numbers.

WTS: $\forall\ n \in \mathbb{N}$, Q(n)

Let $n \in \mathbb{N}$

Let S(n) be the set of all natural numbers containing exactly n sub-strings in its decimal representation which are prime numbers.

From step 1, it is proven that S(n) is not empty. And it's obvious that S(n) is a subset of $\mathbb{N}$.

$\exists\ j \in S(n), \forall\ i \in S(n), i \geq j$ #Well-Ordering Principle

Let $A(n) = j$

A(n) is the smallest number containing exactly n sub-strings in its decimal representation which are prime numbers.

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