**CSC236 Assignment 1**

**Jiahong Zhai (zhaijia3)**

**Due Date: 2020.1.30**

P(n): f(n) = 2(n/2) ⋅ ()!

WTS: ∀ n ∈ ℕ, n is even ⇒ P(n)

Base Case:

f(0) = 1 = (2(0/2)) ⋅ ()!

P(0) holds

Inductive Steps:

Let n be an arbitrary even natural number

Assume P(n) holds:

f(n) = 2(n/2) ⋅ ()!

WTS: P(n+2) i.e., f(n+2) = 2((n+2)/2) ⋅ ()!

f(n + 2) = (n + 2) ⋅ f(n)

= (n + 2) ⋅ 2(n/2) ⋅ ()!

= ( + 1) ⋅ 2 ⋅ 2(n/2) ⋅ ()!

= 2 ⋅ 2(n/2) ⋅ ( + 1) ⋅ ()!

= 2 ⋅ 2(n/2) ⋅ ( + 1)!

= 2((n+2)/2) ⋅ ()!

So P(n) ⇒ P(n + 2) for all even natural number n

So, ∀ n ∈ ℕ, n is even ⇒ P(n) ∎

1. (a)

WTS: ∀ n ∈ ℕ, P(n)

Let n ∈ ℕ

Inductive Hypothesis: If n > 1, P(k) holds for ∀ k ∈ ℕ, 0 < k < n

Case 1: n = 1

P(1) holds # (1)

Case 2: n > 1, n is even

P() holds # ∈ ℕ ∧ I.H

P() ⇒ P(n) # (3)

P(n) holds

Case 3: n > 1, n is odd

P() holds # ∈ ℕ ∧ < n (Since n < 1) ∧ I.H

P() ⇒ P(n + 1) # (3)

P(n + 1) ⇒ P(n) # (2)

P(n) holds

Case 4: n = 0

P(1) holds # (1)

P(1) ⇒ P(0) # (2)

P(0) holds

∀ n ∈ ℕ, P(n) holds

∎

1. (b)

We can only prove P(1) and P(0)

P(1) holds # (1)

P(1) ⇒ P(0) # (2) ∧ 1 ∈ ℕ+

We can’t go any further

∎

Let S be such a set defined in the problem.

P(n): n does not contain substring yh.

Let n ∈ S

WTS ∀ n ∈ S, P(n)

Base Case: n = u

u ∈ S

u does not contain yh

P(u) holds

Inductive steps:

Let s1, s2 ∈ S

Inductive Hypothesis: Assume P(s1) ∧ P(s2)

Lemma 1: y can’t be postfix of any element in S

Because y can only add to one string’s leftmost digit to form a new element in S, and y is not in S

Lemma 2: h can’t be prefix of any element in S

Because h can only add to one string’s rightmost digit to form a new element in S, and h is not in S

Case 1: n = ys1

The first digit in s1 is not h # Lemma 2

So, the first two digits in ys1 is not yh.

So, ys1 does not contain yh.

P(ys1) holds

Case 2: n = s1h

The last digit in s1 is not y # Lemma 1

So, the last two digits in s1h is not yh.

So, s1h does not contain yh.

P(s1h) holds

Case 3: n = s1s2

The last digit in s1 is not y # Lemma 1

The first digit in s2 is not h # Lemma 2

So, s1s2 does not contain yh.

P(s1s2) holds

So, ∀ n ∈ S, P(n)

∎

I will prove this problem by two steps.

First, I will prove that for any natural number n, there exists at least one natural number m, containing exactly n sub-strings in its decimal representation which are prime numbers.

Second, I will prove that for any natural number n, there exists a smallest natural number A(n), containing exactly n sub-strings in its decimal representation which are prime numbers.

Step 1:

P(n): ∃ m ∈ ℕ, m containing exactly n sub-strings in its decimal representation which are prime numbers.

WTS: ∀ n ∈ ℕ, P(n)

Let n ∈ ℕ

Case 1: n = 0

Let m be 8

8 contains exactly 0 sub-strings in its decimal representation which are prime numbers.

P(0) holds.

Case 2: n > 0

Let m be 555555……5555555 (n digit of 5)

5 is a prime number

55……55(k digit of 5, k ∈ ℕ, k > 1) is not a prime number, because it can be divided by 5. Proof below:

# 55……55 = 55……50 + 5 = 10t + 5(let t be such a natural number). WTS: ∃ r ∈ ℕ, 10t + 5 = 5r.

Let r = 2t + 1

5r = 10t + 5 = 55……55

555555……5555555 (n digit of 5) contains exactly n sub-strings in its decimal representation which are prime numbers.

P(n) holds.

So, ∀ n ∈ ℕ, P(n)

Step 2:

Q(n): ∃ A(n) ∈ ℕ, A(n) is the smallest number containing exactly n sub-strings in its decimal representation which are prime numbers.

WTS: ∀ n ∈ ℕ, Q(n)

Let n ∈ ℕ

Let S(n) be the set of all natural numbers containing exactly n sub-strings in its decimal representation which are prime numbers.

From step 1, it is proven that S(n) is not empty. And it’s obvious that S(n) is a subset of ℕ.

∃ j ∈ S(n), ∀ i ∈ S(n), i ≥ j #Well-Ordering Principle

Let A(n) = j

A(n) is the smallest number containing exactly n sub-strings in its decimal representation which are prime numbers.

∎