CSC236 Assignment 3

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1a.

Let Σ be a finite alphabet. The set of extension of regular expression ℛℇ𝒬(over Σ) is the smallest set such that:

Basis: ∅, ϵ and a (for each a ∈ Σ) belong to ℛℇ𝒬.

Induction: If R and S belong to ℛℇ𝒬, then (R+S), (RS), R\* and R? also belong to ℛℇ𝒬.

1b.

P(R): R∈ ℛℇ𝒬 ⟹ ∃R’∈ℛℇ, ℒ(R)= ℒ(R’)

WTS: ∀R∈ ℛℇ𝒬, P(R)

Let R∈ ℛℇ𝒬

Basis: R is ∅ or ϵ or a(for each a ∈ Σ)

R∈ℛℇ, then let R’ = R. We have ℒ(R)= ℒ(R’).

Induction: Assume T and S ∈ ℛℇ𝒬 (defined as in 1a) and P(S), P(T). Which is to say, ∃S’∈ℛℇ, ℒ(S)= ℒ(S’), ∃T’∈ℛℇ, ℒ(T)= ℒ(T’). We want to show P(R) for R = S+T or ST or S\* or S?

Case1: R = S+T

Let R’ = S’ + T’.

By definition 7.7 we know, ℒ(R) = ℒ(S+T) = ℒ(S)∪ℒ(T) = ℒ(S’) ∪ ℒ(T’) = ℒ(S’+T’) = ℒ(R’)

So, ℒ(R) = ℒ(R’)

Case2: R = ST

Let R’ = S’T’

By definition 7.7 we know, ℒ(R) = ℒ(ST) = ℒ(S)∘ℒ(T) = ℒ(S’)∘ℒ(T’) = ℒ(S’T’) = ℒ(R’)

So, ℒ(R) = ℒ(R’)

Case3: R = S\*

Let R’ = S’\*

By definition 7.7 we know ℒ(R) = ℒ(S\*) = ℒ(S)\* = ℒ(S’)\* = ℒ(S’\*) = ℒ(R’).

So, ℒ(R) = ℒ(R’)

Case4: R = S?

Let R’ = (S’ + ε)

ℒ(R) = ℒ(S?) = ℒ(S)∪ℒ(ε) = ℒ(S’)∪ℒ(ε) = ℒ(S’+ε) = ℒ(R’)

So, ℒ(R) = ℒ(R’)

∎

2a.

For all element a in the alphabet, there is a state q1, such that in any state if the next reading element is a, then the automata will transit to q1.

2b.

There is only one q1 for any element a in the alphabet. Because if there are more than one q1 for a, then if the current state is one of the q1s and the next reading element is a, then automata must transit to all q1s. But this contradict to the definition of Deterministic Finite State Automata. So, there is only on q1 for a, let’s call it q1a.

If one string is ending by a, then obviously the last state is q1a. If q1a ∈ F, then the string is accepted. So, ℒ(M) is the set of all strings whose last element’s q1 is in F.

2c.

By 2b we know that for 1 there is a q1. For 0 there is a q1, we call it q0 here.

Case1: q1∈F, q0∈F

ℒ(M) is the set of all strings over {0,1}

Case2: q1∈F, q0∉F

ℒ(M) is the set of strings over {0,1}, whose last element is 1.

Case3: q1∉F, q0∈F

ℒ(M) is the set of strings over {0,1}, whose last element is 0.

Case4: q1∉F, q0∉F

ℒ(M) is ∅

So, in total, 4 languages

∎

3.

No

Let R be 01\* over alphabet {0,1}

Let L be ℒ(R).

L = {0, 01, 011, 0111, 01111, 011111……} all strings that are started by 0 and followed by any number of 1.

WTS S, that L = SS\* does not exist

Case1: ∀x∈S, 0∉x

So, ∀y∈SS\*, 0∉y.

So, 0111 ∉ SS\*, but 0111∈L.

So, L≠SS\*

So, such an S does not exist

Case2: ∃x∈S, 0∈x

Let x be a0b (a and b can be 0 or 1 or ε)

Then xx which is a0ba0b∈SS\*, but a0ba0b∉L, because there are 2 0s in a0ba0b and strings in L could have only one 0.

So, L≠SS\*

So, such an S does not exist

So, the answerer is no

∎