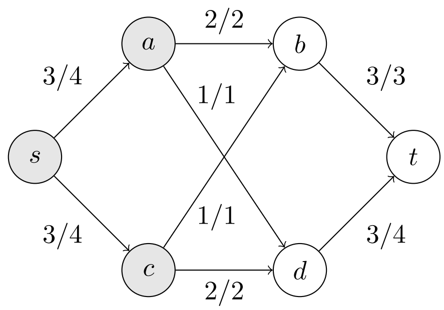
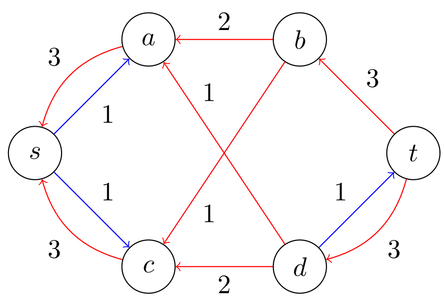
2. (a) A maximum flow is shown as below. A minimum cut of this corresponding maximum flow would be (S, T) where $S = \{s, a, c\}$, $T = \{b, d, t\}$. The value of the flow is 6, and the capacity of the cut is also 6.

   By Corollary 4, for any flow f and any s-t cut (A, B), if $v(f) = cap(A, b)$, then f is a max flow and (A, B) is a min-cut. Therefore our answer is correct.

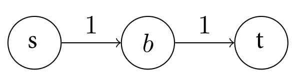


   (b) Residual graph:



   (c) $\{(a, d), (c, d)\}$

   (d) Flow network:



   (e) Find a maximum flow f of (G, s, t, c) and its corresponding residual graph G_f using Ford-Fulkerson Algorithm. For each edge (u, v) in G, if there is a path from s to u and a path from v to t in G_f, then (u, v) is a bottleneck edge. We can find all nodes reachable from s and all nodes that can reach t in G_f, call them A and B respectively. Then we find (u, v) in G such that $u \in A$ and $v \in B$.

   This algorithm is correct because if we increase the capacity of such edge, there will be a s-t path in G_f and if we augment f the maximum flow would increase.

   To find all reachable edges we need to run modified Dijkstra's algorithm which runs in $O(m \log n)$. So the total running time would be the time to run Ford-Fulkerson $+O(m \log n)$. $(m = |E|, n = |V|)$