CSC165H1: Problem Set 1

Due Wednesday October 2 before 4 pm

1.

(a).

r	p	q	$\neg r \Rightarrow (\neg p \Rightarrow q)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

(b).

$$\begin{aligned}
&\neg r \Rightarrow (\neg p \Rightarrow q)\\
&= \neg r \Rightarrow (\neg(\neg p) \vee q)\\
&= \neg(\neg r) \vee (\neg(\neg p) \vee q)\\
&= r \vee (p \vee q)\\
&= r \vee p \vee q
\end{aligned}$$

(c).

Converse:

$$\begin{aligned}
&(\neg p \Rightarrow q) \Rightarrow \neg r\\
&= (\neg(\neg p) \vee q) \Rightarrow \neg r\\
&= \neg(\neg(\neg p) \vee q) \vee \neg r\\
&= \neg(p \vee q) \vee \neg r\\
&= (\neg p \wedge \neg q) \vee \neg r
\end{aligned}$$

2.

(a).

$$
\begin{aligned}
\neg Onto(g) &= \neg\ (\forall n\in\mathbb{N}, \exists m\in\mathbb{N}, g(m)=n, \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}) \\
&= \exists n\in\mathbb{N}, \neg\ (\exists m\in\mathbb{N}, g(m)=n, \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}) \\
&= \exists n\in\mathbb{N}, \forall m\in\mathbb{N}, \neg(g(m)=n, \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}) \\
&= \exists n\in\mathbb{N}, \forall m\in\mathbb{N}, g(m) \neq n, \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}
\end{aligned}
$$

(b).

$$
\begin{aligned}
\neg OneToOne(h) &= \neg\ (\forall n, m\in\mathbb{N}, m\neq n \Longrightarrow h(m) \neq h(n), \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}) \\
&= \neg\ (\forall n, m\in\mathbb{N}, m=n \vee h(m) \neq h(n), \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}) \\
&= \exists n, m\in\mathbb{N}, \neg\ (m=n \vee h(m) \neq h(n), \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}) \\
&= \exists n, m\in\mathbb{N}, m\neq n \wedge h(m) = h(n), \text{ for } g: \mathbb{N}\rightarrow \mathbb{N}
\end{aligned}
$$

(c).

$f(x) = x + 1$

(d).

$f(x) = (0.5)^{x}$

(e).

$f(x) = x \cdot \sin(x)$

(f).

$f(x) = x^2$

3.

(a).

Proof: Let $a, b, c, d, n \in \mathbb{Z}, n \neq 0$

Assume: $a \equiv c \pmod{n} \wedge b \equiv d \pmod{n}$

i.e., $(\exists k_1 \in \mathbb{Z}, k_1 \cdot n = a - c) \wedge (\exists k_2 \in \mathbb{Z}, k_2 \cdot n = b - d)$

WTS: $a + b \equiv c + d \pmod{n}$

i.e., $\exists k_3 \in \mathbb{Z}, k_3 \cdot n = a + b - (c + d)$

Let $k_3 = k_1 + k_2$:

Because: $k_1 \cdot n = a - c, \quad k_2 \cdot n = b - d$

$k_1 \cdot n + k_2 \cdot n = a + b - (c + d)$

$(k_1 + k_2) \cdot n = a + b - (c + d) \cdots\cdots (1)$

$k_3 \cdot n = (k_1 + k_2) \cdot n$

$k_3 \cdot n = a + b - (c + d) \quad \#(1)$

SO: $a + b \equiv c + d \pmod{n}$

■

(b).

Proof: Let $a, b, c, d, n \in \mathbb{Z}, n \neq 0$

Assume: $a \equiv c \pmod{n} \wedge b \equiv d \pmod{n}$

i.e., $(\exists k_1 \in \mathbb{Z}, k_1 \cdot n = a - c) \wedge (\exists k_2 \in \mathbb{Z}, k_2 \cdot n = b - d)$

WTS: $a \cdot b \equiv c \cdot d \pmod{n}$

i.e., $\exists k_3 \in \mathbb{Z}, k_3 \cdot n = a \cdot b - c \cdot d$

Let $k_3 = k_1 \cdot k_2 \cdot n + c \cdot k_2 + d \cdot k_1 \cdots\cdots (1)$

Because assumption:

By multiple of $k_1 \cdot n = a - c$ and $k_2 \cdot n = b - d$:

$a \cdot b = c \cdot d + k_1 \cdot k_2 \cdot n^2 + c \cdot k_2 \cdot n + d \cdot k_1 \cdot n$

$a \cdot b - c \cdot d = n \cdot (k_1 \cdot k_2 \cdot n + c \cdot k_2 + d \cdot k_1)$

$a \cdot b - c \cdot d = n \cdot k_3 \quad \#(1)$

SO: $a \cdot b \equiv c \cdot d \pmod{n}$

■

(c).

To find the unit digit of a big power number, only need to focus on the one's column, which is 7

$7 \cdot 7 = 49$

$9 \cdot 9 = 81$

$1 \cdot 1 = 1$

$1 \cdot 1 = 1$

$\vdots$

$\vdots$

$1 \cdot 1 = 1$

SO: the unit digit of 257^{256} is 1

4.

(a).

Proof: Let $x = 33$

WTS: $x \equiv 3 \pmod 5) \wedge x \equiv 5 \pmod 7$

i.e., $(\exists k_1 \in \mathbb{Z}, k_1 \cdot 5 = x - 3) \wedge (\exists k_2 \in \mathbb{Z}, k_2 \cdot 7 = x - 5)$

Let $k_1 = 6, k_2 = 4$

$$k_1 \cdot 5 = x - 3$$
$$6 \cdot 5 = 33 - 3$$
$$30 = 30$$

$$k_2 \cdot 7 = x - 5$$
$$4 \cdot 7 = 33 - 5$$
$$28 = 28$$

SO: $x \equiv 3 \pmod 5 \wedge x \equiv 5 \pmod 7$

■

(b).

Proof: Let $m_1 = -3$ $m_2 = 2$

WTS: $(m_1 \cdot 7) + (m_2 \cdot 11) = 1$

$$(m_1 \cdot 7) + (m_2 \cdot 11) = -3 \cdot 7 + 2 \cdot 11$$
$$(m_1 \cdot 7) + (m_2 \cdot 11) = -22 + 21$$
$$(m_1 \cdot 7) + (m_2 \cdot 11) = 1$$

■

(c).

Proof: Let $m_1, m_2, a_1, a_2 \in \mathbb{Z}$ and $7 \cdot m_1 + 11 \cdot m_2 = 1$,

WTS: $7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 \equiv a_2 \pmod{11}$

i.e., $\exists k \in \mathbb{Z}$, $11 \cdot k = 7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 - a_2$

Let $k = -a_2 \cdot m_2 + a_1 \cdot m_2$ …… ①

Because: $7 \cdot m_1 + 11 \cdot m_2 = 1$

$$m_1 = \frac{1 - 11 \cdot m2}{7}$$

$$7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 - a_2 = 7 \cdot \frac{1 - 11 \cdot m2}{7} \cdot a_2 + 11 \cdot m_2 \cdot a_1 - a_2$$

$$7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 - a_2 = a_2 - 11 \cdot a_2 \cdot m_2 + 11 \cdot a_1 \cdot m_2 - a_2$$

$$7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 - a_2 = 11 \cdot (-a_2 \cdot m_2 + a_1 \cdot m_2)$$

$$7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 - a_2 = 11 \cdot k \qquad \#①$$

SO: $7 \cdot m_1 \cdot a_2 + 11 \cdot m_2 \cdot a_1 \equiv a_2 \pmod{11}$

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