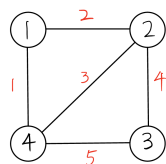


1 a

False

Consider the graph with vertices and weights of edges below. Then MST of G has path connecting vertices 4,1,2,3 in order, with $w(T_1) = 7$. Let T_3 be the spanning tree connecting 2,1,4,3 in order; T_4 be the spanning tree connecting 1,4,2,3 in order. Then $w(T_3) = w(T_4) = 8 = w(T_1) + 1$. But $T_3 \neq T_4$. Thus, this counterexample shows the statement is false.



2 b

True

Definition: A minimum spanning tree is an undirected, connected, acyclic graph $G(V,E)$ that connects all the vertices together, with minimum total edge weights. Moreover, if the edge weights are all positive, it is suffice to define the MST as the subgraph with minimal total weight that connects all the vertices. The statement is the same as this definition. So it is true.

3 c

False

Suppose graph $G=(V,E)$ is a connected graph with all negative edge weights. Consider a spanning subgraph G' with all vertices V and edges $E' \subseteq E$. To reach the minimum weights, G' contains all the negative edges. But the number of edges of a tree is strictly $|V| - 1$, which can be different from $|E|$. Thus, G' is not necessarily a MST in this case. Thus, the statement is false.

Consider graph G shown below as a counterexample. Then $G' = G$ (i.e. G' contains all the edges) to minimize its weight. Clearly, G' is not a MST since it's cyclic and have excess edges (i.e. not $|V| - 1$).

