

Homework Assignment #2

Due: January 30, 2020, by 5:30 pm

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Question 1

We can use Max Binary Heap to solve this problem.

We need four classes to implement this.

Class Node, Class Tree, Class Forest, Class Operation

Class Node:

\\ represent an order

Instance:

1. Incentive (integer) \\ how much cent customer put in this order
2. Order information \\ doesn' t affect our implement
3. Parent (node) \\ the parent of this node, maybe null
4. LeftChild (node) \\ left child of this node, maybe null
5. RightSibling (node) \\ right child of this node, maybe null

Class Tree:

\\ a binomial tree, in the which each node' s incentive is bigger or equal to its children' s

Instance:

1. Root (node) \\ the root of the tree.
2. Height (integer) \\ the height of the tree

Class Forest:

\\ the forest of some binomial trees

Instance:

1. Trees (List) \\ a list contain all binomial trees in the Forest, in increasing order of height
2. Max (integer) \\ the max incentive in this forest.

Class Operation:

\\ all methods to manipulate the data structure

Max_Incentive (f: Forest):

\\ return the max incentive of the forest

Return f.Max

Union_Two_forests (f1: Forest, f2: Forest):

\\ put two forest together, when 1 warehouse is closed

f_new = new Forest

f.Max = Max(f1.Max, f2.Max)

list temp = new list

Tree carry = new Tree

int i = 0

int j = 0

while(i < f1.Trees.lenght && j < f2.Trees.lenght)

if f1.Trees[i].height < f2.Trees[j].height

if carry = null

f_new.Trees.add(f1.Trees[i])

i ++

continue

else

Tree new_carry = new Tree

if carry.root.incentive >= f1.Trees[i].root.incentive:

f1.Trees[i].root.parent = carry.root

f1.Trees[i].root.sibling = carry.root.leftchild

new_carry.height = carry.height + 1

carry = new_carry

i ++

continue

else

carry.root.parent = f1.Trees[i].root

carry.root.sibling = f1.Trees[i].root.leftchild

new_carray.height = carry.height + 1

carry = new_carry

i++

continue

if f1.Trees[i].height > f2.Trees[j].height

if carry = null

f_new.Trees.add(f2.Trees[j])

j++

continue

else

Tree new_carry = new Tree

if carry.root.incentive >= f2.Trees[j].root.incentive:

f2.Trees[j].root.parent = carry.root

f2.Trees[j].root.sibling = carry.root.leftchild

new_carray.height = carry.height + 1

carry = new_carry

j++

continue

else

carry.root.parent = f2.Trees[j].root

carry.root.sibling = f2.Trees[j].root.leftchild

new_carray.height = carry.height + 1

carry = new_carray

j++

continue

if f1.Trees[i].height = f2.Trees[j].height

if carry = null

Tree new_carray = new Tree

iff1.Trees[i].root.incentive >= f2.Trees[j].root.incentive:

f2.Trees[j].root.parent = f1.Trees[i].root

f2.Trees[j].root.sibling = f1.Trees[i].leftchild

new_carray.height = f1.Trees[i].height + 1

new_carray.root = f1.Trees[i].root

carry = new_carray

i++

j++

continue

else

f1.Trees[i].root.parent = f2.Trees[j].root

f1.Trees[i].root.sibling = f2.Trees[j].leftchild

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        new_carray.height = f1.Trees[i].height + 1

        new_carray.root = f2.Trees[j].root

        carry = new_carray

        i++

        j++

        continue

    else

        f_new.Trees.add(carry)

        carry = merged of f1.Trees[i] and f2.Trees[j]. as we
did before

        i++

        j++

    return f_new

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Union_Insertion (f1: Forest, n: Node):

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    new_f = new Forest

    new_t = new Tree

    new_t.root = n

    new_t.height = 0

    new_f.Max = n.incentive

    new_f.Trees.add(new_t)

    return Union_Two_forests (f1, new_f)

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ExtractMax (f: Forest):

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T = new Tree

for(Tree t in f.Trees):

    if t.root.incentive = f.Max

        T = f.Trees.pop(t)

List new_Trees = []

new_Trees.add(T.root.left child)

curr = T.root.left child

while (curr.sibling exist)

    new_Trees.add(curr.sibling)

    curr = curr.sibling

new_f = new Forest

new_f.Trees = new_Trees

new_f.Max = Max(t.incentive in new_Trees)

f = Union_Two_forests (f, new_f)
```

WC MaxIncentive is $O(1)$ because we only need to get the instance Max of the Forest

WC Union is $O(\log n)$ because each of forest have $O(\log n)$ Bk trees. We only do comparison among these trees.

WC insert is $O(\log n)$ because we use Union method to do insert, we consider the new node as a forest with one B0 tree

WC ExtractMax is $O(\log n)$ because we compare the root of each tree to find the one with highest incentive and pop it, this takes $\log n$ time, since there is at most $\log n$ trees. Then we generate new forest with the tree we popped, this takes $\log n_1$ time (n_1 is the # nodes of the tree we popped, $n_1 < n$). Then we merge the origin tree with the new one, which takes $\log(n - n_1)$ times. In total, it's $O(\log n)$

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