1 a

Use adjacency list to represent this weighted undirected graph $G = (V, E)$.

V part: The vertices in G are the integers representing the navigable lakes from list L. Construct adjacency list for each vertex.

E part: Consider the vertex u, check every element (v,x) in P[u], where v is the vertex connected to u and x is the length between u and v. Add all the tuples into the adjacency list of u. Repeat this step for all vertices to contain the information of all edges and weights. In this way, we represent G using adjacency list.

2 b

Assume $G = (V, E)$ contains a cycle, and e=(u,v) is the max weight edge. Consider $G' = (V, E - e)$, to seek contradiction, assume the ratings of G and G' are different,

Then for any 2 lakes (i,j) in this cycle, there exists at least 2 path with distinct toughness between them. Then there exists at least 1 cycle between i,j. Note that one path contains the max weight edge.

Since G and G' differ by the path with edge e by assumption, then this path is chosen in G (otherwise, the ratings of G and G' will be the same). Since e has the max weight and weights are distinct, then this path has the largest toughness. Then there exists another path with smaller toughness (recall there are at least 2 paths due to the cycle). That is, another path with smaller toughness should be chosen for rating evaluation in both G and G'. So we reach a contradiction.

Therefore, the ratings of all pairs of lakes is the same in G and G'.

3 c

In the case there is only 1 path for lakes (i,j). Then the MST T is the same as graph G. Then T and G have the same rating.

Consider the case there are at least 2 paths from i to j. That is, there is at least 1 cycle. To seek contradiction, assume T and G have different ratings for 2 lakes (i,j).

Consider e=(u,v) as the max weight edge. Then T does not contain this edge to minimize its total weight. That is, $T = G(V, G - e)$. Then according to part (b), rating of T = rating of G.

The above theory shows, for each cycle, we cut the path with max weight edge and remain the entire path with smaller toughness. In the case of many paths and many cycles, apply this theory recursively to every 2 paths, and cut the max weight edge on paths until one full path left. Then we get the MST T.

In this case, the rating of T is also the minimum toughness of all paths in the original graph G. By definition, this is the rating of G. Therefore, the rating of T is also the rating of G.