

2. space: we get  $n$  nodes, so, it's  $n$  tree

for each edge  $(i,j) \in G.E$ , there is one node  $i$  in tree rooted by  $j$  and a node  $j$  in tree rooted by  $i$ . So in total,  $n$  root nodes +  $2m$  non-root nodes

Space complexity is  $\Theta(n+2m)$

Time:

$\text{Degree}(G,i)$ : go to tree rooted by  $i$

find # non-root nodes.

do this by BFS starting from root, whenever explore a new node, counter +1.

$\Theta(n)$  for worst case

$\text{AvgDegree}(G)$ : do  $\text{Degree}(G,i)$  for all  $n$  vertices  
get total degree,

for worst case, assume any two  $i,j, i \neq j$   
there is  $(i,j) \in G.E$

So, each tree has  $n-1$  non-root nodes,  
it's  $\Theta(n^2)$

ContainsEdge( $i, j$ ) :

find tree rooted by  $i$ ,  $\Theta(1)$

find  $j$  in tree, by AVL search

$\Theta(\text{height of tree})$

in worst case,  $i$  is incident to all other nodes, so, is  $\Theta(\log_2 n)$

time :  $\Theta(\log_2 n)$

Insert Edge( $G, i, j$ ) :

find tree rooted by  $i$ , insert  $j$ .

by avl insertion

$\Theta(\text{height of tree})$

in worst case,  $i$  is incident to all other nodes, so, is  $\Theta(\log_2 n)$

do the same for tree rooted by  $j$

time :  $\Theta(\log_2 n)$

L: time

Degree( $G, i$ ): find Linked List with head  $i$ ;  
count node in Linked List  
one by one.

In worst case, all nodes are incident  
with  $i$ . So, it's  $\Theta(n)$

AvgDegree( $G$ ): count length of a linked list, one by one.  
for worst case, assume any two  $i, j, i \neq j$   
there is  $(i, j) \in G.E$ , so, count  $n(n-1)$  times  
return  $\frac{n(n-1)}{n}$ ,  $\Theta(n^2)$

ContainsEdge( $G, i, j$ ): find Linked List with head  $i$   
find  $j$  in that Linked List

In worst case, all nodes are incident  
with  $i$ . So, it's  $\Theta(n)$

InsertEdge( $G, i, j$ ): find Linked List with head  $i$   
insert  $j$  in that Linked List  
insert in front (right after head)  
 $\Theta(1)$

Compare

$L$

$L'$

Degree

$\Theta(n)$

$\Theta(n)$

Avg Degree

$\Theta(n)$

$\Theta(n^2)$

Contains Edge

$\Theta(n)$

$\Theta(\log_2 n)$

Insert Edge

$\Theta(1)$

$\Theta(\log_2 n)$