

Homework Assignment #4

Due: Feb 27, 2020 5:30 PM

Written by Jiabao Shen and Jiahong Zhai

Question 1

a. Algorithm

We use forest structure for disjoint set(union/find) with weighted union and path compression.

Step 1: Create two new empty list LS and LD. Loop through L, if encounter S, then put it into LS, if it's D, then put it into LD.

Step 2: Create a new list LF. For 1 to n, make it a disjoint set node and put it into LF.

Step 3: Loop through LS. For each S(i, j) we do the following: $a = \text{find}(i)$, $b = \text{find}(j)$. If $a \neq b$, then $\text{union}(a, b)$

Step 4: Loop through LD. For each D(i, j) we do the following: $a = \text{find}(i)$, $b = \text{find}(j)$. If $a = b$, then return "ERROR FOUND" and terminate the program.

Step 5: return $k = \text{the size of LF}$

b. Worst-Case Time Complexity

Step 1: Loop through a list with m elements, operate constant steps for each element. So, it's $O(m)$

Step 2: From 1 to n, operate constant steps for each number. So, it's $O(n)$

Step 3 and 4: Since there are n disjoint set nodes at the beginning, so, we have at most n-1 union operations. For each S(i, j) and D(i, j) it takes 2 find operations, so, in total, there are $2m$ find operations. So it's $O(2m \log^* n)$

Step 5: For worst case analyze, we assume all bones are from different species. So, we need to count n elements, it's $O(n)$

$O(m) + O(n) + O(2m \log^* n) + O(n) = O(m \log^* n)$, its better than $O(mn)$