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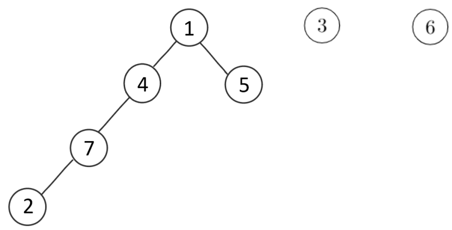
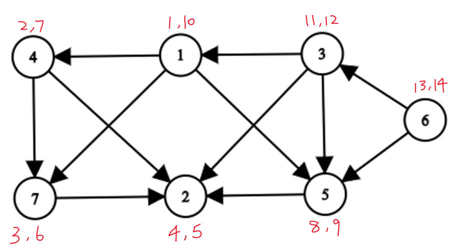
1 a

The Depth-First Search forest of G is shown below:



The graph with all vertices labelled with discovery and finishing time is shown below



2 b

back edges: 0; forward edges: 2; cross edges: 6.
The forward edges are (1,7),(4,2); The cross edges are (3,2),(3,1),(3,5),(5,2),(6,3),(6,5).

3 c

According to white path theorem, for all graphs G and all DFS of G: v becomes a descendant of u if and only if at the time d[u] the DFS discovers u, there is a path from u to v in G that consists entirely of white nodes.
If the node u has a back edge (u,v), then the forward path from its ancester v to itself, and this back edge form a cycle. Also, if there is a cycle, then there exists an edge, say (u,v), connecting 2 nodes which are explored (non-white) before connection. Then v is not the descendent of u (i.e. v is ancestor of u). Therefore, for all directed graphs G and all DFS of G, G has a cycle if and only

if DFS of G has a back edge.

According to part (b), there is no back edge. So there is no cycle in the graph. Then it is possible to do topological sort. Therefore, The dependencies of the courses are correct. That is, every course is not a prerequisite course for any of its prerequisites (i.e. no conflict). Thus, it is possible to take all the courses in a sequential order such that it satisfies all the prerequisite requirements without conflict.

4 d

Method: According to part (a), take courses from largest to smallest finish time. So the order is: 6,3,1,5,4,7,2

5 e

The BFS tree of G starting from node 6 is shown below: