

## Homework Assignment #4

Due: Feb 27, 2020 5:30 PM

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### Question 1

a.

$$1 - \left(\frac{k}{m}\right)^2$$

By SUHA, probability of  $x$  been assigned to any slot by  $h_1$  and  $h_2$  is  $1/m$ . There are  $m-k$  empty slots, so, the probability of  $x$  assigned to an empty slot by  $h_1$  and  $h_2$  is  $\frac{1}{m} \cdot (m-k)$ , to a non-empty slot is  $\frac{1}{m} \cdot k$ . If  $x$  is inserted to an empty slot, we need at least one of  $h_1$  and  $h_2$  assign  $x$  to an empty slot. So, the probability is  $1 - \left(\frac{k}{m}\right)^2$

b.

$$\frac{29}{8}, 3.625$$

By SUHA, probability of  $x$  been assigned to any slot by  $h_1$  and  $h_2$  is  $\frac{1}{4}$ . The probability of any combination of  $T[h_1(x)]$  and  $T[h_2(x)]$  is the same,  $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ . The final slot  $x$  goes to is shown below.

$h_2 \backslash h_1$	T0	T1	T2	T3
T0	T0	T1	T0	T0
T1	T1	T1	T1	T1
T2	T0	T1	T2	T3
T3	T0	T1	T2	T3

$$\text{Prob}(T0) = \frac{5}{16}, \text{length} = 4$$

$$\text{Prob}(T1) = \frac{7}{16}, \text{length} = 2$$

$$\text{Prob}(T2) = \frac{2}{16}, \text{length} = 6$$

$$\text{Prob}(T3) = \frac{2}{16}, \text{length} = 6$$

$$E[\text{length}] = \left(\frac{5}{16}\right) \cdot 4 + \left(\frac{7}{16}\right) \cdot 2 + \left(\frac{2}{16}\right) \cdot 6 + \left(\frac{2}{16}\right) \cdot 6 = 29/8 = 3.625$$