ELE 364: Assignment #5 Solutions

1. (10 pts)

   (a) (2 pts) Let the inputs be a and b. Then $w_{AND,a} = w_{AND,b} = 1$ and the bias $w_{AND,0} = 0$. The perceptron outputs a 1 if $z \geq 1.5$, 0 otherwise.

   (b) (2 pts) Let the inputs be a and b. Then $w_{OR,a} = w_{OR,b} = 1$ and the bias $w_{OR,0} = 0$. The perceptron outputs a 1 if $z \geq 0.5$, 0 otherwise.

   (c) (6 pts) The three perceptrons have weights: (i) $w_{AND,a} = w_{AND,b} = w_{AND,c} = 1$, and the bias $w_{AND,0} = 0$, with an output of 1 if $z \geq 2.5$, 0 otherwise, (ii) $w_{AND,d} = w_{AND,e} = 1$, and the bias $w_{AND,0} = 0$, with an output of 1 if $z \geq 1.5$, 0 otherwise, (iii) $w_{OR,abc} = w_{OR,de} = 1$ and the bias $w_{OR,0} = 0$, with an output of 1 if $z \geq 0.5$, 0 otherwise.

2. (10 pts) $tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$

   $\frac{d(tanh(z))}{dz} = \frac{(e^z+e^{-z})(e^z+e^{-z})-(e^z-e^{-z})(e^z-e^{-z})}{(e^z+e^{-z})^2}$

   $= 1 - \frac{(e^z-e^{-z})^2}{(e^z+e^{-z})^2} = 1 - (tanh(z))^2$

   $logistic(2z) = \frac{1}{1+e^{-2z}} = \frac{e^z}{e^z+e^{-z}}$

   $2logistic(2z) - 1 = tanh(z)$

   $\frac{d(tanh(z))}{dz} = 1 - (2logistic(2z) - 1)^2 = 4logistic(2z)(1 - logistic(2z))$

   $\frac{d(logistic(2z))}{dz} = \frac{2e^{-2z}}{(1+e^{-2z})^2}$

   $1 - logistic(2z) = \frac{e^{-2z}}{1+e^{-2z}} \Rightarrow logistic(2z)(1 - logistic(2z)) = \frac{e^{-2z}}{(1+e^{-2z})^2}$

   $\frac{d(logistic(2z))}{dz} = 2logistic(2z)(1 - logistic(2z))$

   Hence, $tanh'(z) = 2logistic'(2z)$.

3. (10 pts) $\frac{\partial E}{\partial a_4} = -(t - a_4)$

   For the given data instance, $t = 1$

   $z_3 = w_{3,1} \times 1 + w_{3,2} \times 0 + w_{3,0} = 0.1 + 0.1 + 0.2$

   $a_3 = logistic(z_3) = \frac{1}{1+e^{-0.2}} = 0.5498$

   $z_4 = w_{4,3} \times a_3 + w_{4,0} = 0.1 \times 0.5498 + 0.1 = 0.1550$

   $\delta_4 = \frac{\partial E}{\partial a_4}\frac{\partial a_4}{\partial z_4} = 1 \times (-(t - a_4)) = -(t - z_4) = 0.1550 - 1 = -0.8450$

   Note: Neuron 4 has a linear activation. Hence, $z_4 = a_4$.

   $\delta_3 = \frac{\partial E}{\partial a_3}\frac{\partial a_3}{\partial z_3}$

   $\frac{\partial E}{\partial a_3} = \delta_4 \times w_{4,3} = -0.8450 \times 0.1 = -0.0845$

   $\frac{\partial a_3}{\partial z_3} = logistic(z_3)(1 - logistic(z_3)) = 0.2475$

   $\Rightarrow \delta_3 = -0.0845 \times 0.2475 = -0.0209$

   Weight updates:

$w_{3,1}^{(1)} = w_{3,1}^{(0)} - \alpha\delta_3 a_1 = 0.1 + 0.3 \times 0.0209 \times 1 = 0.1063$

$w_{3,2}^{(1)} = w_{3,2}^{(0)} - \alpha\delta_3 a_2 = 0.1 + 0.3 \times 0.0209 \times 0 = 0.1$

$w_{3,0}^{(1)} = w_{3,0}^{(0)} - \alpha\delta_3 a_0 = 0.1 + 0.3 \times 0.0209 \times 1 = 0.1063$

$w_{4,3}^{(1)} = w_{4,3}^{(0)} - \alpha\delta_4 a_3 = 0.1 + 0.3 \times 0.8450 \times 0.5498 = 0.2394$

$w_{4,0}^{(1)} = w_{4,0}^{(0)} - \alpha\delta_4 a_0 = 0.1 + 0.3 \times 0.8450 \times 1 = 0.3535$

4. (10 pts)

   $a' = a \odot DropMax = (0, logistic(-0.30), logistic(0.52), 0).$

   $a'' = \frac{a'}{0.4} = (0, 1.0639, 1.5679, 0).$