Inecuaciones

$3x + 7 = 1$ Sol. finitas

$3x + 7 > 1$ Sol. infinitas

$<$ $>$

$\geqslant$ $\leqslant$

$>$ $\geqslant$

$<$ $<$

Abierto] [

Cerado [ ]

nota1: el $+\infty$ y $-\infty$ siempre son abiertas

nota 2: Cuando multiplico un valor negativo cambia de sentido

$$-2 > -5$$

Nota 3: El denominador siempre es abierto

$$\frac{a}{b} \geqslant c$$

Nota 4: llevar todo al miembro izquierco como suma y resta

Inecuaciones Lineales

(1) $3x + 8 \geqslant 2x - \sqrt{2}$

Solución:

1er llevar todo al miembro izquierdo $\quad 3x + 8 - 2x + \sqrt{2} \geq 0$

2do Factorizar $\quad x + 8 + \sqrt{2} \geq 0$

3er Puntos críticos $\quad x + 8 + \sqrt{2} = 0 \Rightarrow x = -8 - \sqrt{2}$

4to

5to Cs $\quad Cs: x \in [-8 - \sqrt{2}, +\infty[$

Nota: Se lee de izquierda a derecha la solución

(2) $-5 + x < 3x + 1 < 7 + x \qquad a < b < c$

$-5 + x < 3x + 1 \qquad\qquad 3x + 1 < 7 + x$

$-5 - 3x + x - 1 < 0 \qquad\qquad 3x - x + 1 - 7 < 0$

$-2x - 6 < 0 \quad \cdot -1 \qquad\qquad 2x - 6 < 0$

$2x + 6 > 0 \qquad\qquad 2x - 6 = 0$

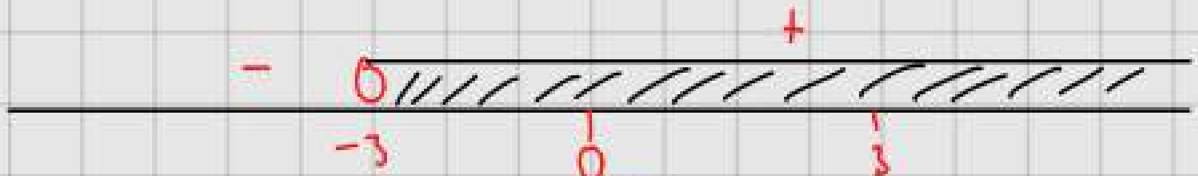
$2x + 6 = 0 \qquad\qquad x = \frac{6}{2}$

$2x = -6$

$x = -\frac{6}{2} \qquad\qquad x = 3$

$x < 3$

$x = -3$



$Cs:]-3, +\infty[$ α

El coeficiente de x no puede ser negativo

○ Solución

$a < b < c$

$a < b \wedge b < c$

$-5 + x < 3x + 1$

$-2x - 6 < 0 \quad // \cdot (-1)$

$2x + 6 > 0$

$2(x+3) > 0$

Puntos críticos

$x + 3 = 0 \Rightarrow x = -3$

③ $3x + 1 < 7 + x$

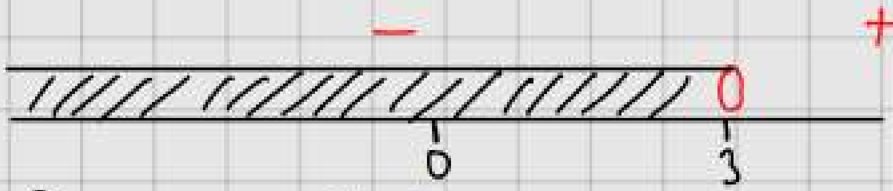
$3x + 1 - 7 - x < 0$

$2x - 6 < 0$

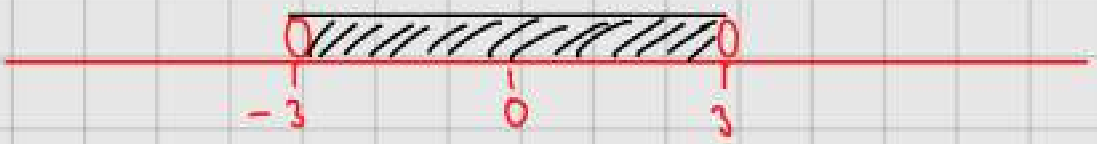
$2(x-3) < 0$

$x - 3 = 0$

$x = 3$



$CS:]-\infty, 3]$ β



$CS: [-3, 3]$

Inecuaciones cuadraticas

$$x^2 + x - 72 \geq 0$$

$x \quad -8 \quad -8x$

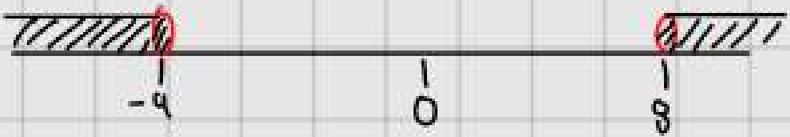
$x \quad 9 \quad 9x$

$\qquad\qquad\quad x$

$$(x-8)(x+9)=0$$

$x - 8 = 0 \qquad x + 9 = 0$

$x = 8 \qquad\quad x = -9$



$$x \in : \,]-\infty, 9] \cup [8, +\infty[$$

④ $\overset{1}{\cancel{(x^2+3)}}(x-1)(x+2) < 0$

Solución:

$$1 \cdot (x-1)(x+2) < 0$$

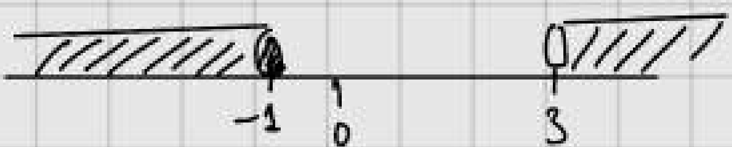
$$(x^3 + \sqrt[3]{3}^3)(x-2) \leq 0$$

⑤ $\dfrac{x+1}{x-3} > 0$

Puntos criticos

numerador

$x + 1 > 0 \qquad x = -1$



denominador

$x - 3 > 0 \qquad x = 3 \qquad Cs = x \in \,]-\infty, -1] \cap [3, +\infty[$

Ejercicio de clase

$$\begin{cases} \dfrac{1-2x}{x+3} \leq -1 \\ \dfrac{x}{3} - 4 \leq \dfrac{x}{4} - 3 \end{cases}$$

$$\frac{x}{3} - 4 - \left(\frac{x}{4} - 3\right) \leq 0$$

$$\frac{x}{3} - 4 - \frac{x}{4} + 3 \leq 0$$

$$\frac{x-12}{3} - \frac{x+3}{4} \quad \frac{(4x-48)-(3x+9)}{12}$$

$$\frac{4x-48-3x-9}{12}$$

$$\frac{x-57}{12} \geq 0$$

$$x - 57 \geq 12$$

$$x \geq 12 + 57$$

$$x \geq 69$$

1

$$1 - 2x \leq -1$$

$$\cancel{1} + 1 - 2x \leq 0$$

$$-2x \leq -2 \quad x(-1)$$

$$2x \geq 2$$

$$x \geq \frac{2}{2} \quad x = 1$$

2

$$x + 3 \leq -1$$

$$x + 3 + 1 \leq 0$$

$$x + 4 \leq 0$$

$$x \leq 4$$

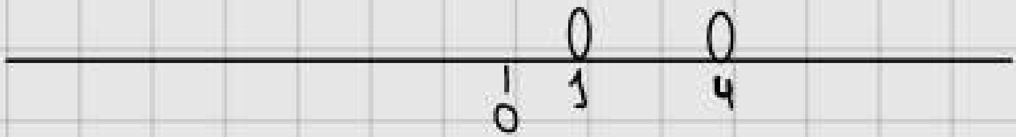
$$x = 4$$

$$\frac{x}{3} - 4 \leq \frac{x}{4} - 3 \quad *(12)$$

$$4x - 48 \leq 3x - 36$$

$$4x - 3x \leq -36 + 48$$

$$x \leq 12$$



$$\frac{(x-2)^{\cancel{2024}\,2023}\,(x-7)^3\,(\cancel{x-6})^{3^{\cancel{2}}}\,(x-3)}{(x-8)\,\cancel{(x-2)}\,\cancel{(x-6)^2}} \geq 0$$

$x-6 \neq 0$
$x \neq 6$

Solución

$$\frac{(x-2)^{2023}\,(x-7)^3\,(x-6)^7\,(x-3)^6}{x-8} \geq 0$$

Pontos criticos

Numerador

$x-2=0 \Rightarrow x=2$

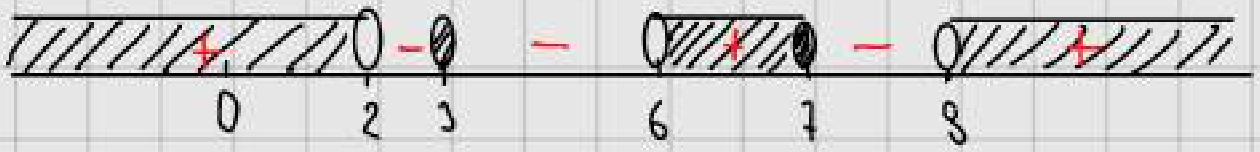
$x-7=0 \Rightarrow x=7$

$x-6=0 \Rightarrow x=6$

$x-3=0 \Rightarrow x=3$

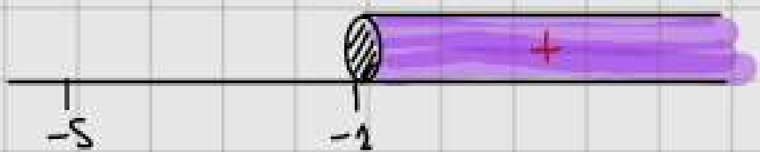
Denominador

$x-8=0 \Rightarrow x=8$



Cs: $x \in \left]-\infty, 2\right[\cup \left]6, 7\right] \cup \left]8, +\infty\right[\cup \{3\}$

$(x+1)^2 \geq 0$



$m_1, m_2, m_3, \ldots, m_n > 0$

$MH \leq MG \leq MA \leq MC$

$$MH = \frac{n}{\frac{1}{m_1} + \frac{1}{m_2} + \cdots + \frac{1}{m_n}}$$

Medioarmonica

$$MA = \frac{m_1 + m_2 + \cdots + m_n}{n}$$

Media aritmetica

$$MG = \sqrt[n]{m_1 \cdot m_2 \cdot m_3 \cdot \ldots \cdot m_n}$$

Medio geometrico

$$MC = \sqrt{\frac{m_1^2 + m_2^2 + \cdots + m_n^2}{n}}$$

Media cuadratica

$$\begin{array}{l} a > b \\ * \; c > d \\ \hline a * c > b * d \end{array} \qquad \begin{array}{l} a > b \\ \div \; c < d \\ \hline a \div c > b \div d \end{array} \qquad \begin{array}{l} 2 > 1 \\ 3 > 2 \\ \hline 2+3 > 1 2 \end{array}$$

29 $x, y, z > 0$ entonces demostrar

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq \frac{2}{x+y} + \frac{2}{y+z} + \frac{2}{x+z}$$

Demostracion

- $x, y > 0$ $\quad MH = \dfrac{2}{\frac{1}{x} + \frac{1}{y}}$
- $y, z > 0$ $\quad MH = \dfrac{2}{\frac{1}{y} + \frac{1}{z}}$
- $x, z > 0$ $\quad MH = \dfrac{2}{\frac{1}{x} \cdot \frac{1}{z}}$
- $x, y > 0$ $\quad MA = \dfrac{x+y}{2}$
- $y, z > 0$ $\quad MA = \dfrac{y+z}{2}$
- $x, z > 0$ $\quad MA = \dfrac{x+z}{2}$

$$MH \leq MA$$

$$\circ \quad \frac{2}{\frac{1}{x}+\frac{1}{y}} \leq \frac{x+y}{2} \quad // \cdot \frac{\frac{1}{x}+\frac{1}{y}}{x+y}$$

$$\frac{2}{x+y} \leq \frac{\frac{1}{x}+\frac{1}{y}}{2} \quad (\alpha)$$

$$\circ \quad \frac{2}{y+z} \leq \frac{\frac{1}{y}+\frac{1}{z}}{2} \quad (\beta)$$

$$\circ \quad \frac{2}{x+z} \leq \frac{\frac{1}{x}+\frac{1}{z}}{2} \quad (\gamma)$$

Demostración

$$(\alpha)+(\beta)+(\gamma)$$

$$\frac{2}{x+y}+\frac{2}{y+z}+\frac{2}{x+z} \leq \frac{1}{2}\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{y}+\frac{1}{z}+\frac{1}{x}+\frac{1}{z}\right]$$

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Paralelo: "C"

$$\frac{x(x-2)^{\cancel{8}}(x-7)^{8}(x-6)^{\cancel{2023}}(2-x)}{\cancel{(x-2)}(x-9)\cancel{(x-6)^{2}}} \geq 0$$

$$\frac{x(x-2)^{4}(x-7)^{8}(x-6)^{2021}(2-x)}{(x-9)} \geq 0$$

$x-2 \neq 0$
$x \neq 2$

Puntos criticos

numerador

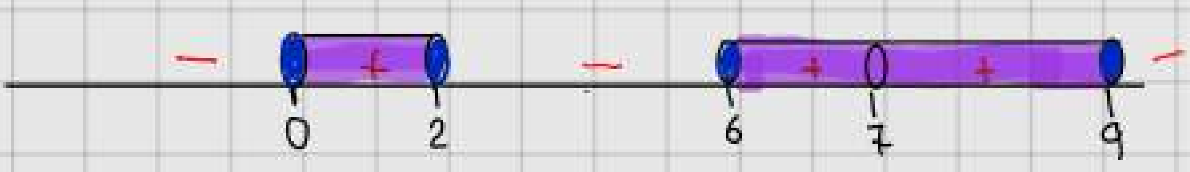
- $x-2=0 \Rightarrow x=2$ ★
- $x-7=0 \Rightarrow x=7$ ★
- $x-6=0 \Rightarrow x=6$ ★
- $2-x=0 \Rightarrow \boxed{-x=-2}$ // $*-1$
  $\downarrow$
  $x=2$ ★
- $x=0$ ★

Denominador

- $x-9=0 \Rightarrow x=9$ ★

★ Impares
★ Pares

Grafico



Cs : $x \in [0,2\rangle \cup \langle 6,9\rangle$

Ej

Si a^2 es imar, entonces a es impar

Prueba:

$$\forall \mathbb{N} \in (2a+1)^2 = 2a+1$$

$$(4-3)(4-2)(4-3)(4-7) \geq 0$$

$$1 \quad 2 \quad 1 \quad -3$$

$$-6 \geq 0$$

F

$$(8-3)(8-2)(8-3)(8-7) \geq 0$$

$$5 \quad 6 \quad 5 \quad 1$$

Ej: Demostrar

$$\underbrace{ab=0}_{p} \text{ si solo si } \underbrace{a=0}_{q} \text{ ó } \underbrace{b=0}_{r}$$

Pruebas i)

Ejercicio

$$\frac{(x-1)^6 (x-7)^8 (x-9)^{25} (2-x)}{(x-10)(x-1)^3 (x-9)^3 (x^3+1)} \geq 0$$

$$\frac{(x-1)^3 \cdot (x-7)^8 \cdot (x-9)^{22} \cdot (-1) \cdot (x-2)}{(x-10)(x+1)\cancel{(x^2-x+1)} \to 1} \geq 0 \quad // \cdot (-1)$$

$$\frac{(x-1)^3 (x-7)^8 (x-9)^{22} (x-2)}{(x-10)(x+1)} \leq 0$$

Puntos criticos

numerador

$x-1=0 \Rightarrow x=1$

$x-7=0 \Rightarrow x=7$

$x-9=0 \Rightarrow x=9$

$x-2=0 \Rightarrow x=2$

Denominador

$x-10=0 \Rightarrow x=10$

$x+1=0 \Rightarrow x=-1$

Grafico

$Cs:]-1,1[\cup [2,10[- \{9\}$

$Cs:]-1,1[\cup [2,9[\cup]9,10[$

Series

$f(x) = x^2 + x^{100} + x^{20} + 100$

Dom : $\mathbb{R}$

$g(x) = \dfrac{x+1+x^2-x^3}{x-2}$

$\dfrac{k(x)}{h(x)}$

$h(x) \neq 0$

$x-2 \neq 0$

$x \neq 2$

Dom : $\mathbb{R} - \{2\}$

$R(x) = \sqrt{2x - x^2 + 1}$

$2x - x^2 + 1 \geq 0$

$-x^2 + 2x + 1 \geq 0 \quad // \cdot (-1)$

$x^2 - 2x + 1 - 1 - 1 \leq 0$

$(x-1)^2 - \sqrt{2}^2 \leq 0$

$(x - 1 - \sqrt{2})(x - 1 + \sqrt{2}) \leq 0$

Dom $[1-\sqrt{2}, 1+\sqrt{2}]$

$P(x) = \underbrace{\sqrt[7]{x + 3x^2 - 1}}$

(-)

(+)

Dom. IR 0

$f(x) = \dfrac{x^2 - 7}{\sqrt{x-3}}$

$x - 3 \geq 0$

Dom $]3, \infty[$

$g(x) = \dfrac{(x-3)\sqrt[10]{x-3}}{(x-7)\sqrt{x-8}}$

Numerador

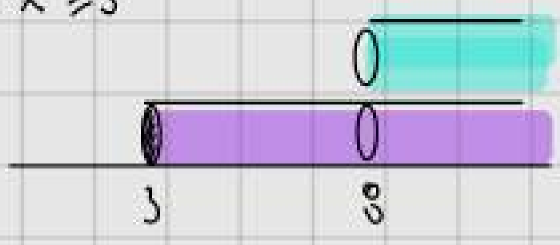
- $x - 3 \geq 0 \Rightarrow x \geq 3$

Denominador

- $x - 7 \neq 0$

  $x \neq 7$

- $x - 8 > 0$



Dom $]8, \infty[$

$R(x) = \dfrac{\sqrt{x^2-1}\,\cancel{(x-6)}^{\mathbb{R}}\,\cancel{\sqrt[3]{x+1}}^{\mathbb{R}}}{(x-8)\sqrt[20]{x^2-2x-24}}$

$x^2 - 1 \geq 0 \wedge x - 8 \neq 0 \wedge x^2 - 2x - 24 > 0$

$R(x) = \sqrt[2n]{f(x)} \quad (n > 1) \; n \in \mathbb{N}^+$

$f(x) \geq 0$

$g(x) = \sqrt[2n+1]{t(x)}$

IR el radical es impar

$R(x) = \dfrac{\sqrt[2n]{W(x)}}{\sqrt[2m]{Z(x)}}$

$W(x) \geq 0$

$\wedge$

$Z(x) > 0$

Dom $[1-\sqrt{2}, 1+\sqrt{2}]$

$P(x) = \underbrace{\sqrt[7]{x + 3x^2 - 1}}$

$x = ?$ (-)

(+)

Dom IR 0

$f_{(x)} = x+1$

$f_{(0)},\ f_{(-3)},\ f_{(100)}$

$f_{(0)} = 1,\ f_{(-3)} = -2,\ f_{(100)} = 101$

Sea $f_{(1)} = 7$, $f_{(x+y)} = f_{(x)} + f_{(y)} + 3$

Determinar $f_{(0)}, f_{(2)}$ y $f_{(4)}$

Solucion $x = 0 \quad y = 1$

$$f_{(0+1)} = f_{(0)} + f_{(1)} + 3$$

$$f_{(0)} = -3$$

$x = 1 \quad y = 1$

$F_{(1+1)}$

Hallar $f_{(x)}$

$$f\Big(\overbrace{\frac{x+e}{x+\pi}}^{x}\Big) = \frac{x^3 + \operatorname{Sen}(x)}{x-7}$$

Solución:

$$\text{C.V.} \quad \frac{x+e}{x+\pi} = m$$

$$x + e = m(x+\pi)$$

$$x - mx = m\pi - e$$

$$\boxed{x = \frac{m\pi - e}{1-m}}$$

Reemplazando

$$f_{(m)} = \frac{\left(\frac{m\pi - e}{1-m}\right)^3 + \operatorname{sen}\left(\frac{m\pi - e}{1-m}\right)}{\frac{m\pi - e}{1-m} - 7}$$

$$f_{(x)} = \frac{\left(\frac{x\pi - e}{1-x}\right)^3 + \operatorname{Sen}\left(\frac{x\pi - e}{1-x}\right)}{\frac{x\pi - e}{1-x} + 7}$$

$f_{(2ux)}$

Hacer Formulario de composición 20 propiedades

Hallar $f^{-1}(x)$

$$f(x) = \frac{x-7}{x-8}$$

$f(x) = y$
$f^{-1}(x) = x$

Solución:

$$y = \frac{x-7}{x-8}$$

$$y(x-8) = x-7$$

$$yx - 8y = x - 7 \Rightarrow yx - x = 8y - 7$$

$$x = \frac{8y-7}{\underbrace{y-1}_{y=x}} \qquad x(y-1) = 8y-7$$

$$\boxed{f^{-1}(x) = \frac{8x-7}{x\ 1}}$$

Si $f(x) = x^2 - 2x - 1$
determinar dos funciones
y tal que $(f \circ g)(x) = x^2 - 3x$

Solución $\boxed{(f \circ h)(x) = f(h(x))}$

$$f(g(x)) = x^2 - 3x \quad // \cdot f^{-1}$$

$$g(x) = f^{-1}(x^2 - 3x)$$

$\boxed{\begin{array}{l} f(x) = y \\ f^{-1}(x) = x \end{array}}$

Con $f(x) = x^2 - 2x - 1$

$$y = x^2 - 2x + 1 - 2$$

$$y + 2 = (x-1)^2 \quad // \sqrt{}$$

$$\sqrt{y+2} = x - 1$$

$$\underbrace{\sqrt{y+2}}_{y=x} + 1 = x$$

$$\sqrt{x+2} + 1 = f^{-1}(x)$$

$$f^{-1}(x^2 - 3x) = \sqrt{x^2 - 3x + 2} + 1$$

$$= g(x)$$

$$f(x) = \frac{x-7}{x-8} \cdot \operatorname{sen} x$$

$$g(x) = \frac{x^3+7}{8}$$

$(f \circ g) = ?$ $\quad (g \circ f)_{(3)} = ?$

$$(f \circ g)(x) = f(g(x)) = \frac{g(x) - 7}{g(x) - 8} \operatorname{sen} g(x)$$

$$(f \circ g)(x) = \frac{\frac{x^3+7}{8} - 7}{\frac{x^3+7}{8} - 8} \operatorname{sen} \frac{x^3+7}{8}$$

$$(f \circ g)_{(3)} = \frac{\frac{3^3+7}{8} - 7}{\frac{3^3+7}{8} - 8} \operatorname{sen}\left(\frac{3^3+7}{8}\right)$$

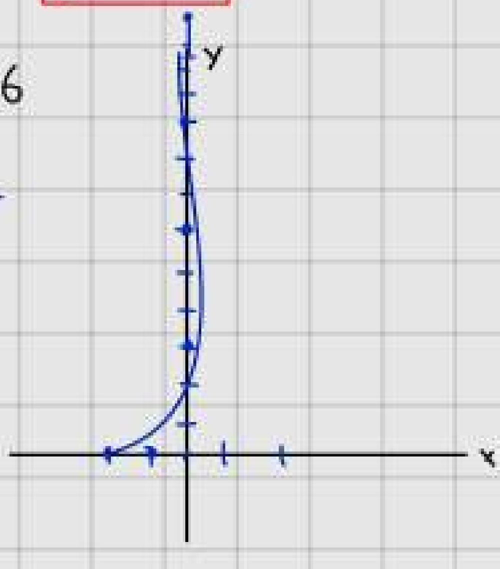
12. Graficar

$$f(x) = 3x + 6$$

Solución: $\boxed{y = f(x)}$

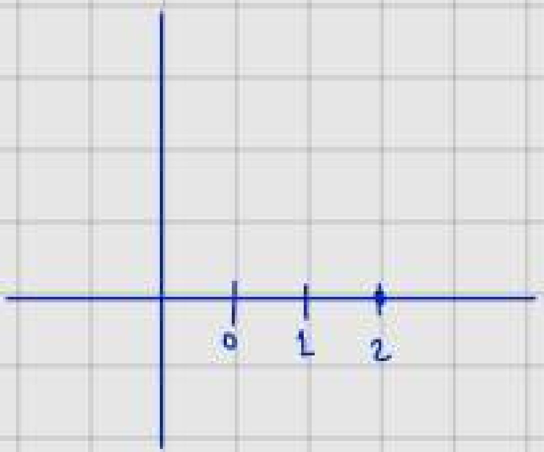
$$y = 3x + 6$$

x	y
0	6
1	9
-1	3
2	12
-2	0



$y = \dfrac{x+4}{x-2}$ ... $x \neq 2$

x	y
3	7
-1	-1
4	4
9	2



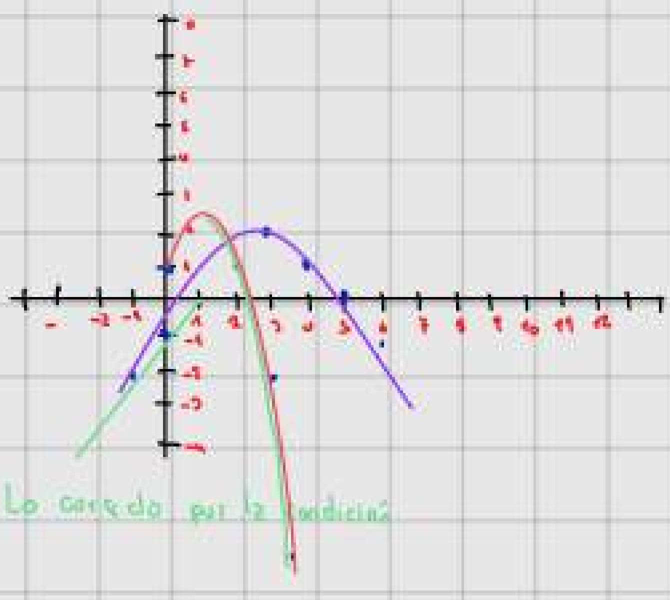
12. Graficar

$$f(x) = \begin{cases} 2-|x-3| \text{ si } -3 < x < 1 \\ 1+2x-x^2 \text{ si } x \geq 1 \end{cases}$$

Solución

$y = 2-|x-3|$

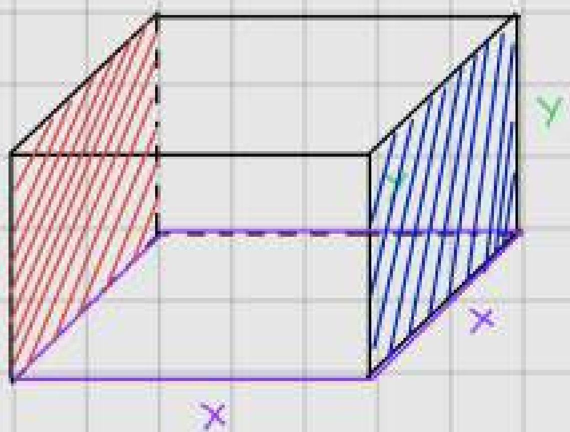
x	y
0	-1
-1	-2
3	2
4	1
5	0
6	-1



$y = 1+2x-x^2$

x	y
0	1
2	1
3	-2
4	-7
5	-14

17.



cte

$$V_{(x,y)} = x \cdot x \cdot y = x^2 y \quad *$$

$$A_B = x^2 = A_T$$

$$A_F = xy = A_{FT} = A_{FI} = A_{FO}$$

$$A_{T(x,y)} = 2x^2 + 4xy \quad \lambda$$

De $*$

$$V = x^2 y$$

$$\frac{V}{x^2} = y \quad \alpha$$

α en λ

$$\boxed{A_{T(\lambda)} = 2x^2 + 4x \cdot \frac{V}{x^2}}$$

$a > 0 \quad b < 0$ demostrar $\frac{b+1}{a} < \frac{1}{a}$

Con $b < 0$

$$b + 1 < 1 \quad // \cdot \frac{1}{a}$$

$$\frac{b+1}{a} < \frac{1}{a}$$

$x, y = 0$

Demostrar $(x+y)(x^{-1} + y^{-1}) \geq 4$

Solución $\quad MH \leq MG \leq MA \leq MC$

$$MA = \frac{x+y}{2} \qquad \frac{2}{x^{-1}+y^{-1}} \leq \frac{x+y}{2} \quad // 2(x^{-1}+y^{-1})$$

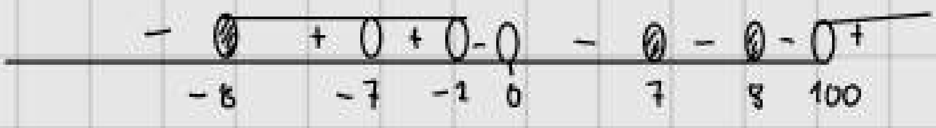
$$MH = \frac{2}{\frac{1}{x}+\frac{1}{y}} \qquad 4 \leq (x+y)(x^{-1}+y^{-1})$$

$$\frac{\cancel{(x+7)^3}(x+8)^5(x-7)^6(x-100)^{-3}}{(x^3+1)\cancel{(x+7)^5}^{2}(8-x)^{-4}(-x^2)}$$

$x \neq -7$

$$\frac{(x+8)^5(x-7)^6(x-6)^4}{(x-100)^3(x+1)(x^2-x+1)(x+7)^2x^2} \geq 0$$

num	Den
$x=-8$	$x=100$
$x=7$ *	$x=-1$
$x=8$ *	$x=-7$ *
	$x=0$ *



$$\frac{x^2}{x-7}+\frac{x}{x-7} \geq \frac{42}{x-7}$$

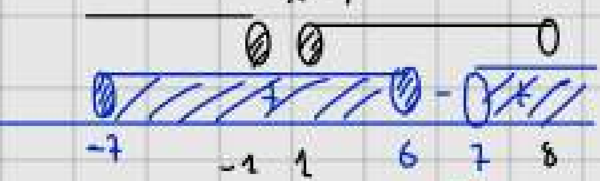
$$\left\{\frac{x^6-1}{x-8} \leq 0\right.$$

Solución

$$\frac{x^2}{x-7}+\frac{x}{x-7}-\frac{42}{7-7} \geq 0$$

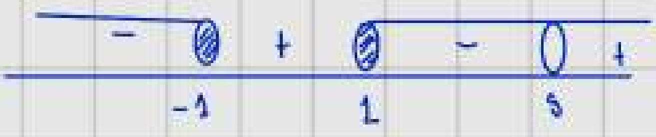
$$\frac{x^2+x-42}{x-7} \geq 0$$

$$\frac{(x+7)(x-6)}{x-7} \geq 0$$



② $$\frac{(x^3+1)(x^3-1)}{x-8} \leq 0$$

$$\frac{(x+1)(x^2-x+1)(x-1)(x^2+x+1)}{x-8} \leq 0$$



CS: $x \in [7,-1] \cup [1,6] \cup]7,8[$

$$f(x)=\frac{\sqrt[3]{x-7}\;\log(x-8)}{\sqrt[20]{x^2-x-30}\;(x-9)}$$

$\sqrt[3]{x-7} \to \mathbb{R}$

- $x=8>0$
- $x^2-x-30>0$
- $x \neq 0$

$$R = \lim_{x \to 0} \frac{|x^2-4| - |x-1| - |x-3|}{|x+2| - 2|x+1|} = \frac{0}{0}$$

Solución: $\boxed{x=0}$

- $|x^2-4| = |0^2-4| = |4| = 4>0$
- $|x-1| = |0-1| = |1| = 1>0$
- $|x-3| = |-(3-x)| = |3-x| = |3-0| = |3| = 3>0$

- $|x+2| = |0+2| = |2| = 2>0$
- $|x+1| = |0+1| = |1| = 1$

Así: $$Q = \lim_{x \to 0} \frac{-(4-x^2) - [-(1-x)] - [-(3-x)]}{(x+2) - 2(x+1)}$$

$$Q = \lim_{x \to 0} \frac{\cancel{-4} + x^2 \cancel{+1} - x + \cancel{3} - x}{\cancel{x} \cancel{+2} - \cancel{2}x \cancel{-2}}$$

$$Q = \lim_{x \to 0} \frac{x^2 - 2x}{-x}$$

$$Q = \lim_{x \to 0} x^2 - x$$

$$Q = \lim_{x \to 0} 0^2 - 0$$

$0 - 0$

$\boxed{Q = 0}$

$\boxed{-2}$

4.1

6 Grafique

$$f(x) = \begin{cases} 2-x & \text{si } x < 2 \\ x-2 & \text{si } 2 < x < 3 \\ 10-x^2 & \text{si } x \geq 3 \end{cases}$$

y luego determine los siguientes limites o establescan que no existen

a) $\lim_{x \to 2} f(x)$ b) $\lim_{x \to 3} f(x)$ c) $\lim_{x \to 1} f(x)$ d) $\lim_{x \to 5} f(x)$

Solución

$$\boxed{\lim_{x \to p^-} g(x) = \lim_{x \to p^+} g(x)}$$

a) $\lim_{x \to 2} f(x)$

• $\lim_{x \to 2^-} 2-x = 2-2 = 0$

Por la derecha

• $\lim_{x \to 2^+} x-2 = 2-2 = 0$ $\therefore$ Existe $\lim_{x \to 0} f(x) = 0$

$x > 2$

b) $\lim_{x \to 3} f(x)$

Por la izquierda

• $\lim_{x \to 3^-} x-2 = 3-2 = 1$

$x < 3$

Por la derecha

$\lim_{x \to 3^+} 10-x^2 = 10-3^2 = 1$ $\therefore$ Existe $\lim_{x \to 3} f(x) = 1$

$x > 3$

$f(x)$ $\lim_{x \to}$

$x = 2$ $f(x) = \frac{x^2-4}{x-2}$

c) $\lim_{x \to 1} f(x)$

• $f(x) = 2-x$

$\lim_{x \to 1} 2-x = 1$

d) $\lim_{x \to 5} f(x)$

• $f(x) = 10-x$

$\lim_{x \to 1} 10-x = 5$

⑥ Determinar el valor de "a", es continua

$$f(x) = \begin{cases} 2ax+1 & si\ x \leq 1 \\ a^2x-2 & si\ x>1 \end{cases}$$

$\lim\limits_{\substack{x \to 1^- \\ x<1}} 2ax+1 = 2a(1)+1 = 2a+1$ (α)

$2(3)+1 = 7$

$= 2(-1)+1 = -1$

$\lim\limits_{\substack{x \to 1^+ \\ x>1}} a^2x-2 = a^2 \cdot 1 - 2 = a^2-2$ (β)

$= 3^2-2 = 7$

$= (-1)^2 - 2 = -1$

$\alpha = \beta$

$2a+1 = a^2-2$

$a^2-2a-3=0$

$(a-3)(a+1)=0$

$a=3$ ✓

$a=1$ ✓

4. Determinar el valor de a y b en continua

$$f(x) = \begin{cases} x^2+1 & si\ x<1 \\ ax+b & 1 \leq x \leq 2 \\ x-5 & x>2 \end{cases}$$

$\lim\limits_{x \to 1}$ $\lim\limits_{\substack{x \to 1^- \\ x<1}} x^2+1 = 1^2+1 = 2$

$\alpha = \beta$

$a+b = 2$ ①

$\lim\limits_{\substack{x \to 1^+ \\ x>1}} ax+b = a(1)+b = a+b$ (β)

$= -5+7$

$= 2$

$\lim_{x \to 2}$ • $\lim_{x \to 2^-}$ $\quad ax + b = a(2) + b = 2a + b$ ②

$x < 2$

$2(-5) + 7$
-3

② = ①

$2a + b = -3$ ②

• $\lim_{x \to 2^+}$ $\quad x - 5 = 2 - 5 = -3$ ①

$x > 2$

Resolver 1 y ②

$a = -5$
$b = 7$

4. Determinar el valor de a y b en continua

$$f(x) = \begin{cases} \dfrac{x^3 + x^2 + x + 1}{x^3 + 1} & \text{si } x < -1 \\ ax^3 + b & \text{si } -1 \leq x \leq 1 \\ \dfrac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} & \text{si } x > 1 \end{cases}$$

$\lim_{x \to 1}$

• $\lim_{x \to 1^-}$ $\quad ax^3 + b = a \cdot 1 + b = a + b$

$x < 1$

• $\lim_{x \to 1^+}$ $\quad \dfrac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} = \dfrac{\sqrt[3]{1} - 1}{\sqrt{1} - 1} = \dfrac{1 - 1}{1 - 1} = \dfrac{0}{0}$

$x > 1$

• $\lim_{x \to -1^-}$ $\quad \dfrac{x^3 + x^2 + x + 1}{x^3 + 1} = \dfrac{-1 + 1 - 1 + 1}{1 + -1} = \dfrac{0}{0} =$

$x < -1$

• $\lim_{x \to -1^+}$ $\quad ax^3 + b = a - 1^3 + b = -a + b$

$x > -1$

Modo axxi

$$\lim_{x\to -1} \quad \lim_{x\to -1^-} \frac{x^2(x+1)+(x+1)}{x^3+1} = \lim_{x\to -1^-} \frac{(x+1)(x^2+1)}{(x+1)(x^2-x+1)} = \frac{2}{3} \quad \alpha$$

$x<-1$

$$\lim_{x\to -1^+} ax^3+b = a(-1)^3+b = -a+b \quad \beta$$

$x>-1$

$x>-1$

$\alpha = \beta$

$-a+b = \frac{2}{3}$ (1)

$$\lim_{x\to 1} \quad \lim_{x\to 1^-} ax^3+b = a(1)^3+b = a+b \quad \lambda$$

$x<1$

$$\lim_{x\to 1^+} \frac{\sqrt[3]{x}-1}{\sqrt{x-1}} \cdot \frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} = \lim_{x\to 1^+} \frac{\sqrt[3]{x^3}-1}{x-1} \cdot \frac{\sqrt{x-1}}{\sqrt[3]{x^2}+\sqrt[3]{x}+1} = 0 \quad \gamma$$

$x>1$

$\sqrt{a}\cdot\sqrt{a} = \sqrt[\cancel{2}]{a^2} = a$

$\lambda = \gamma$

$a+b=0$ (2)

Resolviendo (1) y (2)

$b = \frac{1}{3}$

$a = -\frac{1}{3}$

Hallar el valor de "A" de manera que la funcion sea continua

$$f(x) = \begin{cases} \left(\dfrac{\operatorname{sen} p + \operatorname{sen} x}{\operatorname{sen} p - \operatorname{sen} x}\right)^{\frac{1}{\operatorname{sen} x}} & x \neq 0 \\ A & x = 0 \end{cases}$$

Nombre: Jhamil Calixto Mamani Quea Paralelo: "C"

Halla el valor de "A" de manera que la función sea continua

$$f_{(x)}=\begin{cases} \left(\dfrac{\operatorname{Sen} P + \operatorname{Sen} x}{\operatorname{Sen} P - \operatorname{Sen} x}\right)^{\frac{1}{\operatorname{sen} x}} & x \neq 0 \\ A & x = 0 \end{cases}$$

$f_{(x)}$ $f_{(0)} = A$

con $x = 0$

$$\lim_{x\to 0} \left(\frac{\operatorname{Sen} P + \operatorname{Sen} x}{\operatorname{Sen} P - \operatorname{Sen} x}\right)^{\frac{1}{\operatorname{sen} x}}$$

$$\lim_{\underbrace{x\to 0}} \left(\frac{\operatorname{Sen} P + \operatorname{Sen} x}{\operatorname{Sen} P - \operatorname{Sen} x}\right)^{\frac{1}{\operatorname{sen} x}}$$

$x = 0$

$x - 0 = 0$

C.V

$x - 0 = P \Rightarrow x = P + 0$

$P = 0$

$$\lim_{P\to 0} \left(\frac{\operatorname{Sen} P + \operatorname{Sen}(P+0)}{\operatorname{Sen} P - \operatorname{Sen}(P+0)}\right)^{\frac{1}{\operatorname{Sen}(P+0)}}$$

$$\lim_{P\to 0} \left(\frac{\operatorname{sen} P + \operatorname{Sen} P\cdot\cos(0) + \operatorname{sen}(0)\cdot\cos P}{\operatorname{sen} P - \operatorname{Sen} P\cdot\cos(0) + \operatorname{sen}(0)\cdot\cos P}\right)^{\frac{1}{\operatorname{Sen}(P+0)}}$$

$$\lim_{P\to 0} \left(\frac{\cancel{\operatorname{Sen} P}\,(1\cdot\cos(0)) + \operatorname{Sen}(0)\cdot\cos P}{\cancel{\operatorname{Sen} P}\,((\cos(0) - 1) + \operatorname{sen}(0)\cdot\cos P}\right)$$

$$\lim_{P\to 0} \left(\frac{(1\cdot\cos(0)) + \operatorname{sen}(0)\cdot\cos P}{(\cos(0) - 1) + \operatorname{sen}(0)\cdot\cos P}\right)^{\frac{1}{\operatorname{Sen} P\cdot\cos(0) + \operatorname{sen}(0)\cdot\cos(P)}}$$

$$\left(\frac{(1\cdot 1) + 0\cdot 1}{1 - 1 + 0\cdot 1}\right)^{\frac{1}{0\cdot 1 + 0\cdot 1}}$$

$$\left(\frac{1+0}{0+0}\right)^{\cancel{\frac{1}{0}}\,\infty} \Rightarrow \left(\cancel{\frac{1}{0}}\,\infty\right)^{\infty} \Rightarrow \infty^{\infty} \Rightarrow \boxed{\infty} \quad \therefore A = \infty$$

Nombre: Jhamil Calixto Memani Quea   Paralelo: "C"

Propiedades "Derivadas"

1

1 $(a)' = 0$
2 $(a)' = 0$
3 $(a)' = 0$
4 $(a)' = 0$
5 $(a)' = 0$
6 $(a)' = 0$
7 $(a)' = 0$
8 $(a)' = 0$
9 $(a)' = 0$
10 $(a)' = 0$

$a =$ Constante
$n =$ Constante

2

1 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
2 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
3 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
4 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
5 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
6 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
7 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
8 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
9 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$
10 $(ax^n)' = a(x^n)'$
$(x^n)' = nx^{n-1}$

3

1 $(e^x)' = e^x$
2 $(e^x)' = e^x$
3 $(e^x)' = e^x$
4 $(e^x)' = e^x$
5 $(e^x)' = e^x$
6 $(e^x)' = e^x$
7 $(e^x)' = e^x$
8 $(e^x)' = e^x$
9 $(e^x)' = e^x$
10 $(e^x)' = e^x$

4

1 $(\ln x)' = \frac{1}{x}$
2 $(\ln x)' = \frac{1}{x}$
3 $(\ln x)' = \frac{1}{x}$
4 $(\ln x)' = \frac{1}{x}$
5 $(\ln^x)' = \frac{1}{x}$
6 $(\ln x)' = \frac{1}{x}$
7 $(\ln x)' = \frac{1}{x}$
8 $(\ln x)' = \frac{1}{x}$
9 $(\ln^x)' = \frac{1}{x}$
10 $(\ln x)' = \frac{1}{x}$
1

(5)

(1) $(p \pm q)' = p' \pm q'$

(2) $(p \pm q)' = p' \pm q'$

(3) $(p \pm q)' = p' \pm q'$

(4) $(p \pm q)' = p' \pm q'$

(5) $(p \pm q)' = p' \pm q'$

(6) $(p \pm q)' = p' \pm q'$

(7) $(p \pm q)' = p' \pm q'$

(8) $(p \pm q)' = p' \pm q'$

(9) $(p \pm q)' = p' \pm q'$

(10) $(p \pm q)' = p' \pm q'$

(6)

(1) $(p \cdot q)' = p'q + pq'$

(2) $(p \cdot q)' = p'q + pq'$

(3) $(p \cdot q)' = p'q + pq'$

(4) $(p \cdot q)' = p'q + pq'$

(5) $(p \cdot q)' = p'q + pq'$

(6) $(p \cdot q)' = p'q + pq'$

(7) $(p \cdot q)' = p'q + pq'$

(8) $(p \cdot q)' = p'q + pq'$

(9) $(p \cdot q)' = p'q + pq'$

(10) $(p \cdot q)' = p'q + pq'$

(7)

(1) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(2) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(3) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(4) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(5) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(6) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(7) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(8) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(9) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(10) $\left(\frac{p}{q}\right)' = \frac{p'q - pq'}{q^2}$

(8)

(1) $(\mathrm{Sen}\, x)' = \cos x$

(2) $(\mathrm{Sen}\, x)' = \cos x$

(3) $(\mathrm{Sen}\, x)' = \cos x$

(4) $(\mathrm{Sen}\, x)' = \cos x$

(5) $(\mathrm{Sen}\, x)' = \cos x$

(6) $(\mathrm{Sen}\, x)' = \cos x$

(7) $(\mathrm{Sen}\, x)' = \cos x$

(8) $(\mathrm{Sen}\, x)' = \cos x$

(9) $(\mathrm{Sen}\, x)' = \cos x$

(10) $(\mathrm{Sen}\, x)' = \cos x$

(9)

1. $(\cos x)' = -\operatorname{sen} x$
2. $(\cos x)' = -\operatorname{sen} x$
3. $(\cos x)' = -\operatorname{sen} x$
4. $(\cos x)' = -\operatorname{sen} x$
5. $(\cos x)' = -\operatorname{sen} x$
6. $(\cos x)' = -\operatorname{sen} x$
7. $(\cos x)' = -\operatorname{sen} x$
8. $(\cos x)' = -\operatorname{sen} x$
9. $(\cos x)' = -\operatorname{sen} x$
10. $(\cos x)' = -\operatorname{sen} x$

(10)

1. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
2. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
3. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
4. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
5. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
6. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
7. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
8. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
9. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$
10. $(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1-x^2}}$

(11)

1. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
2. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
3. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
4. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
5. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
6. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
7. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
8. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
9. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
10. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

(12)

① $(\arctan x)' = \dfrac{1}{x^2+1}$

② $(\arctan x)' = \dfrac{1}{x^2+1}$

③ $(\arctan x)' = \dfrac{1}{x^2+1}$

④ $(\arctan x)' = \dfrac{1}{x^2+1}$

⑤ $(\arctan x)' = \dfrac{1}{x^2+1}$

⑥ $(\arctan x)' = \dfrac{1}{x^2+1}$

⑦ $(\arctan x)' = \dfrac{1}{x^2+1}$

⑧ $(\arctan x)' = \dfrac{1}{x^2+1}$

⑨ $(\arctan x)' = \dfrac{1}{x^2+1}$

⑩ $(\arctan x)' = \dfrac{1}{x^2+1}$

(13)

① $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

② $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

③ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

④ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

⑤ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

⑥ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

⑦ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

⑧ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

⑨ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

⑩ $(\text{arccot}\, x)' = -\dfrac{1}{x^2+1}$

(14)

① $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

② $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

③ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

④ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

⑤ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

⑥ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

⑦ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

⑧ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

⑨ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

⑩ $(\text{arcsec}\, x)' = \dfrac{1}{x\sqrt{x^2-1}}$

(15)

(1) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(2) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(3) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(5) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(6) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(7) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(8) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(9) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

(10) $(\text{arc Csc } x)' = -\dfrac{1}{x\sqrt{x^2-1}}$

$$R = \lim_{x \to 0} \frac{|x^2-4| - |x-1| - |x-3|}{|x+2| - 2|x+1|}$$

evaluando

$$\frac{|0^2-4| - |0-1| - |0-3|}{|0+2| - 2|0+1|}$$

$$\frac{4-1-3}{2-2} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{|x^2-4| - |x-1| - |x-3|}{|x+2| - 2|x+1|}$$

$|x^2-4| = |-4| = 4$

$|x-1| = -(x-1) = -x+1$

$|x-3| = -(x-3) = -x+3$

$$\lim_{x \to 0} \frac{(4) - (-x+1) - (-x+3)}{(x+2) - 2(x+1)}$$

$$\lim_{x \to 0} \frac{-2x+6}{x-4}$$

$$\lim_{x \to 0} \frac{1-2}{1} \Rightarrow \boxed{\lim_{x \to 0} = -2}$$

Nombre: Jhamil Calixto Mamani Quea   Paralelo: "C"

Definición:

$\lim_{x \to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0 \ \exists \ \delta > 0 \ / \ 0 < |x-2| < \delta \Rightarrow |f(x) - L| < \varepsilon$

Demostrar por definicion el siguiente limite

$\lim_{x \to 2} (x^2+3x+2) = 12$

$\varepsilon > 0, \ \delta > 0 \quad 0 < |x-2| < \delta \Rightarrow |(x^2+3x+2) - 12| < \varepsilon$

$|x^2+3x+2-12| < \varepsilon$ $\quad]1,3[\quad \delta < 1 \quad 0 < |x-2| < \delta \quad \delta = 1$

$|x^2+3x-10| < \varepsilon$ $\quad X_0 = 2 \quad |x-2| < 1$

$|(x+5)(x-2)| < \varepsilon$ $\quad -1 < x-2 < 1 \ /+7$

$|x+5||x-2| < \varepsilon$ $\quad -1+7 < x-2+7 < 1+7$

$6 < x+5 < 8$

$x+5 < 8 \longrightarrow |x+5| < 8$

$|x+5||x-2| < \varepsilon$

$8|x-2| < \varepsilon \ / \div 8$

$|x-2| < \varepsilon/8$

$\delta = \min(1, \varepsilon/8)$

$0 < |x-2| < \delta$

$\Rightarrow |x-2| < \delta \ / |x+5|$

$\Rightarrow |x-2||x+5| < |x+5| \cdot \delta$

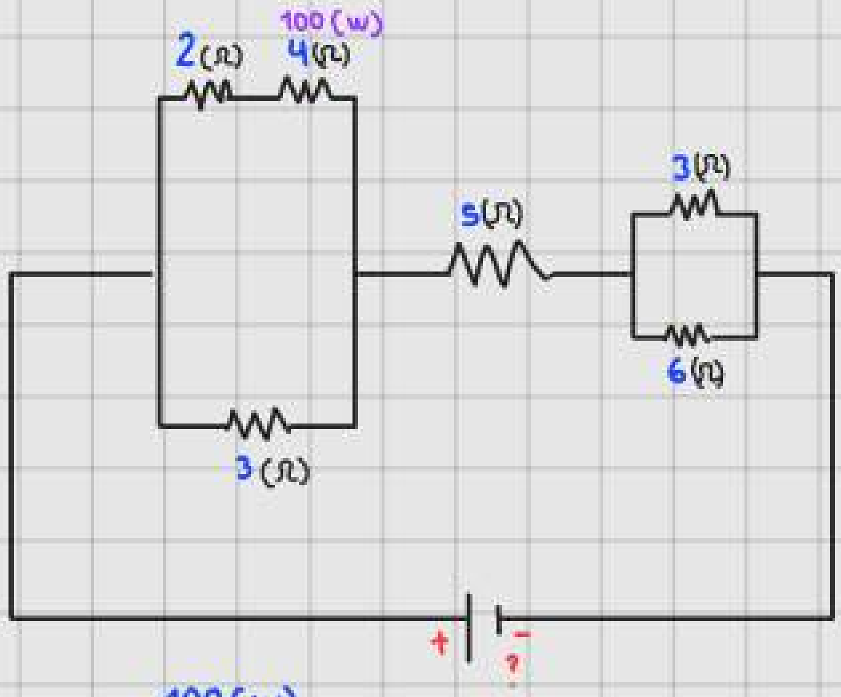
$\Rightarrow |(x-2)(x+5)| < |x+5| \cdot \delta$

$\Rightarrow |x^2+3x-10| < 8 \cdot \delta$

$\Rightarrow |x^2+3x+2-12| < \cancel{8} \cdot \frac{\varepsilon}{\cancel{8}}$

$\Rightarrow |(x^2+3x+2) - 12| < \varepsilon$

$\Rightarrow |f(x) - L| < \varepsilon$



$R_2 =$

Calcula a+b de manera que la función sea continua

$$f(x) \begin{cases} \dfrac{\operatorname{sen}(4(x-1))}{1-x} & \text{si } x<1 \\ ax^2+b & \text{si } 1 \le x \le 2 \\ 4x+3 & \text{si } x>2 \end{cases}$$

$$\lim_{x\to 1^-} \frac{\operatorname{sen}(4(x-1))}{1-x}$$

$$\lim_{\substack{x\to 1 \\ x<1}} \frac{-4\operatorname{sen}(4(x-1))}{-4(1-x)}$$

$$-4 \lim_{x\to 1} \frac{\operatorname{sen}(4(x-1))}{(4(x-1))}$$

C.V.

$y = x-1 \qquad y \to 0$

$y = 1-1 = 0$

$$-4\lim_{y\to 0} \frac{\operatorname{sen}(4y)}{4(y)} \to 1$$

$= -4$

$$\lim_{x\to 1^+} ax^2+b$$

$$\boxed{= a+b}$$

$$\lim_{x\to 1^-} = \lim_{x\to 1^+}$$

$-4 = a+b$

reemplazando "a"

$$-4 = \left(\frac{11-b}{4}\right)+b$$

$$-4 = \frac{11-b+4b}{4}$$

$-4(4) = 11+3b$

$-16-11 = 3b$

$$-\frac{27}{3} = b$$

$-9 = b$

$$\lim_{x\to 2^-} ax^2+b \qquad \lim_{x\to 2^+} 4x+3$$

$= 4a+b \qquad = 11$

$\boxed{4a+b=11} \longrightarrow 4a+(-9) = 11$

$4a = 11-b \qquad 4a = 11+9$

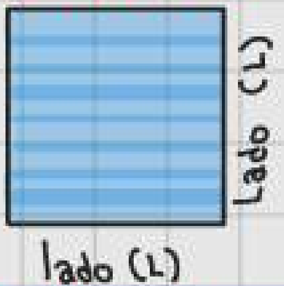
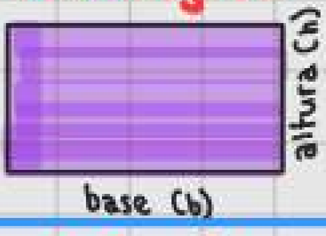
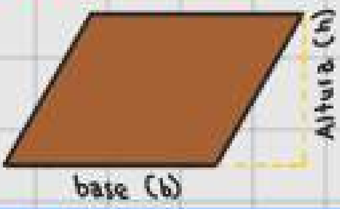
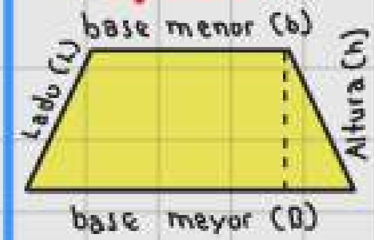
$$a = \frac{11-b}{4} \qquad a = \frac{20}{4} \Rightarrow \boxed{a=5}$$

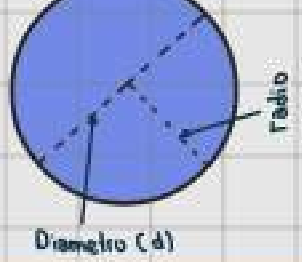
$$\boxed{a+b=-4}$$

$$\begin{cases} a=5 \\ b=-9 \end{cases}$$

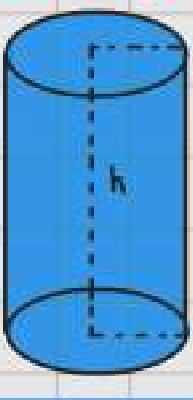
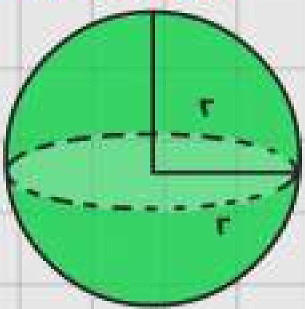
Nombre: Jhamil Calixto Mameni Quea Paralelo: "C"

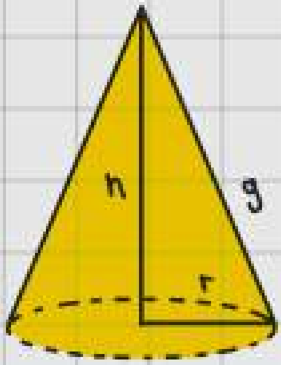
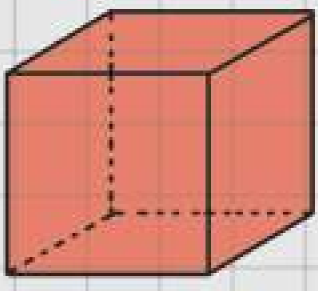
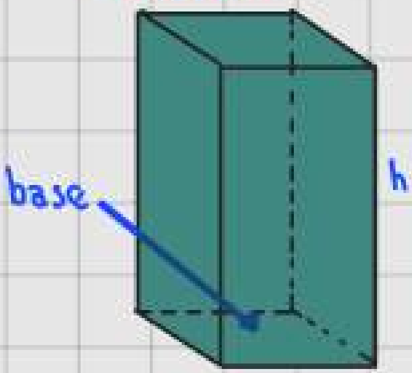
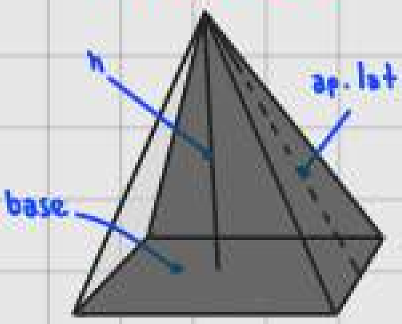
Formulario de Figuras geometricas

	Area	Perimetro
Cuadrado 	$A = L \times L$	$P = L + L + L + L$
Rectengulo 	$A = b \times h$	$P = b + b + h + h$
Triángulo 	$A = \frac{b \times h}{2}$	$P = L + L + L$
Rombo 	$A = \frac{D \times d}{2}$	$P = L + L + L + L$
Romboide 	$A = b \times h$	$P = b + b + h + h$
Trapecio 	$A = \frac{h(B + b)}{2}$	$P = B + b + L + L$

circulo 	Area $A=\pi \times r^2$	Perimetro $C=\pi \times d$
Poligono +5 	$A=\frac{P \times a}{2}$	$P=L \times \#$ lados

Formulas de area y volumen de cuerpos geometricos

Cilindro 	Area $A_t=2\pi r\,(h+r)$	Volumen $V=\pi r^2 \cdot h$
Esfera 	$A_t=4\pi r^2$	$V=\frac{4}{3}\pi r^3$

Cono	Area	Volumen
	$At = \pi r^2 + \pi r g$	$V = \frac{\pi r^2 h}{3}$
Cubo 	$A = 6a^2$	$V = a^3$
Prisma 	$A = (\text{Perim. base} \times h) + 2\,\text{area base}$	$V = \text{área base} \cdot h$
Piramide 	$A = \frac{\text{Perim. base} \cdot \text{ap.lat}}{2} + \text{area base}$	$V = \frac{\text{area base} \times h}{3}$

Nombre: Jhamil Calixto Mamani Quea   Paralelo: "C"

Determine dos numeros positivos cuya suma sea 4, y tal que la suma del cubo del menor con el cuadrado del mayor sea minima

Numeros : x , y

$x + y = 4$

Para minimizar la función

$f(x,y) = x^3 + y^2$

$L(x,y,\lambda) = x^3 + y^2 + \lambda(4 - x - y)$

Ecuaciones

$\frac{\partial L}{\partial x} = 3x^2 - \lambda = 0 \ldots (1)$

$\frac{\partial L}{\partial y} = 2y - \lambda = 0 \ldots (2)$

$\frac{\partial L}{\partial \lambda} = 4 - x - y = 0 \ldots (3)$

Resolviendo sistema de ecuaciones

Para (1)   Para (2)

$3x^2 = \lambda$ $2y = \lambda$

sustituimos (1) y (2) en (3)

$4 - x - y = 0$ (3)

$4 - x - \left(\frac{\lambda}{2}\right) - \left(\frac{3x^2}{2}\right) = 0$

$\boxed{x = 0}$ $\boxed{y = 4}$

otra forma

$y = 4 - x$

$S = x^3 + y^2$

$S' = 3x^2 + (4-x)^2$

$S' = 3x^2 + 2(4-x)'(-1)$

$S' = 3x^2 - 2(4-x)$

$S' = 3x^2 - 8 + 2x$

$S'' = 6x + x$

$S'' = 7x$

$x = -7$

$y = 4 - (-7)$

$y = 4 + 7$

$y = 12$