Question of the Day

Why do you want to study math?

On the Docket

Introductions
Concept Check-in: Basic Vocabulary
Concept Review: Anatomy of a Matrix
Concept Review: Row Operations
Challenge Problems

In your own words, describe

- Linear equations.
- Linear systems.
- Homogneous equations.
- Inhomogeneous equations.
- Consistent systems.
- Inconsistent systems.
- Free variables.
- Solutions to an equation or system.
- Coefficients.
- Constants.
- Variables.
- Vectors.
- A norm of a vector.

Let A be a matrix with n rows and m columns. We may write

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nm} \end{pmatrix}$$

where a_{ij} is the entry in the ith row and jth column.

We could, if so possessed, write A in a different way.

$$A = \{\{a_{11}, a_{12}, a_{13}, \ldots\ a_{1m}\}, \ldots, \{a_{n1}, a_{n2}, a_{n3}, \ldots, a_{nm}\}\}$$

We could even conceive of A as a function $A(i, j) = a_{ij}$.

But matrix notation gives spatial landmarks for computations, and a lot of our language about matrices will reflect that (e.g. row, column, pivot, diagonal matrix, upper triangular matrix, etc.).

Elementary Row Operations

(1) **Exchange**: Two rows may be interchanged.

$$R_i \leftrightarrow R_j$$

(2) **Scaling**: One row may be multiplied by $c \neq 0$.

$$R_i \to c \cdot R_i$$

(3) **Elimination**: A multiple of one row may be added to another row.

$$R_j \to cR_i + R_j$$

We will learn later that these arise naturally from matrix multiplication, but for now we can think of them simply as "things we are allowed to do to matrices."

1. Construct a homogeneous, linear system with three equations and three variables with the non-trivial solution $(1, -2, 4)$.
2. Determine whether the inhomogeneous linear systems have no solution, a unique solution, or infinitely many solutions.

$$\text{(a)}\left(\begin{array}{cccc|c} 4 & -3 & 2 & 0 & -1 \\ 0 & 2 & -1 & 4 & 0 \\ 0 & 0 & 3 & -2 & -5 \\ 0 & 0 & 0 & 4 & 2 \end{array}\right) \quad \text{(b)}\left(\begin{array}{cccc|c} 2 & -1 & -2 & 1 & -1 \\ 0 & 3 & 0 & 5 & 2 \\ 0 & 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & -2 & 9 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right)$$

Let

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix},\ v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix},\ v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix},\ v_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -3 \end{pmatrix}.$$

(a) Verify that each v_i belongs to

$$S = \left\{ v = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4 \ : \ a + b + c + d = 0 \right\}.$$

(b) Explain why there must exist some $b \in \mathbb{R}^4$ which is not a linear combination of v_i.