Math 24 Discussion Section

Concept Review

An ordinary differential equation may be...

Linear	Nonlinear	Separable	Autonomous
Nonautonomous	Exact	Homogeneous	Nonhomogeneous

An ordinary differential equation may admit a(n) ...

Order	Initial Condition	Solution
General Solution	Fundamental Solution	Implicit Solution
Equilibrium Solutions	Critical Points	Stable/Unstable/Semistable Equilibrium
Characteristic Equation	Wronskian	

Important theorems so far include...

(2.4.1), (2.4.2), (2.8.1), (3.2.1) Existence and Uniqueness Theorems

(2.6.1) Criterion for Exactness

(3.2.2) Principle of Superposition

(3.2.3), (3.2.4), (3.2.5) Consequences of Nonzero Wronskian

(3.2.7) Abel's Theorem (Abel's Identity)

Practice

1. For each of the following,
   1. fully categorize the equation by order, linearity, homogeneity, etc.
   2. identify all techniques that could be used to solve the equation.
   3. find the intervals of validity.
   4. solve the equation.

   (a) $2y' - y = 4\sin(3t),\ y(0) = y_0$

   (b) $\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2),\ y(0) = 0$

   (c) $2xy - 9x^2 + (2y + x^2 + 1)y' = 0,\ y(0) = -3$

   (d) $y'' + 8y + 17y = 0,\ y(1) = 1,\ y'(1) = 0$

2. For the equation $x'(t) = x(t)(1 - x(t))^3$,
   1. determine equilibria, and compute x'' at those points. What can you say about the stability of the equilibria?
   2. graph $f(x) = x(1-x)^3$.
   3. draw phase lines and sketch several graphs of solutions in the tx-plane.

3. For the equation $y'' + 4y' + 3y = 0$ and initial point $t_0 = 1$, find the fundamental set of solutions specified by the condition that

$$W(y_1, y_2)(t_0) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}.$$