

## Math 24 Discussion Section

### Warm Up

With your group, agree upon clear definitions of the following terms:

Function

Fundamental Set of Solutions

Differential Operator

Superposition

Wronskian

Discuss the following theorem. In particular, what does the phrase “if and only if” mean in this context?

**Theorem:** Suppose that  $y_1$  and  $y_2$  are two solutions of the initial value problem  $y'' + p(t)y' + q(t)y = 0$ ,  $y(t_0) = y_0$ , and  $y'(t_0) = y'_0$ . Then it is always possible to choose constants  $c_1$  and  $c_2$  such that  $y = c_1 y_1(t) + c_2 y_2(t)$  is a solution if and only if the Wronskian  $W[y_1, y_2]$  is non-zero at  $t_0$ .

### Problems

1. Compute the Wronskian  $W[t^2 e^{-3t}, \cos^2(t)]$ .
2. Verify that  $y_1(x) = x$  and  $y_2(x) = xe^x$  are solutions to  $x^2 y'' - x(x+2)y' + (x+2)y = 0$ ,  $x > 0$ . Do they form a fundamental set of solutions?
3. A complex number is some element  $z \in \mathbb{C}$  which may be expressed as  $z = a + bi$  where the real part,  $\Re(z) = a$ , and the imaginary part,  $\Im(z) = b$ , are both real numbers and where the imaginary unit  $i$  satisfies  $i^2 = -1$ .

If  $z = a + bi$  is some complex number, then it has complex conjugate  $\bar{z} = a - bi$  and modulus  $|z| = \sqrt{a^2 + b^2}$ .

(a) For  $z = a + bi$ , compute  $\bar{\bar{z}}$  and  $z\bar{z}$ .

(b) On the complex plane, graph and label points  $z_1 = -3 + i$ ,  $z_2 = \bar{z}_1$ , and  $z_3 = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}i$ .

Euler's formula tells us that for any real number  $x \in \mathbb{R}$ ,  $e^{ix} = \cos(x) + i \sin(x)$ .

(c) Using Euler's formula, graph and label points  $z_4 = e^{i(\pi/2)}$ ,  $z_5 = e^{i(-\pi/3)}$ ,  $z_6 = e^{i(\pi/3)}$ , and  $z_7 = 3e^{i(\pi/4)}$ .

(d) Compute  $|z_1|$ ,  $|z_3|$ ,  $|z_5|$  and  $|z_7|$ .

4. Find the general solution of  $9y'' + 9y' - 4y = 0$ .
5. Find the solution to  $y'' + 2y' + 2y = 0$ ,  $y(\pi/4) = 2$ ,  $y'(\pi/4) = -2$ .
6. Use Euler's formula to show that

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \quad \text{and} \quad \sin(x) = \frac{e^{ix} - e^{-ix}}{2i}.$$