

Question of the Day

Why do you want to study math?

On the Docket

Introductions

Concept Check-in: Basic Vocabulary

Concept Review: Anatomy of a Matrix

Concept Review: Row Operations

Challenge Problems

In your own words, describe

Linear equations.

Linear systems.

Homogeneous equations.

Inhomogeneous equations.

Consistent systems.

Inconsistent systems.

Free variables.

Solutions to an equation or system.

Coefficients.

Constants.

Variables.

Vectors.

A norm of a vector.

Let A be a matrix with n rows and m columns. We may write

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nm} \end{pmatrix}$$

where a_{ij} is the entry in the i th row and j th column.

We could, if so possessed, write A in a different way.

$$A = \{\{a_{11}, a_{12}, a_{13}, \dots, a_{1m}\}, \dots, \{a_{n1}, a_{n2}, a_{n3}, \dots, a_{nm}\}\}$$

We could even conceive of A as a function $A(i, j) = a_{ij}$.

But matrix notation gives spatial landmarks for computations, and a lot of our language about matrices will reflect that (e.g. row, column, pivot, diagonal matrix, upper triangular matrix, etc.).

Elementary Row Operations

(1) **Exchange:** Two rows may be interchanged.

$$R_i \leftrightarrow R_j$$

(2) **Scaling:** One row may be multiplied by $c \neq 0$.

$$R_i \rightarrow c \cdot R_i$$

(3) **Elimination:** A multiple of one row may be added to another row.

$$R_j \rightarrow cR_i + R_j$$

We will learn later that these arise naturally from matrix multiplication, but for now we can think of them simply as “things we are allowed to do to matrices.”

1. Construct a homogeneous, linear system with three equations and three variables with the non-trivial solution $(1, -2, 4)$.
2. Determine whether the inhomogeneous linear systems have no solution, a unique solution, or infinitely many solutions.

$$(a) \begin{pmatrix} 4 & -3 & 2 & 0 & | & -1 \\ 0 & 2 & -1 & 4 & | & 0 \\ 0 & 0 & 3 & -2 & | & -5 \\ 0 & 0 & 0 & 4 & | & 2 \end{pmatrix} \quad (b) \begin{pmatrix} 2 & -1 & -2 & 1 & | & -1 \\ 0 & 3 & 0 & 5 & | & 2 \\ 0 & 0 & -3 & 0 & | & -3 \\ 0 & 0 & 0 & -2 & | & 9 \\ 0 & 0 & 0 & 0 & | & -1 \end{pmatrix}$$

Let

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -3 \end{pmatrix}.$$

(a) Verify that each v_i belongs to

$$S = \left\{ v = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4 : a + b + c + d = 0 \right\}.$$

(b) Explain why there must exist some $b \in \mathbb{R}^4$ which is not a linear combination of v_i .