Teaching Assitant

Joseph Immel <jhimmel@ucsc.edu>

Office hours:

Wednesday / Friday 11:45 AM - 12:45 PM
McHenry Library Cafe

Website: jhi3.github.io

Reflection

Take a moment to think about what was covered last class and what is due on upcomming assignments. Which concepts, techniques, etc. feel clear and doable? Which, if any, could use more explanation or practice?

Warm Up

Discuss the following with your groups

Give plain language definitions for the following terms:

- Cross Product
- Parameter (and Parametric)

Write...

- the vector form of a line.
- the parametric equations of a line.
- the scalar equation of a plane.
- the vector equation of a plane.

If real vecotrs u and v are orthogonal, what can we say about their dot product and their projection?

Problems 1, 2 and 3 (Cross Product)

Problem 1

For $u = \langle -3, 1, 5 \rangle$ and $v = \langle 2, 2, 9 \rangle$, compute $u \times v$.

Problem 2 (9.4.7)

Find two unit vectors orthogonal to $a = \langle -2, -1, 3 \rangle$ and $b = \langle 4, -2, 3 \rangle$.

Problem 3 (9.4.15(a))

Let $u = \langle u_1, u_2, u_3 \rangle$, $v = \langle v_1, v_2, v_3 \rangle$, and $w = \langle w_1, w_2, w_3 \rangle$. Show that

$$(u \times v) \cdot w = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}.$$

The scalar equation of a plane,

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0,$$

tells us that a plane in $\mathbb{R}^3$ is uniquely determined by a normal vector $\vec{n} = \langle a, b, c \rangle$ and a point $p = (x_0, y_0, z_0)$.

Discuss the following in groups:

- Is this unique to planes in $\mathbb{R}^3$? When $c = 0$, what are we looking at?
- Describe $\vec{n}$.
  - Where might its head or tail be?
  - What effect, if any, does the magnitude $||\vec{n}||$ have?
  - How does it relate, geometrically, to $\vec{v} = \langle x_0, y_0, z_0 \rangle$?

Bonus

The cross product can be described geometrically as constructing a parallelepiped with non-zero volume. For arbitrary $u, v \in \mathbb{R}^3$, compute $u \times v$ and then see what happens (at least in the first component) when you compute $(u \times v) \cdot u$.

Problem 4 (9.5.1)

Rewrite the vector equaton $r(t) = (4-5t)\hat{i} + (-3+5t)\hat{j} + (-4+4t)\hat{k}$ as the corresponding parametric equations for the line.

Problem 5 (9.5.3)

Consider the line which passes through the point $P = (-5,1,5)$ and which is parallel to the line $x = 1+4t$, $y = 2+7t$, $z = 3+4t$. Find the point of intersection of this new line with each of the coordinate planes (i.e. the xy-, (xz-, and yz-planes).

Problem 6 (9.5.5)

Find an equation of a plane containing three points $(-2,4,3)$, $(3,8,2)$, $(3,9,4)$ in which the coefficient of x is 9.