

Math 24 Discussion Section

Warm Up

With your group, give clear definitions for the following terms.

Eigenvalues
Algebraic Multiplicity
The Matrix Exponential
Jordan Form

Eigenvectors
Geometric Multiplicity
Diagonalizable Matrix
Generalized Eigenvector

Eigenpairs
Defective Eigenvalues
Fundamental Matrix

Problems

1. Consider the initial value problem

$$x' = Ax = \begin{pmatrix} 0 & 0 & 4 \\ 1 & 0 & -6 \\ 0 & 1 & 4 \end{pmatrix}x, \quad x(0) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$$

- Identify the eigenvalues and associated eigenvectors of A .
- Write a general solution for $x' = Ax$.
- Write the fundamental matrix Ψ for the system $x' = Ax$, and evaluate $\Psi(0)$.
- Write a solution to the initial value problem.
- Describe how the solution behaves as $t \rightarrow \infty$.

2. Consider the initial value problem

$$x' = Ax = \begin{pmatrix} 3 & 1 \\ -1 & 5 \end{pmatrix}x, \quad x(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

- Identify the eigenpair of A and the algebraic and geometric multiplicity of its eigenvalue.
- Write a solution with respect to the eigenpair (ρ, ξ) of A .
- Write a solution with respect to the generalized eigenvector η where $(A - \rho I)\eta = \xi$ and $(A - \rho I)^2\eta = 0$.
- Write the solution to the initial value problem.
- Sketch the trajectory of the solution in the $x_1 x_2$ -plane, and also sketch the graph of x_1 versus t .