

Problem 3

Friday, October 4, 2024 10:35 AM

Given $K_3 > K_2 > K_1$, $K_3 - K_2 = K_2 - K_1$, and all options have the same maturity.
Then prove,

$$C_2 \leq \frac{C_1 + C_3}{2}$$

By using the hint in the task, I construct that portfolio with expiration S_T .
If,

$S_T \leq K_1 \Rightarrow$ all options expire out of money (profit 0)

If $K_1 < S_T \leq K_2$

$\Rightarrow K_1$ in the money = $S_T - K_1$
 $\Rightarrow K_2$ and K_3 out of money = 0
 $\Rightarrow S_T - K_1$

If $K_2 < S_T < K_3$

$\Rightarrow K_1$ in the money = $S_T - K_1$
 $\Rightarrow K_2$ in the money = $2(S_T - K_2)$
 $\Rightarrow K_3$ out of money = 0
 $\Rightarrow (S_T - K_1) - 2(S_T - K_2) \Rightarrow S_T \leq K_3$

If $S_T > K_3$

\Rightarrow all in money = $(S_T - K_1) + (S_T - K_3) - 2(S_T - K_2) = 0$

Here I can clearly see that the pay-off is always non negative. For there to be no arbitrage opportunity. Thus

$$C_1 + C_3 \geq 2C_2$$

$$\Rightarrow C_2 \leq \frac{C_1 + C_3}{2}$$