Hw4 Task 1

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Given r is the continuous compounding risk, Δt change in time step, U is the up factor and d is the down factor. The following must hold given a risk-neutral world,

$$E(S) = S_0 e^{r\Delta t} = p S_0 U + (1-p) d \cdot S_0$$

$$\Rightarrow e^{r\Delta t} = pU + d - pd$$

$$e^{r\Delta t} - d = p(U - d)$$

$$\Rightarrow \frac{e^{r\Delta t} - d}{U - d} = p$$

Thus given $U = 1{,}1$, $d = 0{,}9$, $S_0 = 100$, $K = 100$, $r = 0{,}08$, $\Delta t = 0{,}5$

$$p = \frac{e^{0{,}08 \cdot 0{,}5} - 0{,}9}{1{,}1 - 0{,}9} = 0{,}704$$

Thus we 4 Schenarios with 3 diffrent outcomes

$$T_{UU} = \max(100 \cdot (1{,}1)^2 - 100, 0) = 21$$

$$T_{Ud} = T_{dU} = \max(100 \cdot 1{,}1 \cdot 0{,}9 - 100, 0) = 0$$

$$T_{dd} = \max(100 \cdot 0{,}9 \cdot 0{,}9 - 100, 0) = 0$$

$$\text{Value} = e^{-2 \cdot 0{,}08 \cdot 0{,}05} (0{,}704^2 \cdot 21 + 2 \cdot 0{,}704 \cdot 0{,}296 \cdot 0 + 0{,}296^2 \cdot 0) = 4{,}61$$