Current stock :$100
strike price :$82
Time to maturity : 1 year
Time steps : 2
Risk free rate : 4.5%, annual continuas compounding
σ : 32%

CRR model :

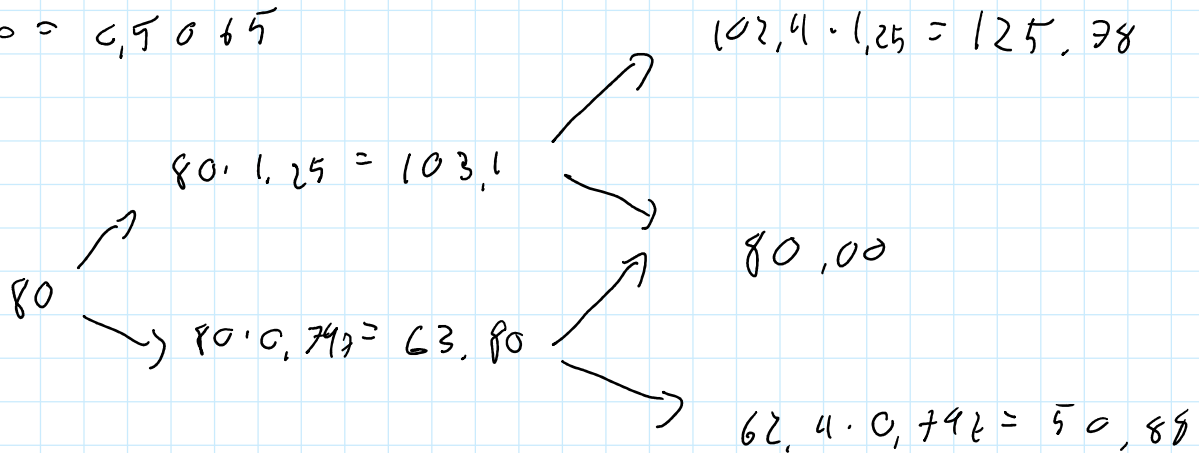$$u = e^{\sigma\sqrt{\Delta t}} \quad , \quad d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$$

$$u = e^{0.32\sqrt{\frac{1}{2}}} = 1.259$$

$$d = \frac{1}{u} = \frac{1}{1.28} = 0.797$$

Risk Neutral Probability :

$$P = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.045 \cdot 0.5} - 0.792}{1.259 - 0.792} \approx 0.4935$$

$1-p = 0.5065$

$102.4 \cdot 1.25 = 125.78$

$80 \cdot 1.25 = 103.1$

80

$80 \cdot 0.797 = 63.80$

80.00

$62.4 \cdot 0.797 = 50.88$

Pay off,

$$P_{uu} = \max(K - S, 0) = \max(82 - 125.78) = 0$$

$$P_{uu} = \max(k-S,0) = \max(82-125, 78) = 0$$

$$P_{ud} = \max(82-80) = 2$$

$$P_{dd} = \max(82-50, 88) = 31.12$$

Expected value european:

$$V_u = e^{-0.45\cdot0.5}(0 + 0.5065\cdot2) = 0.99$$

$$V_d = e^{-0.45\cdot0.5}(0.4935\cdot2 + 0.5065\cdot31.1) = 16.38$$

Option price:

$$V_o = e^{-0.045\cdot0.5}(0.4935\cdot0.99 + 0.5065\cdot16.38) = 8.59$$

The value for 1 year option put is 8,59

b) $$P_{exercise} = \max(k-S_0,0) = \max(82 - 103.1) = 0$$

From a) we know that $V_u = 0.99$

Thus $V_u > P_{exercise}$

For the down node,

for one time step from a V_d is 63.80

$$P_{oy-ds} = \max(82-63.80) = 18.20$$

From a the value V_d is 16.87

Thus payoff > V_f

This payoff $>$ V_8

At present the value is,
$$e^{-0.048 \cdot 0.5} (0.49 \cdot 16.37 + 0.5464 \cdot 18.20) = 9.49$$

The 1 year american put value is 9,49