Problem 3

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$$dS = \mu S\,dt + \sigma S\,dW$$

Given a function $G(S,t)$, it follows,

$$dG(S,t) = \left(\frac{\partial G}{\partial t} + \mu \frac{\partial G}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 G}{\partial S^2}\right)dt + \sigma S \frac{\partial G}{\partial S}\,dW$$

Since $G(S,t) = S^n$, and does not explicitly depend on t, thus

$$\frac{\partial G}{\partial t} = 0$$

$$\Rightarrow \frac{\partial G}{\partial S} = \frac{\partial}{\partial S} S^{n} = nS^{n-1}$$

$$\frac{\partial^2 G}{\partial^2 S} = \frac{\partial}{\partial S}(nS^{n-1}) = n(n-1)S^{n-1}$$

Thus,

$$dG = \left(0 + \mu S \cdot nS^{n-1} + \frac{1}{2}\sigma^2 S^2 \cdot n(n-1)S^{n-2}\right)dt + \sigma S \cdot nS^{n-1}\,dW$$

$$= S^n\left(n\mu + \frac{1}{2}n\sigma^2(n-1)\right)dt + n\sigma S^n\,dW$$

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Here $G = S^n$, thus

$$d(S^n) = \left(n\mu + \frac{1}{2}n(n-1)\sigma^2\right)S^n\,dt + n\sigma S^n\,dW$$

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$$dY = \mu' Y\,dt + \sigma Y\,dW$$

where $Y = S^n$, $\mu' = n\mu + \frac{1}{2}n(n-1)\sigma^2$, $\sigma' = n\sigma$

Thus,

Drift coefficient for S^n,

$$\hat{\mu} = n\mu + \frac{1}{2}n(n-1)\sigma^2$$

Volitility for S^n,

$$\bar{\sigma} = n\sigma$$

This confirms that S^n follows,

$$d(S^n) = \hat{\mu} S^n\,dt + \bar{\sigma} S^n\,dW$$