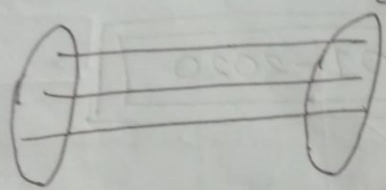
"Ayub Sir" Date: 15.01.2020

One-one ও one-two হলে Inverse function নির্ণয় করা যায়।

Inverse function:

$f: \mathbb{R} \longrightarrow \mathbb{R}$

x y



$f: X \longrightarrow Y$

$f(x) = x^2 - x - 2$

$f([1,2])$, $f^{-1}([-2,0])$, $f^{-1}(\{0\})$

(i) $f(1) = 1 - 1 - 2 = -2$

$f(2) = 2^2 - 2 - 2 = 0$ (Ans)

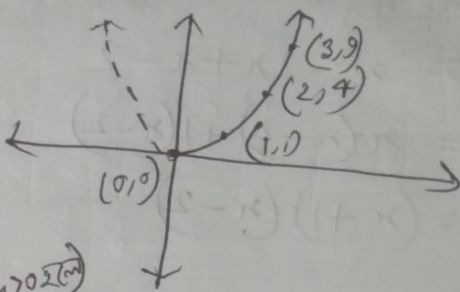
$f([1,2]) = [-2,0]$

$\Rightarrow [1,2] = f^{-1}([-2,0])$

$\therefore f^{-1}([-2,0]) = [1,2]$ (Ans)

$\boxed{y = x^2; \; x > 0}$

x	0	+1	2	3
y	0	+1	4	9

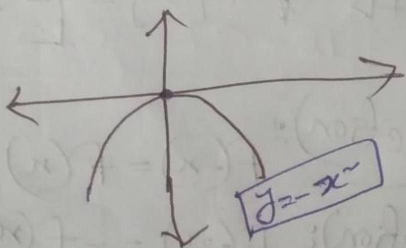


ফাংশনের ডোমেন ও রেঞ্জ নির্ণয় কর:

$Df = [0, \infty)$ (x>0 হলে) $= (-\infty, \infty) = \mathbb{R}$ (x>0 না হলে)

$Rf = [0, \infty)$

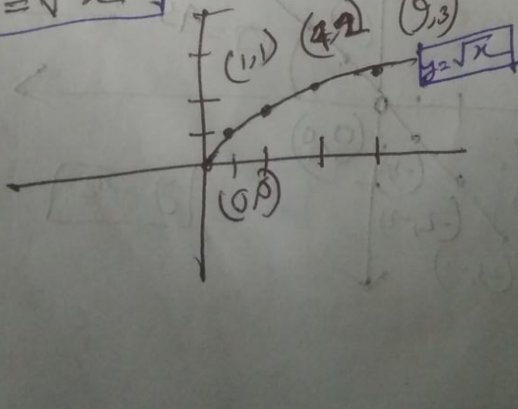
$\boxed{y = -x^2}$



$\therefore Df = \mathbb{R}$

$\therefore Rf = (-\infty, 0]$

$\boxed{y = \sqrt{x}}$



$Df = [0, \infty)$

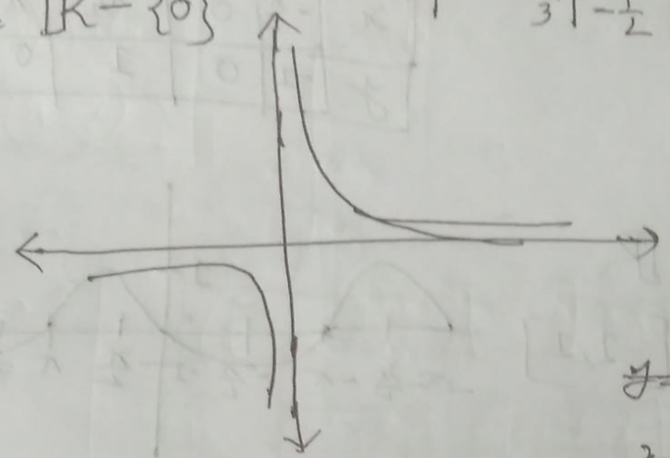
$Rf = [0, \infty)$

x	0	1	4	9
y	0	1	2	3

\# $\boxed{F(x) = \frac{1}{x}}$

x	... -3	-2	-1	0	1	2	3
y	... $-\frac{1}{3}$	$-\frac{1}{2}$	-1	∞	1	$\frac{1}{2}$	$\frac{1}{3}$

$D_f = \mathbb{R} - \{0\}$

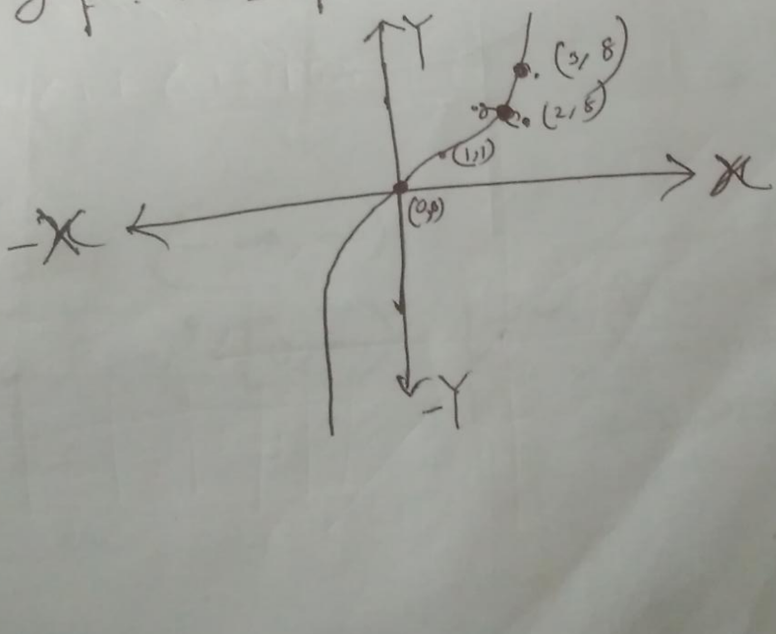


$y = \frac{1}{x}$

$x = \frac{1}{y}$

\# $\boxed{y = x^3}$

x	... 0	1	2	3	...	-1	-2	-3
y	... 1	1	8	27	...	-1	-8	-9



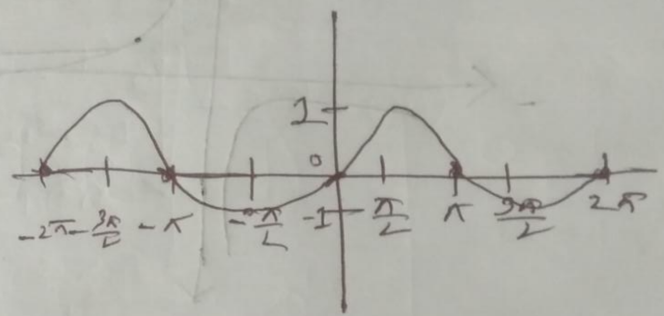
$y = \sin x$

x	0	$\dots$	-2π	$-\frac{3\pi}{2}$	[illegible]
y	0	$\dots$	0	1	[illegible]

x	$-\frac{\pi}{2}$	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
y	-1	0	1	0	-1	0

$D_f = \mathbb{R}$

$R_f = [-1, 1]$



21-01-2020

"Ayub Ali Sir"

~~64~~

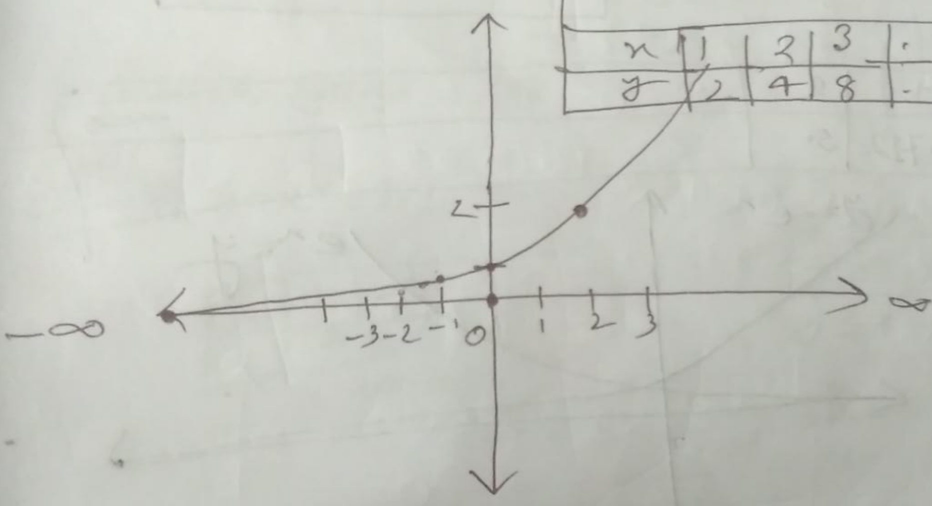
(*) a^x ; $a > 1$

$a = 2$

$y = 2^x$

x	$-\infty$ ...	-3	-2	-1	0
y	0 ...	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1

x	1	2	3	... ∞
y	2	4	8	... ∞



D_f $(-\infty, \infty)$

$R_f =$ $[0, \infty)$

$-\infty$, (open interval)
x-এর মানের y '0' হয়
কি '0' open interval

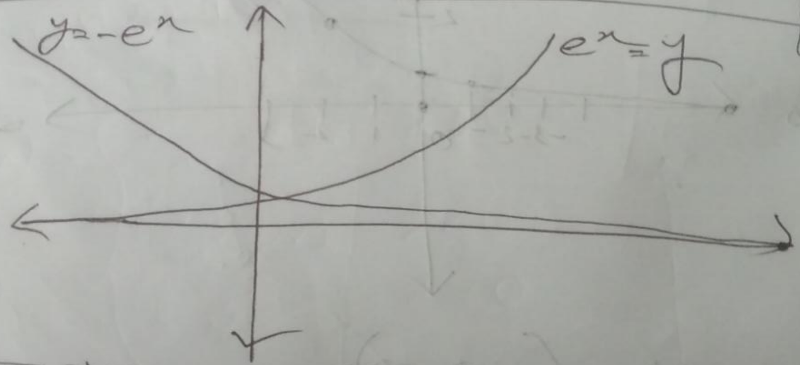
$2<e<3$

$0.1^{\infty}=0$	$2^{\infty}=\infty$
$0.1^{3}=0.0001$	$2^{3}=2\times2\times2$
$0.1^{\infty}=0.1\times0.1\times\cdots$	$2^{4}=2\times2\times2\times2$
	$2^{\infty}=2\times2\times\cdots\infty$

$y=e^x$ এর graph 2^x এর মতোই।

$$\boxed{-2^{\infty}=(-1)^{\infty}(2)^{\infty} = -\infty}$$

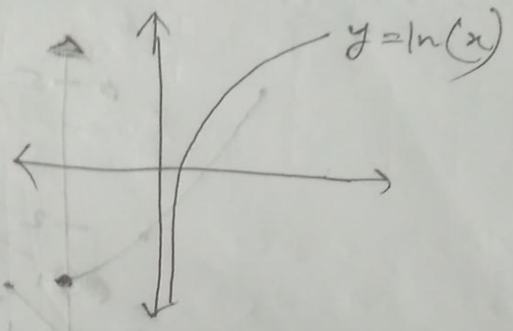
x	0	1	2	....	$-\infty$
y	1	2.712	5...		



$\boxed{\ln(1)=0}$ $\boxed{\ln(0)=-\infty}$

$\boxed{y=\ln x}$

x	0	1	2	3	4
y	0	0	0.693	1.09	1.386



page: $\boxed{164}$ $\boxed{Ex:4}$

Ex 13 iii)

$$f(x) = \begin{cases} x^2+1 \; ; & x<0 \\ x \; ; & 0 \le x \le 1 \\ \frac{1}{x} \; ; & 1<x \end{cases}$$

$\boxed{\text{step function}}$

লেখচিত্র, ডোমেন ও রেঞ্জ

Sol^n: $y = x^2+1 \; ; \; x<0$

x	0	-1	-2	. . .
y	1	2	5	. . .

$x<0 \Rightarrow (-\infty, 0)$

$R_f = ($

$y = x; \; 0 \le x \le 1$

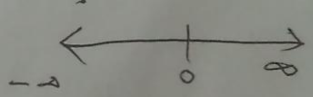
x	0	1
y	0	1

$0 \le x \le 1 \Rightarrow [0,1]$, $x>1$

$R_f = [0,1]$

$y = \frac{1}{x}$

x	1	2	3	. . . . +∞
y	1	$\frac{1}{2}$	$\frac{1}{3}$	 0



$D_f = (-\infty, 0) \cup [0,1] \cup (1, \infty)$

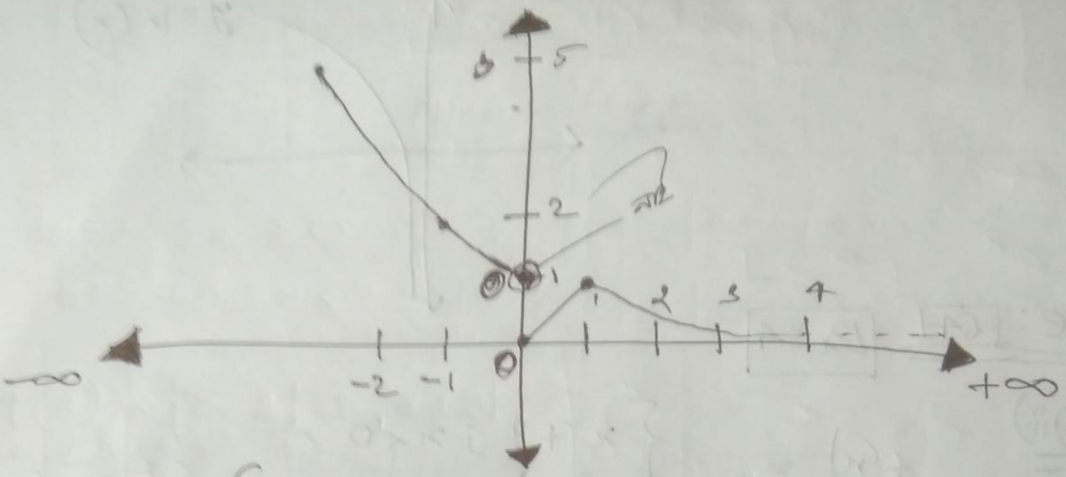
$= (-\infty, \infty)$

$= \mathbb{R}$

$R_f = \cancel{0, 1}$

$(1,\infty) \cup [0,1] \cup (0,1)$

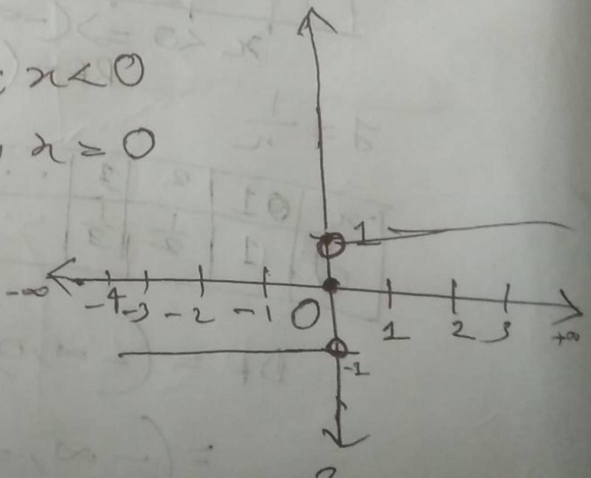
$= (0, \infty)$



$f(x) = \begin{cases} \frac{x}{|x|} ; & x \pm 1 \\ 0 ; & x = 0 \end{cases}$

ডোমেন, রেঞ্জ ও লেখচিত্র নির্ণয় কর।

Soln: $f(x) = \begin{cases} \frac{x}{x} = 1 ; & x > 0 \\ \frac{x}{-x} = -1 ; & x < 0 \\ 0 ; & x = 0 \end{cases}$



$f(x) = \begin{cases} 1 ; & x > 0 \\ 0 ; & x = 0 \\ -1 ; & x < 0 \end{cases}$

$\therefore D_f = (-\infty, 0) \cup \{0\} \cup (0, \infty) = (-\infty, \infty)$

$R_f = \{-1, 0, 1\}$

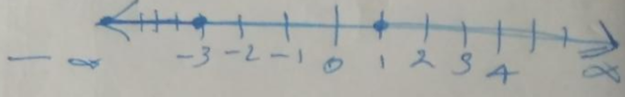
Date: 22. 01. 2020

"Ayub Ali Sir"

$|x| = x \; ; \; x \geq 0$ $\qquad$ $|x-0| = x-0 \; ; \; x-0 \geq 0$

$|x| = -x \; ; \; x < 0$ $\qquad$ $|x-0| = -x-0 \; ; \; x-0 < 0$

$f(x) = |x-1| + |x+3|$



$= |x-1| + |x-(-3)|$

$f(x) = -(x-1) - (x+3) \; ; \; \boxed{x < -3}$

$= -(x-1) - \{x-(-3)\}$

$= -2x-2 \; ; \; (x < -3)$

$x-1 \geq 0$

$x \geq 1$

$x+3 < 0$

$x < -3$

$-3 > x \geq 1$

$f(x) = -(x-1) + \{x-(-3)\}$ $\qquad$ $\boxed{-3 \leq x < 1}$

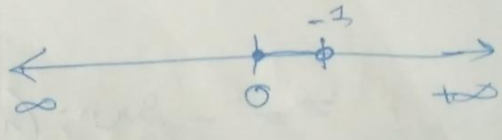
$= 4$

$f(x) = x - 1 + x - (-3) \; ; \; x \geq 1$

$= 2x + 2$

$$f(x) = \begin{cases} -2x-2; & x < -3 \\ 4; & -3 \le x < 1 \\ 2x+2; & x \ge 1 \end{cases}$$ (Ans)

\# $f(x) = |x| + |x-1|$



$= -x - (x-1); \quad x < 0$

$= -2x + 1; \quad x < 0$

$f(x) = |x| + |x-1| \quad (0 \le x < +1)$

$= 1$

$f(x) = |x| + |x-1| \quad (x \ge 1)$

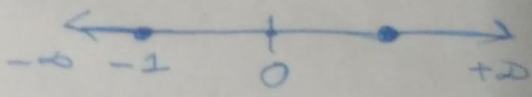
$= 2x - 1$

$$f(x) = \begin{cases} -2x+1; & x < 0 \\ 1; & 0 \le x < 1 \\ 2x-1; & x \ge 1 \end{cases}$$

(Ans)

$f(x) = |x+1| + |x| + |x-1|$ $\boxed{-1, 0, 1}$

$= -(x+1) - x - (x-1)$



$\boxed{x < -1}$

$= -3x ; (x < -1)$

$f(x) = (x+1) - x - (x-1) ; \boxed{-1 \le x < 0}$

$= x + 2 ; \quad (-1 \le x < 0)$ $x \to 1$

$f(x) = -(x+1) + x - (x-1) \quad \boxed{x > 0}$

$= -x + 2 ; \quad (-1 \le x < 0)$

$f(x) = x + 1 + x + x - 1 ; \quad (x \ge 1)$ $x \ge 1$

$= 3x ; (x \ge 1)$

$$f(x) = \begin{cases} -3x ; & \boxed{x < -1} \\ x + 2 ; & (-1 \le x < 0) \\ x + 2 ; & (-1 \le x < 0) \\ 3x ; & (x \ge 1) \end{cases}$$

(A)

$|\sin x| \leq 1$

$\boxed{64(23)}$

(6) ii)

$$f(x) = \begin{cases} x \sin \frac{1}{x} \;; & x \neq 0 \\ 0 \;; & x = 0 \end{cases}$$

এই function-এর ডোমেন, রেঞ্জ ও মোটামোটি লেখচিত্র অঙ্কন কর।

Sol^n:

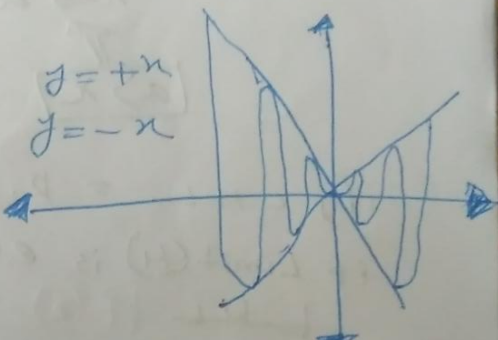
$y = \left| x \sin \frac{1}{x} \right|$

$= |x| \left| \sin \frac{1}{x} \right|$

$|y| \leq |x|$

$y = +x$

$y = -x$



$D_f = |$

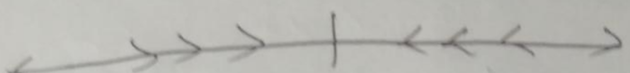
$= (-\infty, +\infty)$

$= \mathbb{R}$

$$\boxed{\begin{array}{l} x > 0 \Rightarrow (0, \infty) \\ x < 0 \Rightarrow (-\infty, 0) \\ x = 0 \Rightarrow \{0\} \end{array}}$$

$R_f = \mathbb{R}$

28.01.2020

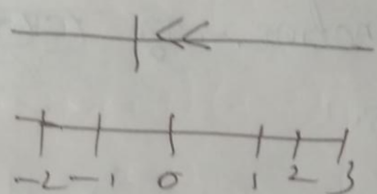


$\boxed{x = 0.999}$

R.H.S. $= \underset{x \to a^+}{Lt} f(x)$

↓

Limit $x \to a$



L.H.L. $= \underset{x \to a^-}{Lt} f(x)$

$\boxed{a > x}$

$x = 1.0001$

if, L.H.L. = R.H.L. = ~~High~~ l (Finite)

if Limit (H) is equal and finite ~~then~~ limit,

~~L.H.L.~~

$\underset{x \to a}{Lt} f(x) = \underset{x \to a^+}{Lt} f(x) = l$, then

$= \underset{x \to a}{Lt} f(x) = l$

$\exists \to$ There exist

Page: 83

$$f(x) = \frac{x^3 - 1}{x - 1}$$

$$|x - a| = 1$$

$$|x-1| < \delta + |f(x) - 3| < \varepsilon$$

Now, $|f(x) - 3| = \left|\frac{x^3-1}{x-1} - 3\right|$

$$= \left|\frac{(x^2+x+1)(x-1)}{(x-1)}\right|$$

$$= |x^2 + x + 1|$$

$$= |x^2 + 2x + 1| + |x - 3|$$

$$= |(x-1)^2 + 3(x-1)^2|$$

$$\cancel{|f(x) - 3| = x-1)^2 +}$$

$$|f(x) - 3| < |(x-1) + 3(x-3)|$$

$$\Rightarrow |f(x) - 3| < |4(x-1)|$$

$$\Rightarrow |f(x) - 3| < 4|x-1|$$

$$\Rightarrow |f(x) - 3| < 4\delta$$

$$\Rightarrow |f(x) - 3| < \varepsilon$$

α

$$\boxed{\begin{aligned} \varepsilon &= 4\delta \\ \delta &= \frac{\varepsilon}{4} \end{aligned}}$$

Language - C : ~~Dr.~~ Zulfiquar Sir ? 26.01.2020

Power $250

Minimum
Row 1
$n_1 + n_2$

Arithmetic operators

2nd X

Relational
↓
Logical

a = 5
b = 3
c = 10
d = 25

a > b && c < d
= 5 > 3 && 10 < 25
= True && True
= True (Ans)

6532~~8~~ 8 $s = 65328 \backslash 10$
6532 $s = 65328 \% 10$
653
65
6

Page-81

$$f(x) = \begin{cases} e^{-\frac{|x|}{2}} ; & -1<x<0 \\ x^2 ; & 0 \le x \le 2 \end{cases}$$

$\boxed{\begin{array}{l} |x| = x ; x>0 \\ |x| = -x ; x<0 \end{array}}$

$\lim_{x \to 0} = a = 0$

$$L.H.S. = \lim_{x \to 0^-} e^{-\frac{|x|}{2}}$$

$$= \lim_{x \to 0^-} e^{-\left(\frac{-x}{2}\right)} \quad [x<0]$$

$$= \lim_{x \to 0^-} (e)^{0} = 1$$

$$R.H.L. = \lim_{x \to 0^+} f(x)$$

$$= \lim_{x \to 0^+} x^2$$

$= 0$

$R.H.S. \neq L.H.S.$

So, $\lim_{x \to 0} f(x)$ doesn't exist.

~~but~~

~~So, $\lim_{x \to 0} f(x)\ x \to 0 \quad f(x)$ de~~

Ayub Ali Sir

Date: 29.01-200

$$f(x) = \begin{cases} x\cos\left(\frac{1}{x}\right); & x \neq 0 \cdot x > 0, x < 0 \\ 0 & x = 0 \end{cases}$$

$$L.S. = \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$

$$= \lim_{x \to 0} (x^2) \mathop{Lt}_{x \to 0} \cos\left(\frac{1}{x}\right)$$

$= 0 \times$ (Any values -1 and $+1$)

$= 0$

$$R.S. = \mathop{Lt}_{x \to 0^+} f(x)$$

$$= \mathop{Lt}_{x \to 0^+} x^2 \cos\left(\frac{1}{x}\right)$$

$$= \lim_{x \to 0^+} x^2 \lim_{x \to 0^+} \cos\left(\frac{1}{x}\right)$$

$= 0 \times$ (Any values: -1 and $+1$)

$\therefore$ L.H.L = R.HL $= 0$

$\therefore \lim_{x \to 0} f(x)$ exist

83 page

$f(x) = \begin{cases} \tan \frac{x}{2}; & x < \frac{\pi}{2} \\ \sin 3 - \frac{\pi}{2}; & x = \pi/2 \\ \dfrac{x^3 - \frac{\pi^3}{8}}{x - \frac{\pi}{2}} & x > \frac{\pi}{2} \end{cases}$

$$\text{R.H.L.} = \underset{x \to \frac{\pi}{2}^+}{Lt} f(x)$$

$$= \underset{x \to \frac{\pi}{2}^+}{Lt} \left\{ \frac{(x)^3 - \left(\frac{\pi}{2}\right)^3}{x - \frac{\pi}{2}} \right\}$$

$$= \underset{x \to \frac{\pi}{2}^+}{Lt} \left(x^2 + \frac{\pi}{2} x + \frac{\pi^2}{4} \right)$$

$$= \frac{\pi^2}{4} + \frac{\pi^2}{4} + \frac{\pi^2}{4}$$

$$= \frac{3\pi^2}{4}$$

$$\text{L.H.L.} = \underset{x \to \frac{\pi}{2}^-}{Lt} f(x)$$

$$= \underset{x \to \frac{\pi}{2}^-}{Lt} \tan \frac{x}{2}$$

$$= \tan \frac{\pi}{4} = 1$$

$\therefore$ L.H.L. $\neq$ R.H.L

Hence $\underset{x \to \frac{\pi}{2}}{} f(x)$

Hence, $\underset{x \to \pi/2}{Lt} f(x)$ Limit does not exist.

"Ayub Ali Sir" Date: 04.02.2020

অবিচ্ছিন্নতা (Continuity)

$\underset{x \to a}{Lt} f(x) = f(a)$

$\underset{x \to a^+}{Lt} f(x) = \underset{x \to a^-}{Lt} f(x) = f(a) = l$

Then $f(x)$ is continious at $x = a$

Otherwise $f(x)$ is $\boxed{\text{discontinious}}$ at $x = a$;

$|x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$

$$\boxed{\begin{aligned} \underset{x \to a^+}{Lt} f(x) &= \underset{h \to 0}{Lt} f(a+h) = R.H.S. \\ \underset{x \to a^-}{Lt} f(x) &= \underset{h \to 0}{Lt} f(a-h) = L.H.S. \end{aligned}}$$

$\underset{h \to 0}{Lt} f(a+h) = \underset{h \to 0}{Lt} f(a-h) = f(a) = l$ (finite)

$f(x)$ is continious at $x = a$.

$$\begin{cases} \lim\limits_{h \to 0^+} f(a+h) = \lim\limits_{x \to a^+} f(x) \\ \cancel{f(x)} = \lim\limits_{x \to a^-} f(x) = \lim\limits_{h \to 0^-} f(a+h) \end{cases}$$

$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & ; x \neq 2 \\ 3 & ; \text{for } x = 2 \end{cases}$

$\boxed{\text{Sol}^n:}$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} (x+2)$$

$$= \cancel{\lim_{x \to 2^+}} (2+2) = 4$$

$$\lim_{x \to 2^-} f(x)$$

$$\boxed{\therefore f(2) = 3}$$

$$= \lim_{x \to 2^-} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^-} (x+2)$$

$$= (2+2) = 4$$

$$\therefore \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) \neq f(2) \quad \underline{\text{(Ans)}}$$

$f(x) = \dfrac{1}{5 + e^{\frac{1}{x-2}}}$

dicuss continuity at $x = 2$;

$\boxed{\text{Sol}^n:}$

$$R.H.S. = \lim_{x \to 2^+} f(x)$$

$$= \lim_{h \to 0} f(2+h)$$

$$= \lim_{h \to 0} \frac{1}{5 + e^{\frac{1}{2+h-2}}}$$

$$= \frac{1}{5 + e^{\frac{1}{h}}}$$

$$= \frac{1}{5 + e^{\infty}}$$

$$= \frac{1}{\infty} = 0$$

$$L.H.S. = \lim_{x \to 2^-} f(x)$$

$$= \lim_{h \to 0} f(2-h)$$

$$= \lim_{h \to 0} \frac{1}{5 + e^{\frac{1}{2-h-2}}}$$

$$= \lim_{h \to 0} \frac{1}{5 + e^{\frac{1}{-h}}}$$

$$= \frac{1}{5 + e^{\frac{-1}{0}}} = \frac{1}{5 + e^{-\infty}} = \cancel{\frac{1}{5}} = 0$$

$$= \frac{1}{5 + \frac{1}{e^{\infty}}} = \frac{1}{5 + 0} = \frac{1}{5}$$

$$f(2) = \frac{1}{5 + e^{\frac{1}{2-2}}}$$

$$= \frac{1}{5 + e^{\frac{1}{0}}} = \frac{1}{5 + e^{\infty}} = \frac{1}{\infty} = 0$$

$\therefore \ \underset{x \to 2^+}{Lt} f(x) \neq \underset{x \to 2^-}{Lt} f(x) \neq \underset{x \to 2}{Lt} f(2)$

$\therefore$ $f(x)$ is discontinious at $x = 2$.

$[x] \rightarrow$ greatest integer function

বড় ~~[illegible]~~ নয় পূর্ণসংখ্যা

$\left[\frac{3}{2}\right] = 1$

$[0-] = -1$

"Ayub Ali Sir"

Continuity

Differenciivity:

L.H.L. = R.H.L. = l (Limit exists)

L.H.L. = R.H.L. = $f(a) = l$

L.H.D. = R.H.D. = l (finite)

$$R.H.D. = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

$$L.H.D. = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

$$L.H.D. = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$$ ~~= L.H.D = Lt h→0⁻~~

$$R.H.D. = \lim_{h \to 0} \frac{f(a-h) \mp f(a)}{\mp h}$$

If R.H.D. = R.H.D $= l$, Then $f(x)$ is differentiable at $x=a$,

Otherwise $f'(a)$ does not $x=a$

#

$\boxed{\text{135 Page}}$

$\boxed{3.32}$ If $f(x)$ is differentiable of at.

We know,

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$\lim_{h\to 0} \left[f(a+h) - f(a)\right]$$

$$= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \times h$$

$$= \lim_{h\to 0} \cancel{\left[f(a+h) - f(a)\right]}$$

$$= f(a)$$

$$= f'$$

$$= f'(a) \times \lim_{h\to 0} h$$

$$= f'(a) \times 0$$

$$= 0$$

For differenciavity,

$$R.H.D. = \underset{h \to 0^+}{Lt} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h}$$

$$= \lim_{h \to 0^+} \frac{1-(\frac{1}{2}+h) - (1-\frac{1}{2})}{h} = \underset{h \to 0^+}{Lt} \frac{-h}{h}$$

$$= -1$$

$$= \cancel{1-h = 1-\frac{1}{2} = \frac{1}{2}}$$

$$L.H.D. = \underset{h \to 0^-}{Lt} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h}$$

$$= \underset{h \to 0}{Lt} \frac{\frac{1}{2}+h - (1-\frac{1}{2})}{h}$$

$$= \underset{h \to 0^-}{Lt} \frac{\frac{1}{2}+h - \frac{1}{2}}{h}$$

$$= \underset{h \to 0^-}{Lt} \frac{h}{h} = 1$$

$\therefore$ R.H.D $\neq$ L.H.D $\therefore f'(\frac{1}{2})$ does not exist.

$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$

$$R.H.D = \underset{h \to 0}{Lt} \frac{f(0+h) - f(0)}{h}$$

$$= \underset{h \to 0^+}{Lt} \frac{(0+h)^2 \sin\frac{1}{0+h} - 0}{h}$$

$$= \underset{h \to 0}{Lt}\ h \sin\frac{1}{h} = 0 \times \text{Any value of } -1 \text{ to } 1$$
$$= 0$$

$$L.H.D. = \underset{h \to 0^-}{Lt} \frac{f(0+h) - f(0)}{h}$$

$$= \underset{h \to 0^-}{Lt} \frac{(0+h)^2 \sin\frac{1}{0+h} - 0}{h}$$

$$= \underset{h \to 0}{Lt}\ h \sin\frac{1}{h}$$

$$= 0 \times \text{Any value } -1 \text{ to } 1$$

$$= 0$$

$\therefore$ L.H.D. = R.H.D = 0 $\therefore$ so, $f'(0)$ exists
so, $f(x)$ is continuity at $x = 0$.

$$\underset{h\to 0}{Lt}\left[f(a+h)-f(a)\right]=0$$

$$\underset{h\to 0}{Lt}\ f(a+h)=f(a)$$

#math:

$$f(x)=\begin{cases} x, & 0\le x<\frac{1}{2} \\ 1-x, & \frac{1}{2}\le x<1 \end{cases}$$

~~$x=\frac{1}{2}$~~

For continity,

$$R.H.L.=\underset{x\to\frac{1}{2}^+}{Lt}f(x)=\underset{x\to\frac{1}{2}^+}{Lt}(1-x)$$

$$=1-\frac{1}{2}=\frac{1}{2}$$

$$L.H.L.=\underset{x\to\frac{1}{2}^-}{Lt}f(x)=\underset{x\to\frac{1}{2}^-}{Lt}(x)=\frac{1}{2}$$

$$\therefore f\left(\frac{1}{2}\right)=1-\frac{1}{2}=\frac{1}{2}$$

$$\therefore L.H.L.=R.H.L.=f\left(\frac{1}{2}\right)$$

$\therefore f(x)$ is continuous at $x=\frac{1}{2}$.

$f'(x) = \begin{cases} \left(2x \sin\frac{1}{x} + x^2 \cos\frac{1}{x} \cdot \frac{1}{x^2}\right) ; & x \neq 0 \\ 0 ; & x = 0 \end{cases}$

$f'(x) = \begin{cases} \left(2x \sin\frac{1}{x} - \cos\frac{1}{x}\right) ; & x \neq 0 \\ 0 ; & x = 0 \end{cases}$

$$\text{L.H.L} = \lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} \left\{2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)\right\}$$

$= 0 \times$ Any value of -1 to 1

$-$ Any value of -1 to 1

~~doesnot exist~~

$=$ does not exist (অনির্ণেয়)

$$\text{R.H.L.} = \lim_{x \to 0^+} f(x)$$

$$= \lim_{x \to 0^+} \left(2x \sin\frac{1}{x} - \cos\frac{1}{x}\right)$$

$=$ does not exist (অনির্ণেয়)

$f(0) = 0$

$\therefore$ R.H.L. $\neq$ L.H.L.

$\therefore$ So, $f(x)$ ~~does not~~ is discontinuous at $x=0$ ।

Date: 21.02.2020

$$L.H.D. = \lim_{h\to 0} \frac{f(a-h)-f(a)}{h} = \lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h}$$

$$R.H.D. = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h}$$

Page: 135:

অন্তরীকরণযোগ্য ফাংশন অবিচ্ছিন্ন হবে কিন্তু অবিচ্ছিন্ন ফাংশন অন্তরীকরণযোগ্য নাও হতে পারে।

$x=a$ বিন্দুতে ফাংশনটি অন্তরীকরণযোগ্য।

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Now, $\lim_{h\to 0} \left[f(a+h)-f(a)\right]$

$$= \lim_{h\to 0} \left[\frac{f(a+h)-f(a)}{h}\right]$$

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \times \lim_{h\to 0} h$$

$= f'(a) = 0$

$\therefore \underset{h \to 0}{Lt} f(a+h) = f(a)$

$R.H.L. = \underset{h \to 0^+}{Lt} f(a+h)$

$L.H.L. = \underset{h \to 0^-}{Lt} f(a-h)$

$\boxed{\underset{h \to 0}{Lt} f(a+h)}$

∴ লিমিট ও ফাংশনের value সমান, তাই ফাংশন continuous.

∴ Diffentiable হলে continous হবে।

137 Page

অন্তরীকরণযোগ্য ফাংশন অবিচ্ছিন্ন হবে, অবিচ্ছিন্ন হলেও ফাংশন অন্তরীকরণযোগ্য হবে না।

$$f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ 1-x; & \frac{1}{2} \le x < 1 \end{cases}$$

For continity,

$$R.H.L. = \underset{h \to 0^+}{Lt} f\left(\frac{1}{2}+h\right)$$

$$= \underset{h \to 0^+}{Lt} \left\{1 - \left(\frac{1}{2}+h\right)\right\}$$

$$= 1 - \frac{1}{2} - h = \frac{1}{2}$$

$$\text{L.H.L.} = \underset{h \to 0^-}{Lt} f\left(\tfrac{1}{2}+h\right)$$

$$= \underset{h \to 0^-}{Lt} \left(\tfrac{1}{2}+h\right)$$

$$= \frac{1}{2}$$

$$\therefore f\left(\tfrac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

R.H.L. = L.H.L. = $f\left(\frac{1}{2}\right)$. So, $f(x)$ is continuous at $x = \frac{1}{2}$.

$$\text{R.H.L.} = \underset{x \to \frac{1}{2}^+}{Lt} f(x)$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{L.H.L.} = \underset{x \to \frac{1}{2}^-}{Lt} f(x)$$

$$= \frac{1}{2}$$

$$\therefore f\left(\tfrac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

For differentiability:

b.

$$\text{R.H.D.} = \underset{h \to 0^+}{Lt} \frac{f\left(\frac{1}{2}+h\right) - f\left(\frac{1}{2}\right)}{h}$$

$$= \underset{h \to 0^+}{Lt} \frac{\left\{1-\left(\frac{1}{2}+h\right)\right\} - \left(1-\frac{1}{2}\right)}{h}$$

$$= \frac{\frac{1}{2} - h - \frac{1}{2}}{h} = \underset{h \to 0^+}{Lt} \frac{-h}{h} = -1$$

$$\text{L.H.D.} = \underset{h \to 0^-}{Lt} \frac{f\left(\frac{1}{2}+h\right) - f\left(\frac{1}{2}\right)}{h}$$

$$= \underset{h \to 0^-}{Lt} \frac{\left(\frac{1}{2}+h\right) - \frac{1}{2}}{h} = \underset{h \to 0^-}{Lt} \frac{\frac{1}{2}+h-\frac{1}{2}}{h} = \frac{h}{h} = 1$$

$\therefore$ R.H.D. $\neq$ L.H.D.

$\therefore$ So, $f(x)$ is not differentiable at $x = \frac{1}{2}$ or

or, $f'(x)$ does not exist at $x = \frac{1}{2}$

Hence, $f(x)$ is continuous at $x = \frac{1}{2}$ but not differentiable.

#

$$f(x) = \begin{cases} 1 & ; x < 0 \\ 1+\sin x & ; 0 \le x < \frac{\pi}{2} \\ 2+\left(x-\frac{\pi}{2}\right)^2 & ; x \ge \frac{\pi}{2} \end{cases}$$

For continuity at $x = \frac{\pi}{2}$:

$$\text{R.H.L.} = \lim_{x \to \frac{\pi}{2}^+} f(x)$$

$$= \lim_{x \to \frac{\pi}{2}^+} \left[2 + \left(x - \frac{\pi}{2}\right)^2\right] \quad \left[\begin{array}{l}\pi/2 \text{ থেকে} \\ x \text{ বড়}\end{array}\right]$$

$$= 2 + \left(\frac{\pi}{2} - \frac{\pi}{2}\right)^2 = 2 + 0 = 2$$

$$\text{L.H.L.} = \lim_{x \to \frac{\pi}{2}^-} f(x)$$

$$= \lim_{x \to \frac{\pi}{2}^-} (1 + \sin x) = 1 + \sin \pi/2 = 1 + 1 = 2$$

$\therefore f\left(\frac{\pi}{2}\right) = 2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2$

$= 2+0^2 = 0$

$\therefore$ L.H.L. = R.H.L. = functional value.

$\therefore f(x)$ is continuous at $x=\frac{\pi}{2}$.

for differentiability at $x = \pi/2$:

$$f(x) = \begin{cases} 1; & x<0 \\ 1+\sin x; & 0 \le x < \frac{\pi}{2} \\ 2+\left(x-\frac{\pi}{2}\right)^2; & x \ge \frac{\pi}{2} \end{cases}$$

~~R.H.D. = $\lim_{h \to \frac{\pi}{2}^+} f$~~

$$\text{R.H.D.} = f'\left(\frac{\pi}{2}\right) = \lim_{h\to 0^+} \frac{\left(\frac{\pi}{2}+h\right) - \frac{\pi}{2}}{h}$$

$$= \lim_{h\to 0^+} \frac{2+\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)^2 - \left\{2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2\right\}}{h}$$

$$= \lim_{h\to 0^+} \frac{2+h^2-2}{h} = h = 0$$

$$\text{L.H.D.} = \lim_{h\to 0^-} \frac{f\left(\pi/2+h\right) - f\left(\pi/2\right)}{h} \quad \left[\begin{matrix}\pi/2 \\ h\end{matrix}\right]$$

$$= \lim_{h\to 0^-} \frac{1+\sin\left(\frac{\pi}{2}+h\right) - \left(2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2\right)}{}$$

$$= \lim_{h\to 0^-} \frac{1+\cos h - 2}{h} = \frac{(1-\cos h)}{h}$$

$$\therefore L.H.D. = \underset{h\to 0^-}{Lt} \frac{2\sin^2 h/2}{h}$$

$$= 2 \underset{h\to 0}{Lt} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h}{4}$$

$$= \frac{1}{2} \underset{h\to 0^-}{Lt} \left(\frac{\sin h/2}{h/2}\right)^2 \times \underset{h\to 0}{Lt} h$$

$$= \frac{1}{2} \times 1 \times 0 = 0$$

$$R.H.D. = L.H.D. = 0$$

$$\frac{d}{dx}(uv) = u\frac{du}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{dv}{dx} - u\frac{du}{dx}}{v^2}$$

$\boxed{\text{Page: 212}}$

$y = \sec x^\circ$

$y = \sec \frac{\pi x}{180}$

$$\boxed{\begin{aligned} 180^\circ &= \pi \\ 1^\circ &= \frac{\pi}{180} \\ x &= \frac{\pi x}{180} \end{aligned}}$$

D.W.R. to x

$$\frac{dy}{dx} = \sec\left(\frac{\pi x}{180}\right) \cdot \tan\left(\frac{\pi x}{180}\right) \frac{d}{dx}\left(\frac{\pi x}{180}\right)$$

$$= \frac{\pi}{180} \sec x^\circ \tan x^\circ \text{ (Ans.)}$$

(vi) $y = \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)^{1/2}$

$$= \tan^{-1}\left(\frac{2\sin^2 x/2}{2\cos^2 x/2}\right)^{1/2}$$

$$= \tan^{-1} \tan x/2$$

$$y = x/2$$

(4) 802

∴ D.W.R. to x:

$$\frac{dy}{dx} = \frac{1}{2} \text{ (Ans.)}$$

(vii) অব্যক্ত ফাংশন : (Implicit function) :

$y = x^{x^x}$

$\cancel{\Rightarrow \ln y = x}$

Taking ln on both sides,

$$\ln y = x^x \ln x^x$$

$$\Rightarrow \ln(\ln y) = x \ln(x \ln x)$$

$\Rightarrow \frac{1}{\ln y}\frac{dy}{dx}\frac{1}{y} = x\ln x + x\ln(\ln x)$

$\Rightarrow \frac{dy}{dx}\frac{1}{y} = \ln y \left(\cancel{x \ln x}\ \frac{x}{\cancel{x}} + \cancel{\ln x} + 1 + \frac{1}{\ln x}\cdot\frac{1}{x}\right)$

$\Rightarrow \frac{dy}{dx} = y \cdot \ln y \left(\cancel{y + \frac{1}{x} + \frac{1}{x\ln x}}\right)$

$\left(1 + \ln x + \frac{1}{x\ln x}\right)$

$= x^{x^x} \cdot \ln x^{x^x} \left(1 + \ln x + \frac{1}{x\ln x}\right)$

$\therefore \frac{dy}{dx} = x^{x^x} \cdot x^x \ln x \left(1 + \ln x + \frac{1}{x\ln x}\right)$

(Ans)

#

$y = (\cancel{\sec}\cdot\sin x)^{\cos x} + (\cos x)^{\sin x}$

$\boxed{y = u + v}$ $\qquad u = (\sin x)^{\cos x}$

$\therefore \ln u = \cos x \ln \sin x$

$\therefore \frac{1}{u}\frac{du}{dx} = \cancel{\cos x}\ \frac{1}{\sin x} \cdot \cos x - \sin x \ln(\sin x)$

$\frac{du}{dx} = u\left[\cos x \cdot \cot x - \sin x \ln(\sin x)\right]$

$= (\sin x)^{\ln x}\left[\cos x \cdot \cot x - \sin x \ln(\sin x)\right]$

$\ln v = \sin \ln \cdot (\cos x)$

$\frac{1}{v}\frac{dv}{dx} = \sin x \cdot \frac{1}{\cos x}(\sin x) + \cos x \ln(\ln x)$

$\frac{dv}{dx} = v\left[\cos x \ln(\cos x) - \sin x \cdot \tan x\right]$

$= (\cos x)^{\sin x}\left[\cos x \ln(\cancel{\cos x}\ \sin x) - \sin x \cdot \tan x\right]$

$y = u + v$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ (Ans)

$y = e^{\ln(\sin x)^{\cos x}} + e^{\ln(\cos x)^{\sin x}}$

$y = e^{\cos \ln(\sin x)} + e^{\sin x \ln(\cos x)}$

D.W.R. to x:

$\frac{dy}{dx} = e^{\cos x \sin (\ln x)}\left[\cos x \cdot \frac{1}{\sin x} \cdot \cos x - \sin x \ln(\sin x)\right]$

$+ e^{\sin x \ln(\cos x)}\left[\cancel{\cos}\ \sin x \cdot \frac{1}{\cos x} + \cos x \cdot \ln(\cos x)\right]$

$= (\sin x)^{\ln x}\left[\cancel{g} \cdot \sin x \cdot \cot x - \sin x \cdot \ln \frac{1}{x}(\sin x)\right]$
$+ (\cos x)^{\sin x} - \left[\cos x \ln(\sin x) - \sin x \cdot \tan x\right]$

page – 257

$z = \ln x$ এর সাপেক্ষে $y = x^{\sin^{-1} x}$ কে diff করো,

$$\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} =$$

$\frac{dz}{}$

$$\ln y = \sin^{-1}(x) \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right)$$

$$\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(1+x)}\sqrt{(1+x^2)}\cos}}}}$$

P-239

(ii) $x^y = e^{x-y}$

Taking ln of both sides,

$y \ln x = (x-y) \ln e$

$\Rightarrow y.\ln x = x-y$

$\Rightarrow y.\frac{1}{x} + \ln x . \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow (1+\ln x)\frac{dy}{dx} = 1 - \frac{y}{x} = \frac{x-y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\ln x)}$ (Ans.)

P- 256:

$$y = (\sin x)^{(\sin x)^{(\sin x)^{\cdots\infty}}}$$

$$\ln y = \ln (\sin x)^{(\sin x)^{(\sin x)^{\cdots}}}$$

Taking ln on both sides:

$$\ln y = (\sin x)^{(\sin x)^{\cdots\infty}} \ln (\sin x)$$

Differenciation w.r. to x,

$$\frac{1}{y}\frac{dy}{dx} = y \cdot \frac{\cos x}{\sin x} + \ln(\sin x)\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y} - \ln(\sin x)\frac{dy}{dx} = y \cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \cot x}{\frac{1}{y} - \ln(\sin x)} = \frac{y^2 \cot x}{1 - y\ln(\sin x)}$$

Successive differentiation:

$y \longrightarrow y_5$

$\frac{dy}{dx} = y_1$

$\frac{d^2y}{dx^2} = y_2$

$y_n = \frac{d^n y}{dx^n}$

$y = x^4$

$y_1 = 4x^3$

$y_2 = 12x^2$

$y_3 = 24x$

$y_4 = 24$

$y_5 = 0$

x^4, y_4 } হলে

$y_4 = 4!$

$y_5 = 0$

x^n হলে, $y_n = n!$

$y_{(n+1)} = 0$

৬ষ্ঠ অধ্যায়:

Page: 301:

$(uv)_n = u_n v + {}^nc_1 u_{n-1} v_1 + {}^nc_2 v_{n-2} v_2 + \dots + uv_n$

Let $y = uv$

Differentiation with respect to x:

${}^nc_r = \frac{n!}{r!(n-r)!}$

$y_1 = v\frac{du}{dx} + u\frac{dv}{dx} = u_1 v + uv_1$

$y_2 = u_2 v + v_1 u_1 + v_2 u + u_1 v_1$

$= u_2 v + 2u_1 v_1 + uv_2$

$= u_2 v + {}^2c_1 u_2 v_1 + uv_2$

$y_3 = u_3 v + u_2 v_1 + 2 u_2 v_1 + 2 u_1 v_2 + u_1 v_2 + u v_3$

$= u_3 v + 3 u_2 v_1 + 3 u_1 v_2 + u v_3$

$=$

$n = 1, 2, 3;$

$y_m = u_m v + {}^mC_1 v$

$y_m = u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \cdots + u v_m$

$y_{m+1} = u_{(m+1)} v + u_m \cancel{v_1} + {}^mC_1 (u_{m-1+1} v_1) + {}^mC_1 u_{m-1} v_2 + {}^mC_2 u_{m-2+1} v_2 + {}^mC_2 u_{m-2} v_3 + \cdots + u_1 v_m + u v_{m+1}$

$= u_{m+1} v + (1 + {}^mC_1) u_{m+1-1} v_1 + ({}^mC_1 + {}^mC_2) u_{(m+1-2)} v_2 + \cdots + u v_{m+1}$

$\boxed{1 + {}^mC_1 = {}^{m+1}C_1}$

$y_{m+1} = u_{m+1} v + {}^{m+1}C_1 u_{m+1-1} v_1 + {}^{m+1}C_2 u_{m+1-2} v_2 + \cdots + u v_{m+1}$

for $n = m+1$, m এর জন্য $(uv)_n$ সত্য।

~~$y_{m+1} = u_{m+1} v + {}^{(m+1)}C_1 u_{(m+1-1)} +$~~

$\therefore$ n এর সকল যোগাত্মক পূর্ণ সংখ্যার জন্য এটি সত্য.

Page - 305 $\Rightarrow$ (সূত্র)

$D^n \left(y \right.$

Page-310

(2)

$y = x^n \ln x$

$y_{n+1} = \frac{\lfloor n}{x}$ $\boxed{x > 0}$

$y = x^n \ln x$

Diff - - .

$y_1 = x^n \cdot \frac{1}{x} + \ln x \cdot x^{n-1}$

$\Rightarrow x y_1 = x^n + n x^n \ln x$

$\Rightarrow \underset{v}{x} \underset{u}{y_1} = x^n + n y$

Ans

$x \rightarrow v$
$y \Rightarrow u$

Diff. . . . .

$\Rightarrow \quad y_{1+n}\, x + {}^nC_1\, y_{1+x-1}\, 1 + 0 = \lfloor n + x y_n$ power's diff. eqn.

$\Rightarrow$ $y_{1+n}\, x + {}^nC_1\, y_1 \cancel{+}\, x$

$x\, y_{n+1} + x y_n = \lfloor n + n y_n$

$\therefore y_{n+1} = \dfrac{\lfloor n}{x}$ (Ans)

—— X ——

318

$x = \sin\left(\frac{1}{m} \ln y\right)$

$(1-x)\, y_{n+2} - (\cancel{2n}\,2m+1)\, x\, y_{n+1}$

$- (x^2 + m^2)\, y_n = 0$ $\qquad (y_n)_0 = ?$

$\Rightarrow \frac{1}{m} \cdot \ln y = \sin^{-1} x$

$\Rightarrow \ln y = m(\sin^{-1} x)$

$\Rightarrow y = e^{m(\sin^{-1} x)}$

(1) w.r. to x;

$\therefore y_1 = e^{m \sin^{-1} x} \frac{m}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\, y_1 = m y$

$\Rightarrow (1-x^2)\, y_1^2 = m^2 y^2$

$\Rightarrow (1-x^2) \cdot 2y_1 y_2 - 2x y_1^2 = 2m^2 y$

$\Rightarrow (1-x^2)\, y_2 - x y_1 - m^2 y = 0$

D. up to n times by L. theorem,

$$y_{2+n} \cdot (1-x^2) + {}^nc_1\, y_{1+n-1}\,(-2x) + {}^nc_2 \cdot y_{2+n-2}\,(-2) + 0 - y_{(1+n)} \cdot x - {}^nc_1\, y_{1+n-1} \cdot 1$$
$$+ 0 - m^2 y_n = 0$$

$\Rightarrow$

$$u_n v + {}^nc_1\, v_1 + {}^nc_2\, u_{n-2} v_1 + \cdots \Big[$$
$$+ u v_n$$

\# $(1-x^2)\, y_{n+2} - 2xn\, y_n + \frac{n(n-1)!}{2!} \cdot 2$
$$- x y_{1+n} - n\, y_n - x y_n - m^2 y_n = 0$$

$\Rightarrow$ $(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} - (n^2 - n + n + m^2)\, y_n = 0$

$\Rightarrow$ $(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} - (n^2 + m^2)\, y_n = 0$

When 'n' is odd;

\# $y = e^{m \sin^{-1} x}$ ——— (i)

$$\left(\sqrt{1-x^2}\right) y_1 = my \text{ ——— (2)}$$

$(\sqrt{1-x^2})y_2 - xy_1 - m^2 y = 0$ —— (3)

$(1-x^2)y_{m+2} - (2m+1)xy_{(m+1)} - (n^2+m^2)(y_m)$

$(y_1)_0 = m \quad (y_2)_0 = m^2 \quad (y_3)_0 =$

$n = 1, 3, 5, \ldots (n-2)$

$(y_3)_0 = (1^2+m^2)(y_1)_0 = (1^2+m^2)m$

$(y_5)_0 = (3^2+m^2)(y_3)_0 = (3^2+m^2)(1^2+m^2)m$

$(y_n)_0 = \{(n-2)^2 + m^2\}(y_{n-2})_0$

$$\boxed{(y_n)_0 = \{(n-2)^2+m^2\} \ldots (3^2+m^2)(1^2+m^2)m}$$

When 'n' 'is' even,

$n = 2, 4, 6, \ldots, (n-2)$ in (5)

$(y_4)_0 = (2^2+m^2)(y_2)_0 = (2^2+m^2)m^2$

$(y_6)_0 = (4^2+m^2)(y_4)_0 = (4^2+m^2)(2^2+m^2)m^2$

~~$(y_6)_0 = (6$~~

$(y_8)_0 = (6^2+m^2)(y_6)_0 = (6^2+m^2)(4^2+m^2)(2^2+m^2)m^2$

$\ldots\ldots\ldots\ldots$

$(y_n)_0 = \{(n-2)^2+m^2\}\{(n-4)^2+m^2\}\ldots\ldots$
$\ldots\ldots(4^2+m^2)(2^2+m^2)m^2$

$\boxed{\text{page - 319}}$ $\boxed{\text{8 no. math}}$

321 page (2.1):

$y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$

D. W. r. to x:

$\frac{1}{m} y^{\frac{1}{m}-1} \cdot y_1 - \frac{1}{m} y^{-\frac{1}{m}-1} \cdot y_1 = 2$

$\Rightarrow \left(\frac{y^{\frac{1}{m}}}{y} - \frac{y^{\frac{1}{n}}}{y}\right) y_1 = 2m$

$(x^2-1) y_{m+2}$
$\cancel{+2}$
$+ (2m+1) x y$

$+ (n^2-m^2) y_n = 0$

$\Rightarrow \left(y^{\frac{1}{m}} - y^{-\frac{1}{m}}\right) y_1 = 2my$

$\Rightarrow \left(y^{\frac{1}{m}} - y^{-\frac{1}{m}}\right)^2 y_1^2 = 4m^2y^2$

$\Rightarrow \left\{\left(y^{\frac{1}{m}} + y^{-\frac{1}{m}}\right)^2 - 4 \cdot y^{\frac{1}{m}} \cdot y^{-\frac{1}{m}}\right\} y_1^2 = 4m^2y^2$

$\Rightarrow \left\{(2x)^2 - 4\right\} \cancel{y^{\frac{1}{m}} \cdot y^{-\frac{1}{m}}} \; y_1^2 = 4m^2y^2$

$\Rightarrow (x^2-1) y_1^2 = m^2y^2$

D.w.r. to x:

$(x^2-1) 2y_1 y_2 + 2xy_1^2 = 2m^2 y y_1$

$(x^2-1)y_2 + xy_1 - m^2 y = 0$

D.w.r. to x n times by L. Theorem,

$(x^2-1)y_{n+2} + {}^nC_1 \cdot y_n \cdot 1 - m^2 y_n = 0$

$\Rightarrow (x^2-1)y_{n+2} + {}^nC_1\, y_n \cdot 1 - m^2 y_n = 0$

$\Rightarrow (x^2-1)y_{n+2} + 2nx\, y_{n+1} + n(n-1)y_n + xy_{n+1} + ny_n - m^2 y_n = 0$

$\Rightarrow (x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$

page-453:

$\boxed{\text{Role's Theorem}}$

$f(x) \longrightarrow [a,b] \longrightarrow$ continity

$(a,b) \longrightarrow$ differentiable

$f(a) = f(b)$

$$\boxed{\begin{array}{l} a < c < b \\ f'(c) = 0 \end{array}}$$

$f(x)$

$\boxed{a < c < b}$

$[a,b] \longrightarrow$ ~~di~~ cont.

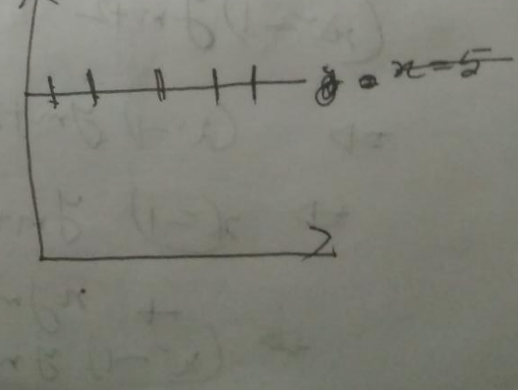
$(a,b) \longrightarrow$ diff.

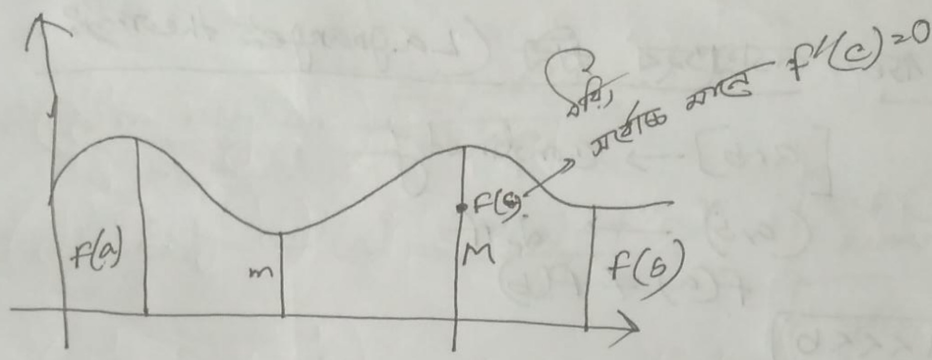
$f(a) = f(b)$

$f'(c) = 0$

সর্বোচ্চ ও সর্বনিম্ন মান সমান (যখন $f(x) = constant$)

$m = M$





$\boxed{M \neq m}$

$M = f(c)$

$f(c+h) - f(c) \leq 0 \; ; \; \boxed{h \cancel{>} 0, \; h < 0}$

$h \to 0$

$\Rightarrow \dfrac{f(c+h) - f(c)}{h} \leq 0$

$\Rightarrow$

$\lim_{h \to 0} \dfrac{f(c+h)}{h} \geq 0$

$\begin{cases} f'(c) \leq 0 \\ f'(c) \geq 0 \end{cases}$ | $f'(c) = 0$

Page: 136: ল্যাগ্রাঞ্জের সূত্র (Lagrange's theorem):

$[a,b] \rightarrow$ continuity

$(a,b) \longrightarrow$ diff

$f(a) \neq f(b)$

$\boxed{a<c<b}$

$$f'(c) = \frac{f(a)-f(b)}{b-a}$$

$$\therefore f(b)-f(a) = (b-a)\, f'(c)$$

Continatary function:

Associated function/complicated/Auxillary:

$$F(x) = f(x) - Ax \quad \cdots\cdots (1)$$

[এখানে, $F(a)=F(b)$]

$$F(a) = F(b)$$

$$\Rightarrow \therefore f(a) = f(b) - Ab$$

$$\Rightarrow F(a) = F(b)$$

$$\Rightarrow f(a) - Aa = f(b) - Ab$$

$$\Rightarrow f(b) - f(a) = Ab - Aa$$

$$\Rightarrow A = \frac{f(b)-f(a)}{b-a}$$

$$F(x) = F(x) - \frac{f(b)-f(a)}{b-a}x$$

$$\therefore F'(x) = f'(x) = \frac{f(b)-f(a)}{b-a}$$

$\because F(x)$ rolle's theorem মেনে চলে

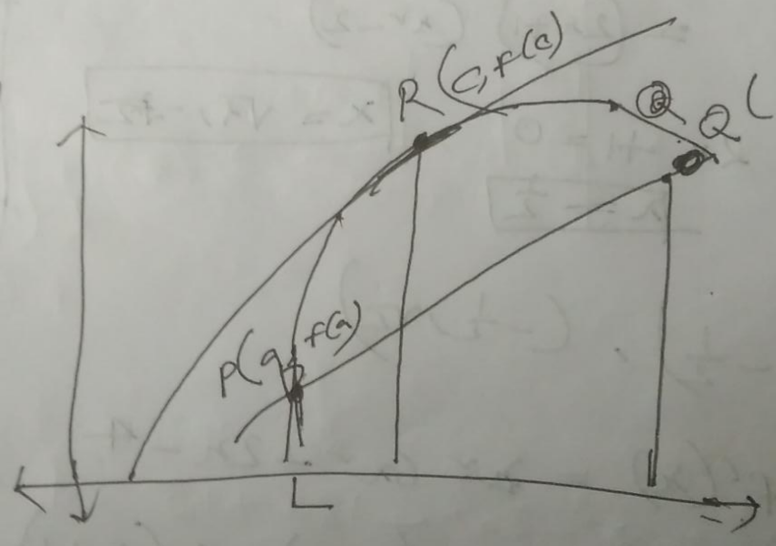
$\therefore F(a) = F(b)$

$$F'(c) = f'(c) - \frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow 0 = f'(c) - \frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow f'(c) = \frac{f(b)-f(a)}{b-a}$$

Page: 459



$F'(c) = \frac{f(b)-f(a)}{b-a}$

~~Step~~ Slape of PQ = $\frac{f(b)-f(a)}{b-a}$

Stepmethods

$f(a+h) = f(a) + hf'(a) = f''(a+\theta h);\ 0<\theta<1$

$\boxed{\text{Page: 464}}$

(অনু: 2.2, 10 – same)

$f(x) = 2x^3 + x^2 - 4x - 2$ → continuous & differentiable

$\left[\begin{array}{l} D_f = (-\infty, +\infty) \\ R_f = (-\infty, +\infty) \end{array}\right]$

$f(x) = 2x(x^2-2) + 1(x^2-2)$

$= (2x+1)(x^2-2)$

$2x + 1 = 0$

$\boxed{x = -\frac{1}{2}}$

$\boxed{x = \sqrt{2}, -\sqrt{2}}$

$\boxed{\begin{array}{l} f(\sqrt{2}) = 0 \\ f(-\sqrt{2}) = 0 \\ f(-\frac{1}{2}) = 0 \end{array}}$

$\left(-\sqrt{2}, -\frac{1}{2}\right), \left(-\frac{1}{2}, \sqrt{2}\right)$

$f'(x) = \cancel{x^2}\ 6x^2 + 2x - 4$

$= 6x(x+1) - 4(x+1)$

$= (x+1)(6x-4)$ $\therefore \boxed{x = -1, \frac{2}{3}}$

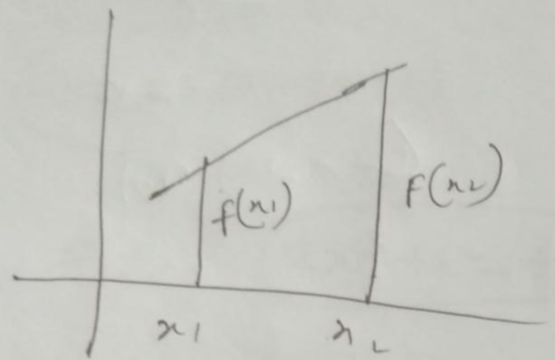
$-1.4 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \cancel{\frac{2}{3}}\ 0.66$

$\left(-\frac{1}{2}, 0.66\right)$

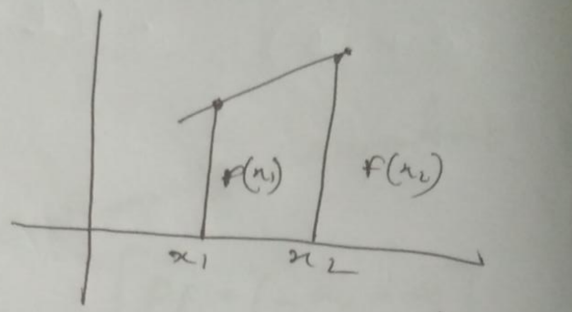
04-03-2020

Page-486:

$f(x_1) < f(x_2)$



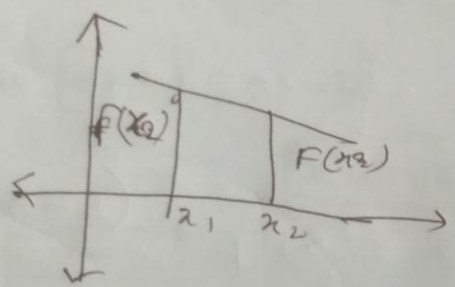
$x_1 < x_2$



$f(x_1) < f(x_2)$

$f(x_1) > f(x_2)$

$x_1 < x_2$



$f'(x) > 0$ increasing

$f'(x) < 0$ decreasing

= $f(x) = x^3 - 3x^2 + 18x + 15$; if $f(x)$ is increasing

~~of f(x)~~

$f'(x) = 3x^2 - 6x + 18$

$= 3(x^2 - 2x + \cancel{18}) + 15$

$= 3(x-1)^2 + 15 > 0$ [∵ x এর সকল মানের জন্য $f'(x)$ ধনাত্মক]

4

#

$f(x) = 1 - x - x^3$

$\therefore f'(x) = 0 - 1 - 3x^2$

$= -(1+3x^2) = -(1+3x^2) < 0$

$\boxed{\therefore f'(x) \text{ is decreasing}}$

~~= -(1+3x²)~~

#

$f(x) = x^4 - 8x^2$

$\boxed{x=1 \text{ বিন্দুতে ফাংশনটি decreasing}}$

$\therefore f'(x) = 4x^3 - 16x$

$= 4x(x^2 - 4)$

$x = 1 = 4 - 16 = -12 < 0$

$\therefore$ ফাংশনটি $x=1$ বিন্দুতে decreasing.

কোন ব্যবধিতে increasing ও কোন ব্যবধিতে decreasing,

$f(x) = x^3 - 3x + 5$

$f'(x) = 3x^2 - 3$

$= 3(x-1)(x+1)$

$f'(x) = 0$

$\Rightarrow 3(x-1)(x+1) = 0$ $\boxed{\therefore x = 1, -1}$

(Ans)

#

$x < -1$, $\boxed{-1 < x < 1}$, $\boxed{x > 1}$

$-\infty$ — -1 — 0 — $+1$ — $+\infty$

$(-\infty, -1)$, $(-1, +1)$, $(1, +\infty)$

for $(x < -1)$

$\boxed{\text{for } (x < -1),\ f'(x) = 3(-)(-) = (+ve),\ f'(x) > 0}$

$f(x)$ is increasing in $(-\infty, -1)$

for $(-1 < x < 1)$, $f'(x) = 3(-)(+)$

$= 3(-ve)$

$f'(x) < 0$

$f'(x)$ is decreasing in ~~$(-1, \infty)$~~ $(-1, +1)$

for $(x > 1)$, $f'(x) = 3(+)(+) = 3(+ve) > 0$

$f'(x)$ is increasing in $(1, \infty)$