NPTEL Online Certification Courses

Indian Institute of Technology Kharagpur

Course: Signal Processing for mm Wave communication for 5G and beyond

Assignment:

Week -4

TYPE OF QUESTION: MCQ/MSQ

Number of questions: 10 **Total mark: 10 X 1 = 10**

Q.1 Consider the random signal, $x(t) = \sin(\omega t + \phi)$ varies over time t where, the ϕ follows uniform distribution in $(0, 2\pi)$ i.e., $\phi \sim U(0,2\pi)$. The mean of the parameter x is given by

a. π
b. 0
c. $sinc\ (\omega t + \phi)$
d. $sinc(\omega t)$

Answer: b. 0

Hints: Compute expectation of $x(t)$ using definition.

Q.2 The auto-correlation of a stationary signal $x(t)$ is given by $R_x(\tau) = 16e^{-5|\tau|} \cos 20\,\pi\tau + 8 \cos \pi\tau$. The variance of the signal is given by

a. 16
b. 8
c. 24
d. 17.88

Answer: c. 24

Explanation: The variance is $R_x(0) = 16 + 8 = 24$.

Q.3 A weakly stationary process $x(t)$ has autocorrelation given by $R_x(k) = (-0.5)^{|k|}$ with mean zero. The PSD for $f = 0$ is

a. $\frac{1}{3}$
b. 3
c. $\frac{1}{2}$
d. 2

Answer: a. $\frac{1}{3}$

Explanation: The PSD is computed by taking Fourier transform of the autocorrelation function, which is given by

$$S_x = \frac{\frac{3}{4}}{cos(2\pi f) + \frac{5}{4}}$$

For $f = 0$, $S_x(0) = \frac{1}{3}$.

Q.4 Suppose, a discrete-time random process representation of the channel $h[n]$ has zero mean, and represented by
$h[n] = ah[n-1] + bw[n]$,
where, the input process $\{w[n]\}$ is weakly stationary and Gaussian distributed, with zero-mean and variance $\sigma_w^2 = 3$. Given that, the autocorrelation function for $h[n]$, is given by $R_h(k) = 4\left(\frac{1}{3}\right)^{|k|}$. The value of a and b are

a. $a = \sqrt{\frac{32}{27}}$, $b = \frac{1}{3}$
b. $b = \sqrt{\frac{32}{27}}$, $a = \frac{1}{3}$
c. $a = 3, b = \sqrt{\frac{32}{27}}$
d. $b = 3, a = \sqrt{\frac{32}{27}}$

Answer: b. $\boldsymbol{b} = \sqrt{\frac{32}{27}}$, $\boldsymbol{a} = \frac{1}{3}$

Explanation: The transfer function is given by, $H(z) = \frac{b}{1-az^{-1}}$, and $R_h(k) = \frac{b^2\sigma^2}{1-a^2}a^{|k|}$.
Comparing with the given information, $a = \frac{1}{3}$, and $b = \sqrt{\frac{32}{27}}$.

Q.5 A zero-mean random process $h(t)$ has PSD $S_h(f) = \delta(f) + 2$, for $f \in [-0.5, 0.5]$, and it is periodic with periodicity 1. The process is also

a. Mean ergodic process
b. Not a mean ergodic process
c. Cannot be predicted from the given data

Answer: b. Not a mean ergodic process

Explanation: The auto-correlation for the process is $R_h(k) = 2\delta(k) + 1$. Let, k takes values from $-2N$ to $2N$. Hence, $\lim_{N\to\infty} \frac{1}{2N+1}\sum_{k=-2N}^{2N}\left(1 - \frac{|k|}{2N+1}\right)R_h(k) = 1 \neq 0$. As the time-average and ensemble average did not match here, hence not ergodic.

Q.6 The characteristic polynomial of AR (p) model of a random process X_t, is given by $\phi(B) = \prod_{i=1}^{p}(1 - \alpha_i B)$. The process X_t cannot be stationary for which of the given choice of α_i

a. $\alpha_i = 2$
b. $\alpha_i = -0.75$
c. $\alpha_i = 0.75$
d. $\alpha_i = 0$

Answer: a. $\boldsymbol{\alpha_i} = \mathbf{2}$
Explanation: The poles of the AR model must lie within the unit circle of a stationary process X_t, $||\alpha_i|| < 1, for\ i = 1, 2, \dots, p$.

Q.7 An AR (1) process is given by
$h_t = 0.75\ h_{t-1} + w_t$, where, w_t is white noise with zero mean, and variance 0.5. The variance of h_t is given by

a. 0.55
b. 0.57
c. 0.59
d. 0.5

Answer: b. 0.57
Explanation: The variance of AR(1) process h_t is given by, $\sigma^2 = \frac{\sigma_w^2}{1-a^2}$. Here, $a = 0.75, \sigma_w^2 = 0.5$, hence, $\sigma^2 = 0.5714$.

Q.8 The Yule-Walker equation for some AR process h_t is given as
$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & -0.5 \\ 0.25 & 0.25 & -0.25 \\ 0 & 0 & -0.125 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$. The order of the given process $h_t$is

a. 1
b. 2
c. 3
d. None

Answer: b. 2
Explanation: The system matrix has rank of 2. We will get two independent column vectors only, for two roots of the polynomial. Hence, it is a AR(2) process.

Q.9 Given a set of pilot data (x) and received data (y) in a communication system as follows.

x	1	0.9	0.75
y	1.1	1	0.72

The relationship between x and y is given by $y = \alpha + \beta x + w$, where w is the noise term. What are the approximate values of α and β using the tabular data?

a. 0.42, 1.54
b. -0.42, 1.54
c. 0.42,-1.54
d. -0.42,-1.54

Answer: b. $\boldsymbol{\alpha = -0.42, \beta = 1.54}$
Explanation: Try least square estimation method, explained in Lecture 21.

Q.10 In a communication system, which of the following order is more suitable for equalizer design?

a. Signal model -> data detection -> Channel Estimation
b. Data detection -> Signal Model -> Channel Estimation
c. Signal Model -> Channel Estimation -> Data Detection
d. Channel Estimation -> Signal model -> Data detection

Answer: c. Signal Model -> Channel Estimation -> Data Detection

Explanation: We assume a suitable signal model first. With the pilot, channel estimation is performed. Using the estimated channel, data detection is performed which is the ultimate goal.
