

1. Given the $n + 1$ dimensional vector given $\tilde{\mathbf{x}} = \begin{bmatrix} \bar{\mathbf{x}} \\ x_{n+1} \end{bmatrix}$. The set $S = \{\tilde{\mathbf{x}} | \|\tilde{\mathbf{x}}\| \leq x_{n+1}\}$ represents a Norm cone

Ans a

2. The l_∞ norm ball for the set of two dimensional vectors $[x \ y]^T$ is a square with sides parallel to x -axis or y -axis

Ans b

3. The l_2 norm ball for the set of two dimensional vectors $[x \ y]^T$ is a circle centered at origin

Ans c

4. Given X is a real-valued random variable with $\Pr(X = a_i) = p_i, i = 1, 2, \dots, n$. The condition $E\{X^2\} \leq \alpha \Rightarrow \sum_{i=1}^n a_i^2 p_i \leq \alpha$, which can be seen to be a halfspace

Ans d

5. Each constraint $\|\bar{\mathbf{x}} - \bar{\mathbf{x}}_0\| \leq \|\bar{\mathbf{x}} - \bar{\mathbf{x}}_i\|$ can be represented as

$$(\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_0)^T \bar{\mathbf{x}} \leq \frac{\|\bar{\mathbf{x}}_i\|^2 - \|\bar{\mathbf{x}}_0\|^2}{2},$$

which is a halfspace. The set V is the intersection of K such halfspaces. Hence, it is a polyhedron

Ans d

6. A polyhedron is formed from the intersection of several hyperplanes and halfspaces

Ans b

7. The l_1 norm ball for the set of two dimensional vectors $[x \ y]^T$ is a tilted square with diagonals on x -axis or y -axis

Ans a

8. As shown in the lectures, the set of points $S = \{[x_1 \ x_2]^T | x_1 x_2 \geq 1, x_1, x_2 \geq 0\}$ can be represented as the intersection of the halfspaces

$$\alpha^2 x_2 + x_1 \geq 2\alpha$$

Ans c

9. Let X be a real-valued random variable with $\Pr(X = a_i) = p_i, i = 1, 2, \dots, n$. Consider the condition $\Pr\{X^2 \leq \alpha\} = \beta$. This can be expressed as

$$\Pr\{X^2 \leq \alpha\} = \beta \Rightarrow \sum_{i: a_i^2 \leq \alpha} p_i = \beta$$

Hence, the set of all vectors $\bar{\mathbf{p}}$ that satisfy this property is a hyperplane

Ans b

10. The given set is a norm ball, which can be seen as follows

$$\begin{aligned} \|\bar{\mathbf{x}} - \bar{\mathbf{a}}\| \leq \theta \|\bar{\mathbf{x}} - \bar{\mathbf{b}}\| &\Rightarrow \|\bar{\mathbf{x}} - \bar{\mathbf{a}}\|^2 \leq \theta^2 \|\bar{\mathbf{x}} - \bar{\mathbf{b}}\|^2 \\ &\Rightarrow (1 - \theta^2) \bar{\mathbf{x}}^T \bar{\mathbf{x}} - 2(\bar{\mathbf{a}} - \theta^2 \bar{\mathbf{b}})^T \bar{\mathbf{x}} + (\bar{\mathbf{a}}^T \bar{\mathbf{a}} - \theta^2 \bar{\mathbf{b}}^T \bar{\mathbf{b}}) \end{aligned}$$

which is a norm ball with center and radius given as

$$\bar{\mathbf{x}}_0 = \frac{(\bar{\mathbf{a}} - \theta^2 \bar{\mathbf{b}})}{(1 - \theta^2)}, \left(\frac{\theta^2 \|\bar{\mathbf{b}}\|^2 - \|\bar{\mathbf{a}}\|^2}{1 - \theta^2} + \|\bar{\mathbf{x}}_0\|^2 \right)^{1/2}$$

Ans a