Applied Optimization for Wireless, Machine Learning Big Data
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Lesson - 45
Least Norm Signal Estimation

Keywords: *Least Norm Signal Estimation*

Hello welcome to another module in this massive open online course. So we have looked at the least squares paradigm, let us look at its analogue or a counterpart which is known as the Least Norm Paradigm.

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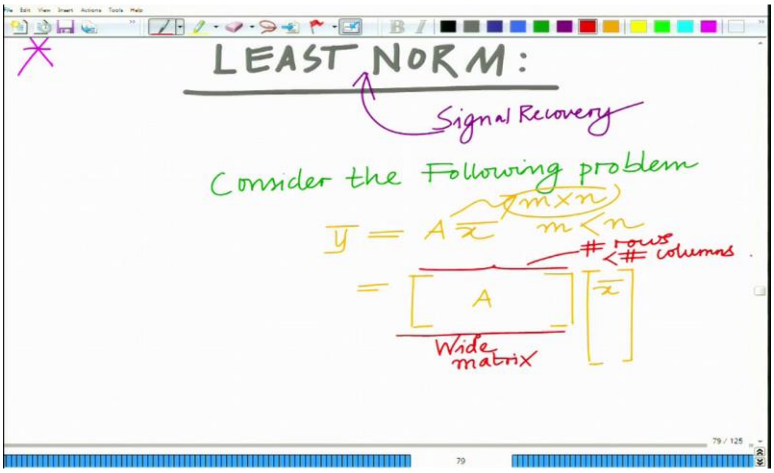

So consider the following problem where we have $\bar{y} = A\bar{x}$ where A is an m $\times$ n matrix. But while previously m > n, in the least norm framework we will consider m < n, that is the number of rows is much lower than the number of columns, so this can be called as a wide matrix.

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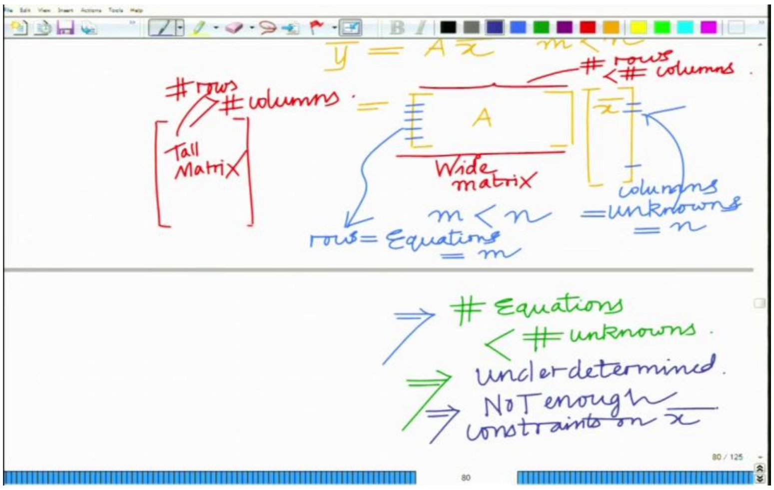

This basically implies that the number of equations is smaller than the number of unknowns which implies that the system is under determined.

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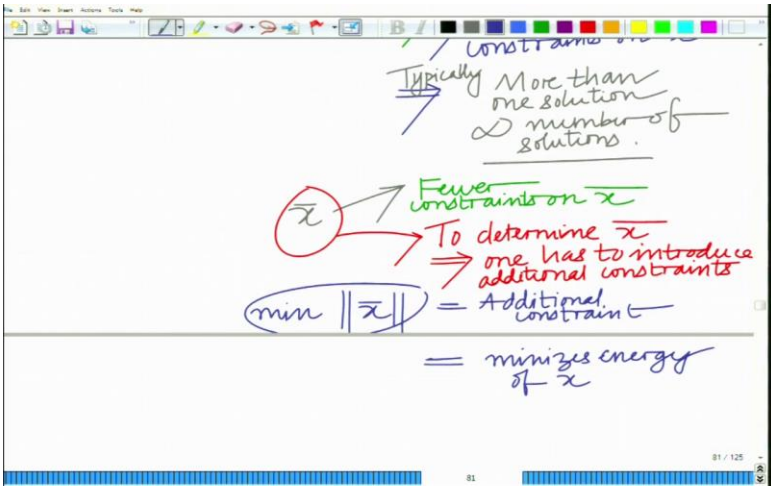

This means that there is an infinite number of solutions and if there are fewer constraints then the only way to determine the possible value of $\bar{x}$ is to introduce additional constraints. So this can be $\min\|\bar{x}\|$, so this is the additional constraint. It basically minimizes the energy of $\bar{x}$ which implies that you are trying to find an energy efficient solution. And this is precisely known as the least norm problem.

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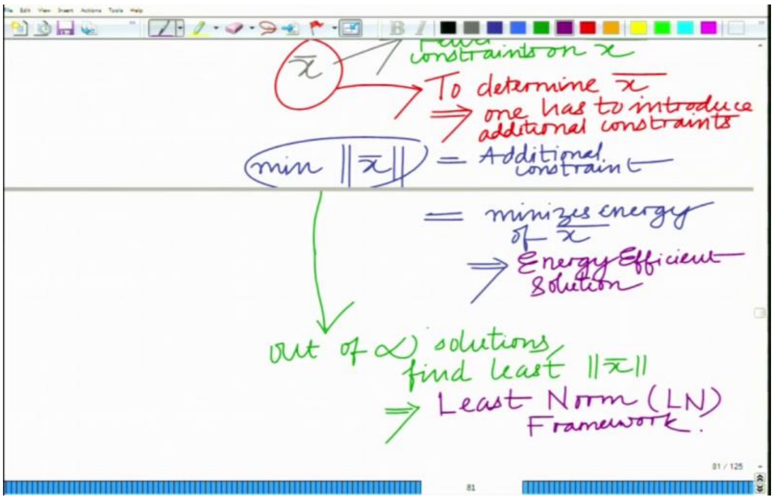

So this means out of infinite solutions, find the one that has least norm.

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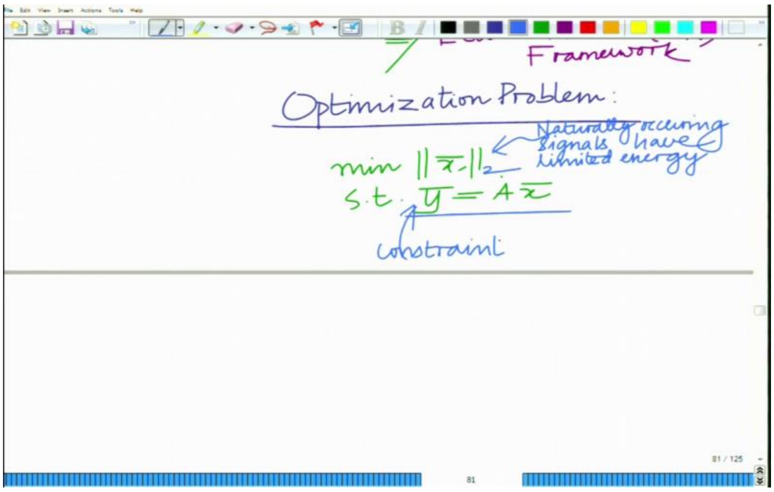

The relevant optimization problem for this can be given as $\min \left\| \bar{x} \right\|_2$ and this is the objective function and the constraint is $\bar{y} = A\bar{x}$. So this is justified because naturally occurring signals have limited energy.

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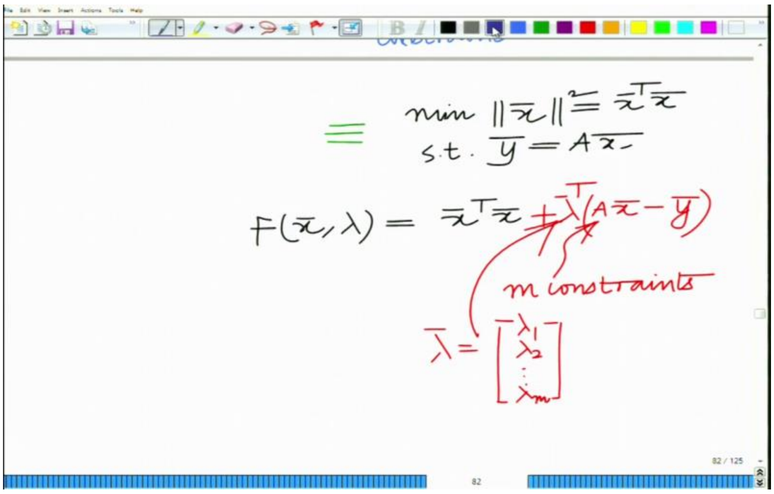

Therefore, one can form the Lagrangian which is $f(\bar{x}, \lambda) = \bar{x}^T \bar{x} + \bar{\lambda}^T (A\bar{x} - \bar{y})$ because we have m constraints, each row is an equation. So there has to be one Lagrange multiplier for each constraint.

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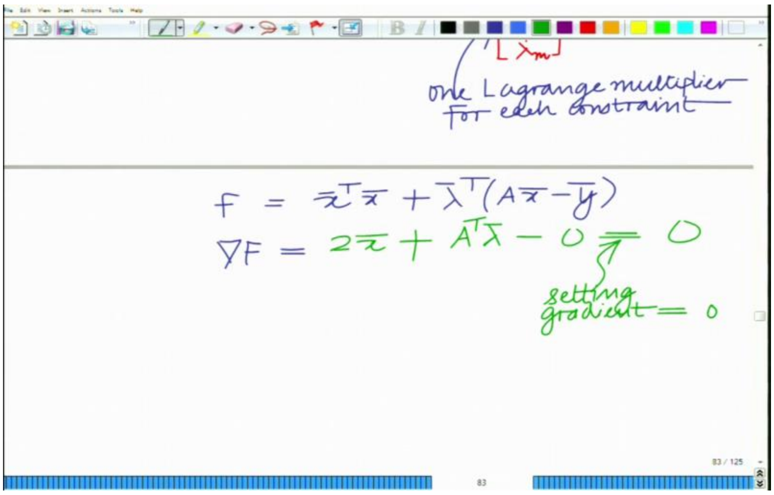

So we can solve this as shown in slide by taking the gradient with respect to $\bar{x}$ and setting it to 0.

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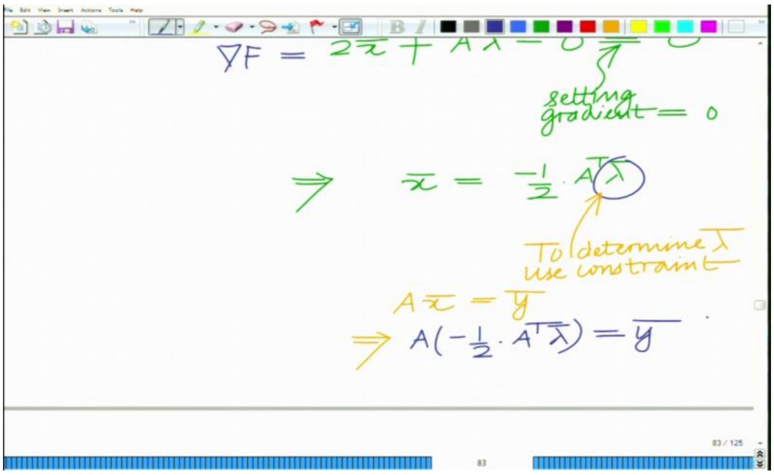

So on solving, finally you will get $x = A^T \left(A A^T \right)^{-1} \bar{y}$.

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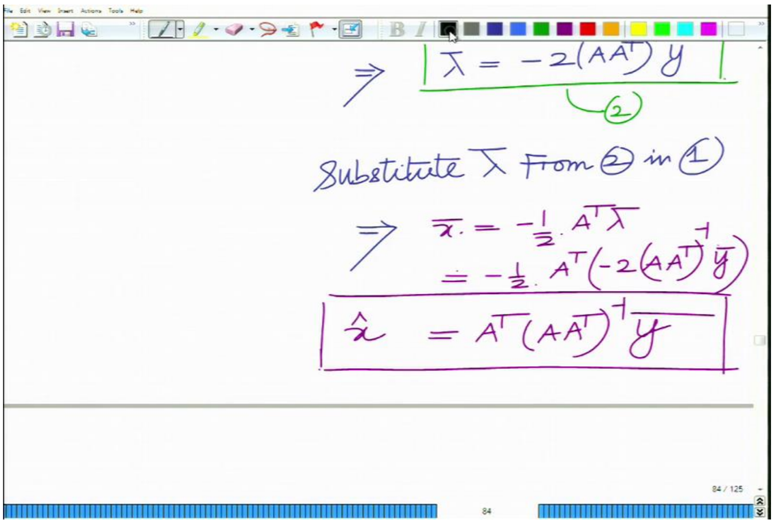

So this is the least norm signal estimate and this also known as the least norm solution.

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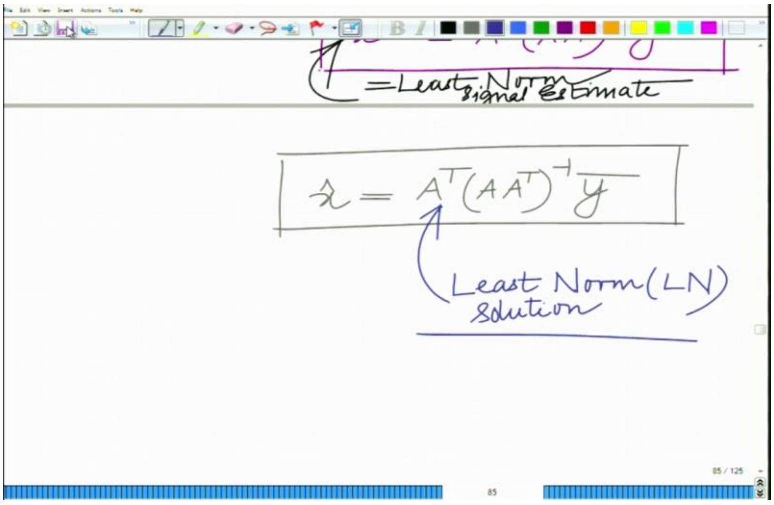

So this is suitable for scenarios where there are under constrained systems. So we will stop here. Thank you very much.