

**Applied Optimization for Wireless, Machine Learning, Big Data**  
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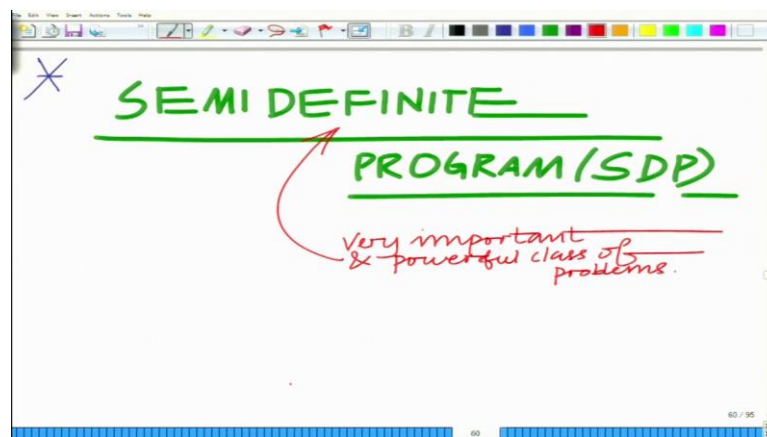
**Lecture – 75**

**Semi Definite Program (SDP) and its application: MIMO symbol vector decoding**

**Keywords:** *Semi Definite Program*

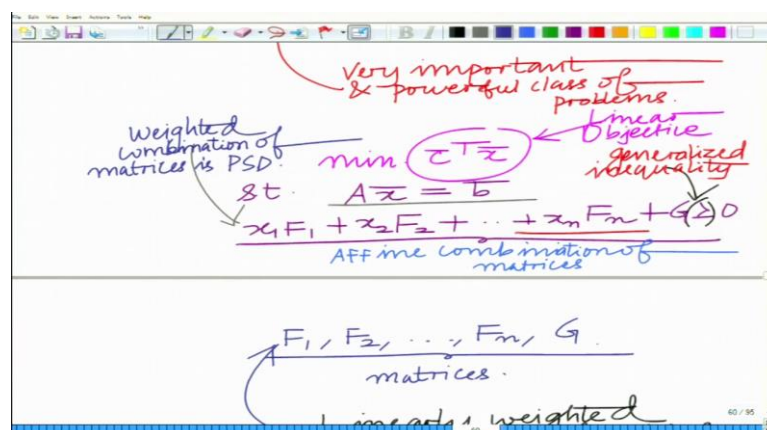
Hello, welcome to another module in this massive open online course. So we are looking at convex optimization problems and their applications. Let us start looking at what is known as Semi Definite Programming.

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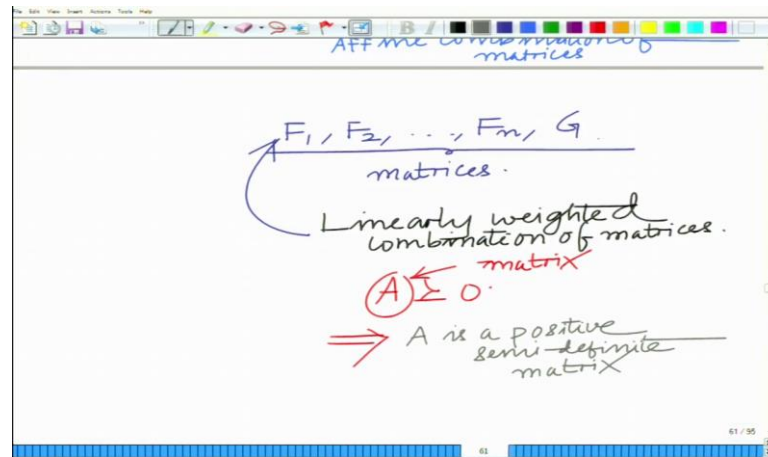
So it is a very important and powerful class of problems and a semi definite program is the following where you are minimizing a seemingly simple objective that is the objective is still a linear objective.

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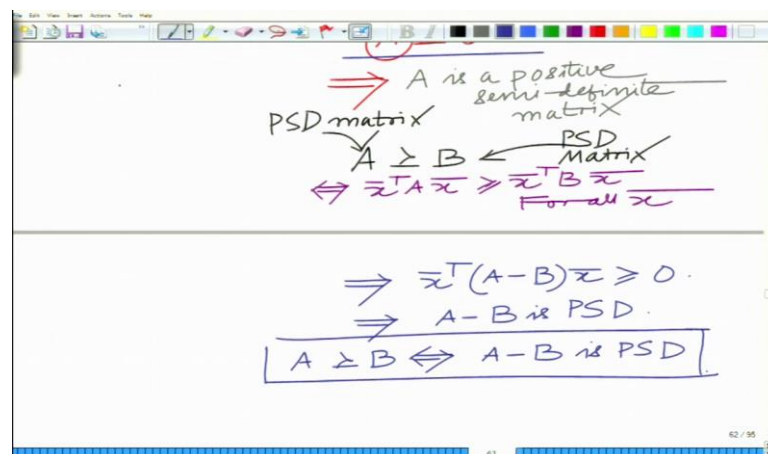
The equality constraint is something that is very similar. But the inequality constraint is a weighted combination of matrices or rather an affine combination.

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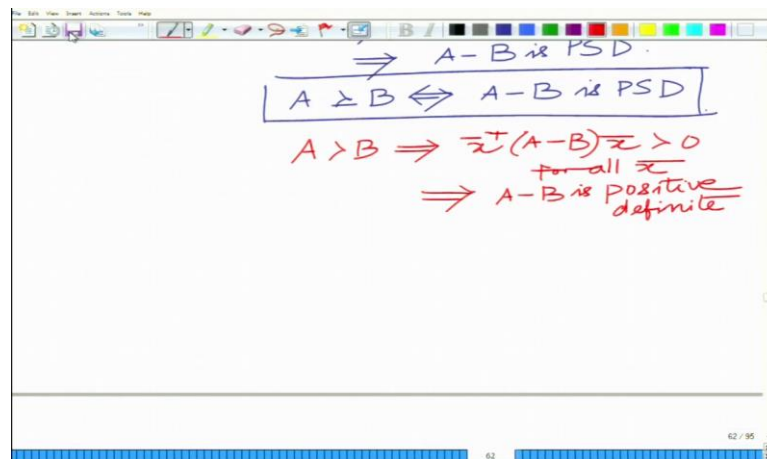
You are saying this matrix has to be greater than equal to 0 which implies that this matrix has to be positive semi definite.

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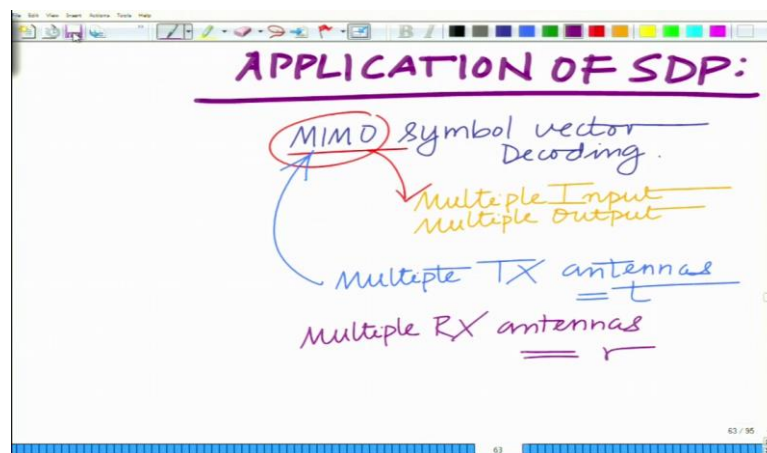
We say two positive semi definite matrices that is  $A \geq B$  where  $A$  is a positive semi definite matrix, if and only if  $\bar{x}^T A \bar{x} \geq \bar{x}^T B \bar{x} \Rightarrow A - B$  is PSD, for all  $\bar{x}$ .

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$$\Rightarrow A - B \text{ is PSD}$$
$$A \succeq B \Leftrightarrow A - B \text{ is PSD}$$
$$A \succ B \Rightarrow \mathbf{x}^T(A - B)\mathbf{x} > 0 \text{ for all } \mathbf{x}$$
$$\Rightarrow A - B \text{ is positive definite}$$

Now, similarly if you have a strict inequality  $A > B$ , naturally this implies  $A - B$  is positive definite. So this is the notion of this generalized inequality on the set of positive semi definite matrices.

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So let us look at an application of SDP and the application is as follows. So let us consider a MIMO system and we want to perform MIMO symbol decoding. So in this MIMO system, you have multiple TX antennas and multiple RX antennas.

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The image shows a handwritten MIMO system model. At the top, the scalar equation  $y = Hx + n$  is written. Below it, the vector equation is shown:  $\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = H \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}$ . Annotations include:  $r = \# \text{output symbols.}$  pointing to the output vector,  $t = \text{transmit symbols.}$  pointing to the input vector, and  $n_{r \times 1}$  indicating the noise vector dimensions.

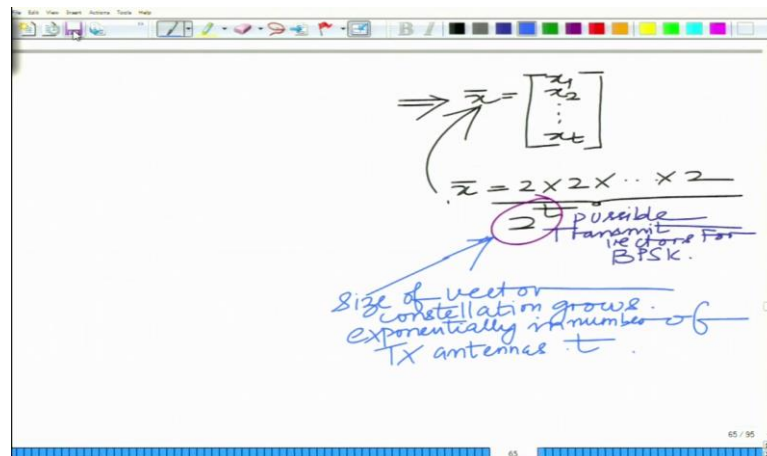
And I can represent this MIMO system model as  $\bar{y} = H \bar{x} + \bar{n}$ , where you have  $t$  transmit symbols and  $r$  output symbols.

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This slide contains handwritten notes explaining the nature of the input symbols  $x_i$ . It states: "Each  $x_i$  is drawn from a suitable Digital Constellation". An example is given: "Ex: BPSK = Binary Phase Shift Keying". Below this, it specifies the possible values:  $x_i \in \{\pm 1\}$  and notes that there are "2 Possible values". The vector notation and annotations from the previous slide are also present at the top of this slide.

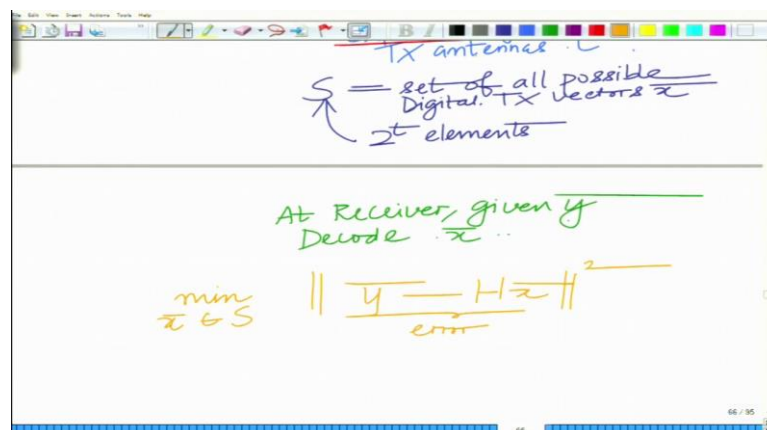
Now, if you look at each symbol  $x_i$  this has to be drawn from a constellation that is it cannot take any possible value. So each  $x_i$  is drawn from a suitable digital constellation for a digital wireless system, example, BPSK this is basically your binary phase shift keying, which implies there are two phases that means, each  $x_i$  has two possible values. So each  $x_i$  can be plus or minus 1, that means, each  $x_i$  has two possible values.

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So basically the size of this set of the vector constellation is  $2^t$  which is growing exponentially in the number of transmit antennas  $t$ .

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So now, let us say we have the set  $S$  which is the set of all possible digital transmit vectors  $\vec{x}$  and this set has  $2^t$  elements and therefore the problem is at the receiver once you receive  $\vec{y}$  we have to decode  $\vec{x}$ . So therefore you use the best possible decoder which is known as the ML decoder. So we have  $\min_{\vec{x} \in S} \|\vec{y} - H\vec{x}\|^2$ . The problem is you have to search over all  $t$  possible vectors to find  $\vec{x}$  which basically minimizes this error and this is known as the maximum likelihood decoder.

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Decode  $x$

$$\hat{x} = \min_{x \in S} \|y - Hx\|^2$$

Search over  $2^t$  Possible vectors!

maximum Likelihood (ML) Decoder  
= Best performance

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maximum Likelihood (ML) Decoder  
= Best performance

increasingly complex as  $t$  increases!

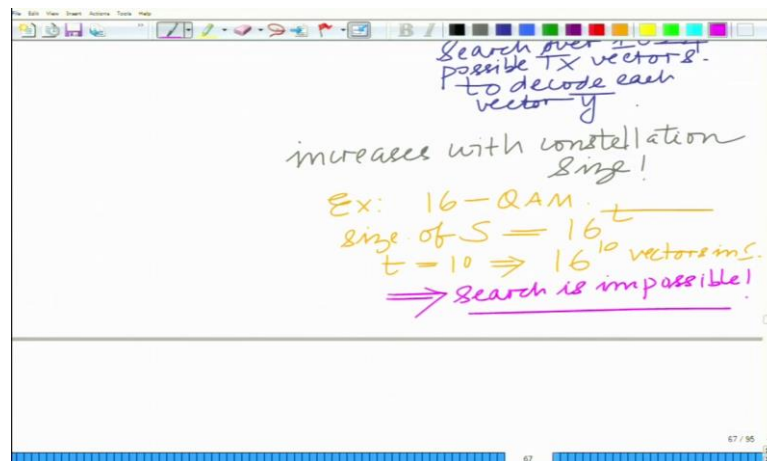
For example if  $t = 10$ ,  
then  $2^t = 2^{10} = 1024$

Search over 1024 possible Tx vectors to decode each vector  $y$

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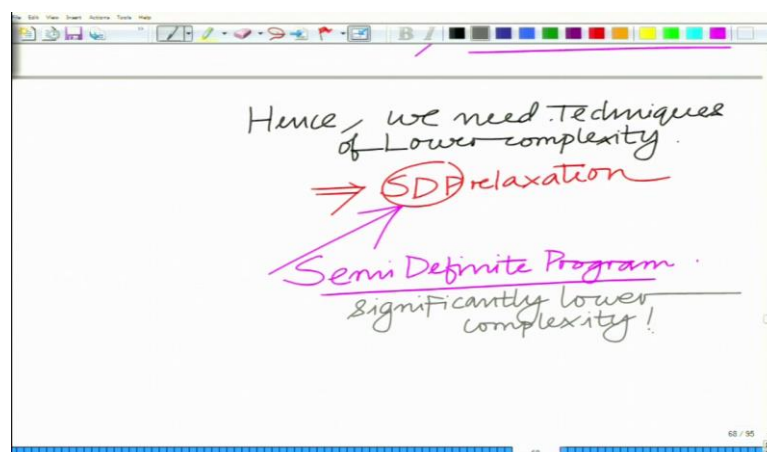
Now this search this increases with the constellation. For instance, if you have 16 QAM, then the number of vectors becomes  $16^t$ .

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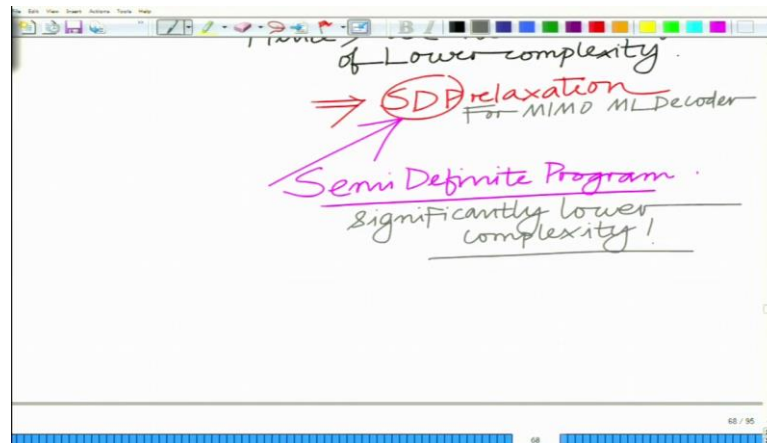
So this implies that this search is impossible or next to impossible and therefore, we have to come up with low complexity techniques.

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Hence, one such technique is basically what is termed as SDP relaxation we relax this ML decoder problem as a Semi - Definite Program. We are going to formulate this MIMO ML decoder as a Semi - Definite Program which has a significantly lower complexity. So this is very useful for practical implementation.

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So this has significantly lower complexity in comparison to the optimal ML decoder which has exponential complexity which is virtually impossible for a large number of transmit antennas and large constellation sizes. So the resulting ML decoder or the approximate ML decoder has a significantly lower complexity. So we will stop here and we will look at this in the subsequent module. Thank you very much.