Applied Optimization of Wireless, Machine Learning, Big data

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Lecture - 71
Examples: l_1 minimization with l_∞ norm constraints, Network Flow problem

Keywords: *l_1 minimization, l_∞ norm constraints, Network Flow problem*

Hello welcome to another module in this massive open online course. So we are doing examples on convex optimization, so let us continue the discussion.

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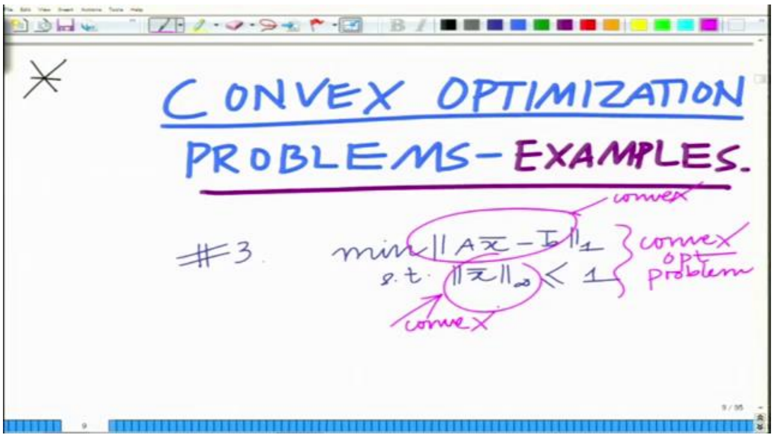

So we are doing a few examples which will illustrate how to formulate these problems as convex optimization problems so that we can use the convex solver to solve these optimization problems. So let us look at another example, example number 3 that is $\begin{array}{l} \min \left\| A\bar{x} - \bar{b} \right\|_1 \\ s.t \left\| \bar{x} \right\|_\infty \leq 1 \end{array}$. Now, this is a convex optimization problem because if you look at this l_1 norm, this is convex, so we have convex objective, convex inequality constraint and this is the convex optimization problem so you can directly solve it. But we want to recast it into a form that is more intuitive or more amenable to analysis.

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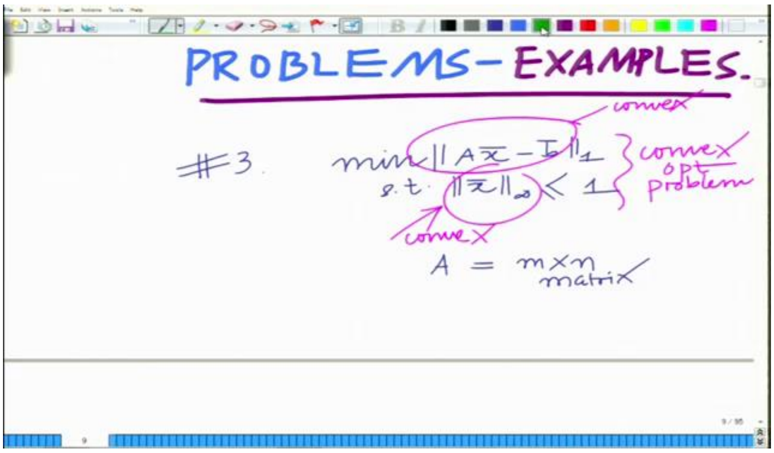

So let us assume that A is an m $\times$ n matrix.

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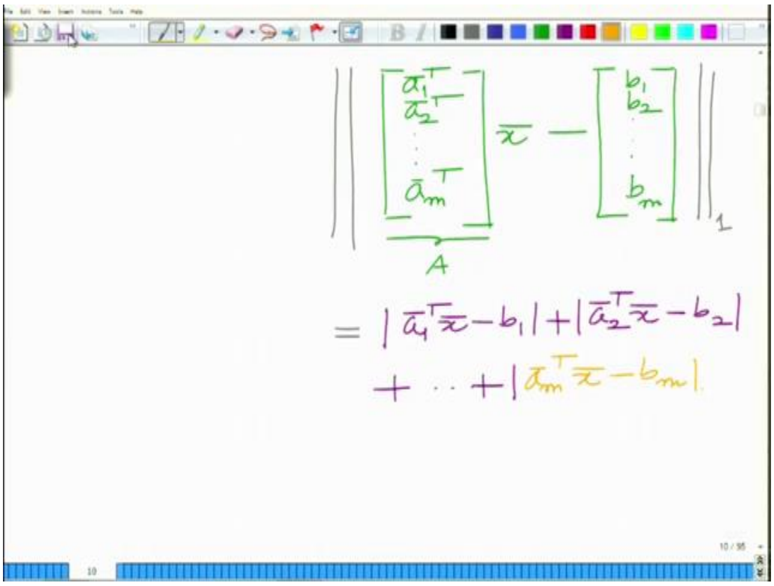

Then you can rewrite this as shown in slide. So this matrix A has m rows and you are taking the l_1 norm of this which is nothing but the sum of the magnitudes of these elements of this vector, which is given as $\left|\bar{a}_1^T\bar{x}-b_1\right| + \left|\bar{a}_2^T\bar{x}-b_2\right| + \ldots. + \left|\bar{a}_m^T\bar{x}-b_m\right|$.

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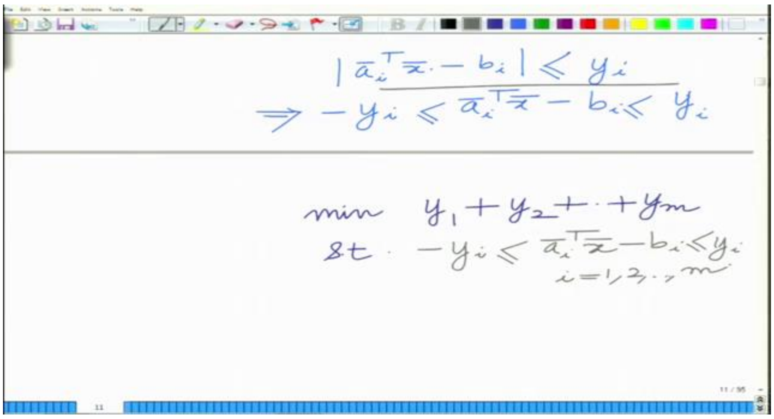

And now we want to introduce the constraint that is $\left|\bar{a}_i^T \bar{x} - b_i\right| \leq y_i$. So this implies that $-y_i \leq \bar{a}_i^T \bar{x} - b_i \leq y_i$. So using the epigraph form you can write this as

$$\begin{aligned} &\min\ y_1 + y_2 + .. + y_m \\ &s.t - y_i \leq \bar{a}_i^T \bar{x} - b_i \leq y_i \\ &\qquad\quad i = 1, 2, .., m \end{aligned}.$$

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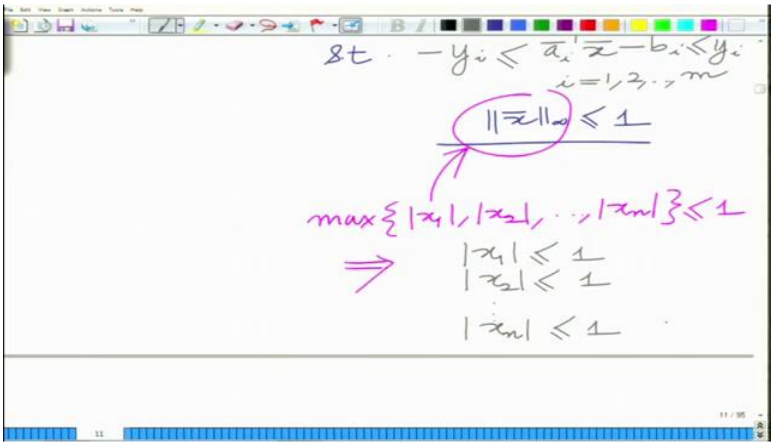

And now you still have the other constraint as it is. Now we also know that this l_∞ norm constraint can also be written as a set of linear constraints and this is as shown in slide.

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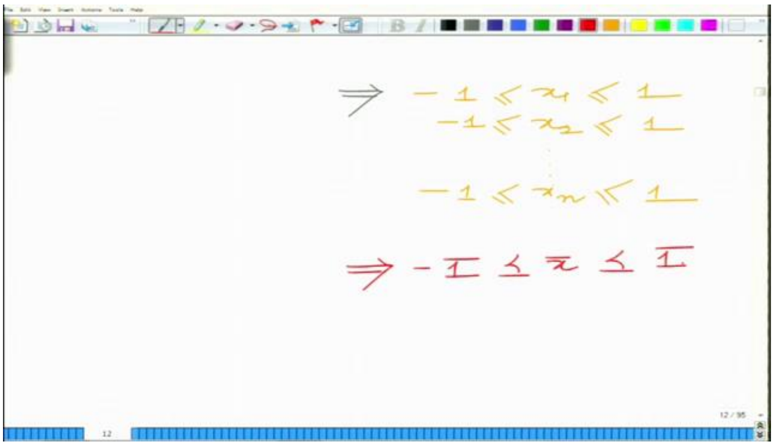

This can be written in a compact fashion as $-\bar{1} \leq \bar{x} \leq \bar{1}$.

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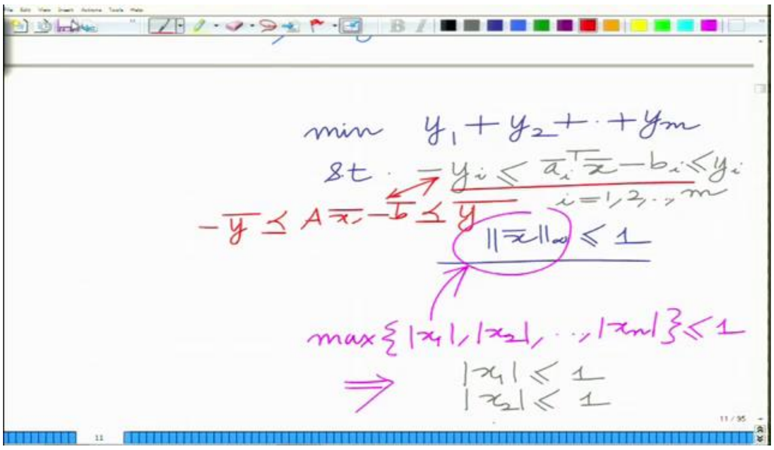

So this reduces to this equivalent representation as shown in slide.

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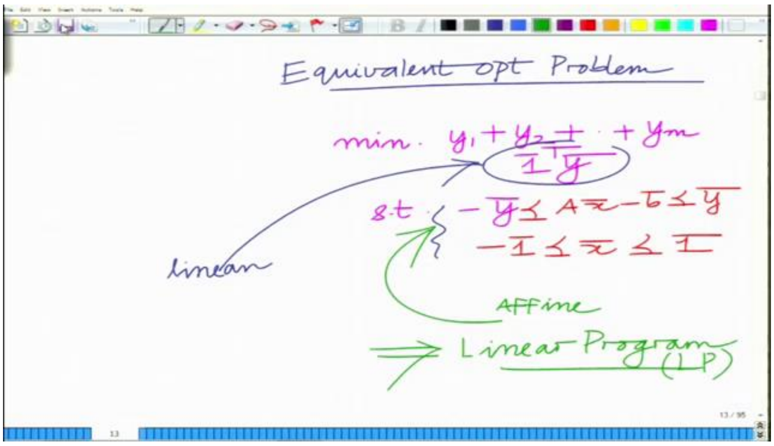

So the equivalent optimization problem can be written as $\begin{array}{l} \min\ y_1 + y_2 + .. + y_m \\ s.t\ \begin{array}{l} -\bar{y} \le A\bar{x} - b \le \bar{y} \\ -\bar{1} \le \bar{x} \le \bar{1} \end{array} \end{array}$. Here the objective is linear and constraints are also linear, so this is a Linear Program, LP.

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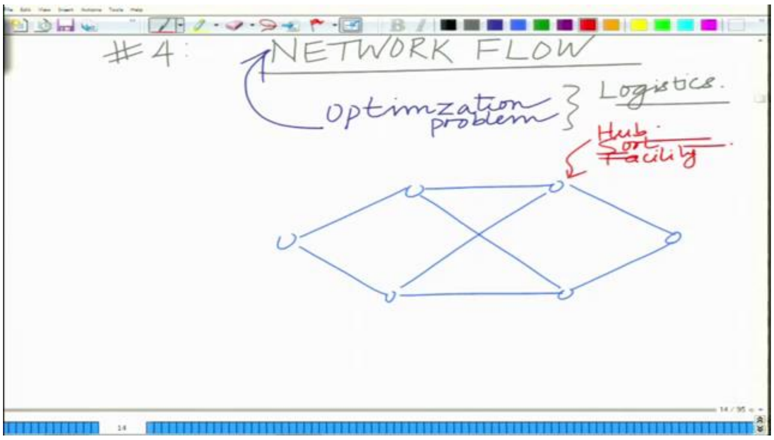

Let us look at another problem which is termed as network flow. This is one of the most important kind of optimization problems that arises in various fields such as for logistics management, supply chain management etc. Consider a network of hubs or sort facilities. So this is our network, so each of these nodes is a hub or a sort facility, for instance in the distribution network of Etailer, in an E commerce company, to distribute these products you need a network of hubs or sort facilities, where you have a lot of these products that are brought into, sorted and dispatched to other hubs and ultimately delivered to the end user.

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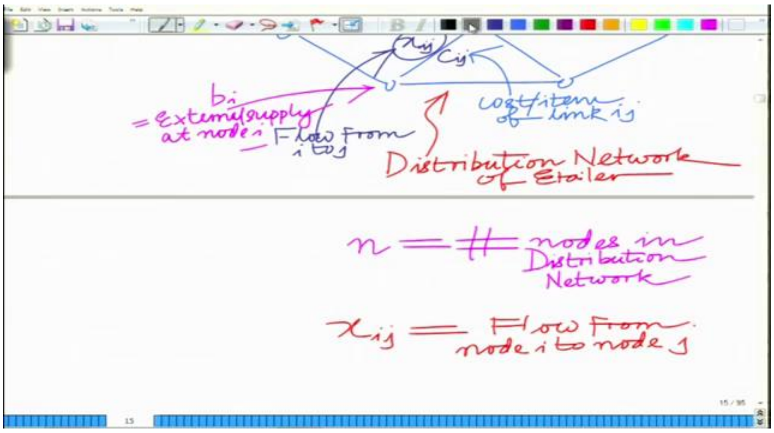

So you have this network of connected hubs and we have flows between these hubs. So x_{ij} denotes the number of items that represents the flow between load i and load j and each flow has a cost C_{ij} which is the cost per item of link or the path between i and j. So these are basically your sort facilities. Now in addition you will have external supply b_i. So if you look at each hub or each load you might see supply flows that are coming from other loads, flows going to other loads and in addition you might have an independent supply b_i. So this b_i indicates the supply that is coming into load i, if b_i is positive and on the other hand if b_i is negative it means that the commodities are leaving from that load. So let us say n equals the number of loads in the distribution network. Let us denote the upper bound and lower bound, since each sort facility has a certain limit in terms of the total outflow in flow products.

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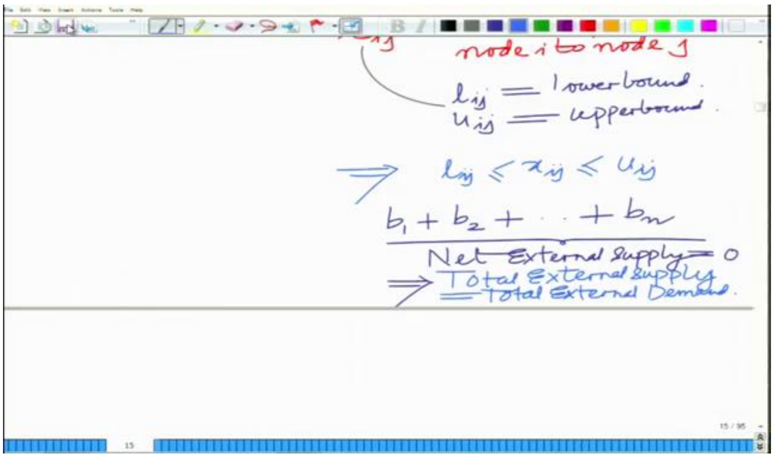

So we have the lower bound as l_{ij} and u_{ij} equals the upper bound which basically implies that each flow x_{ij} has to be between l_{ij} and u_{ij}. So we will enforce another condition that is if we look at the total external flow for all nodes, the total external supply equals the total external demand. So it cannot happen that a large number of commodities are entering and only a few commodities are leaving, which means that these commodities are getting lost or it does not mean that only some commodities are entering and a large number of commodities are leaving which means commodities are somehow being magically generated. So it just means that whatever commodities entering at the various hubs are eventually leaving the network at possibly the same or different hubs that depends on the flow. So the net external supply must be 0 which means that the total external supply equals the total external demand.

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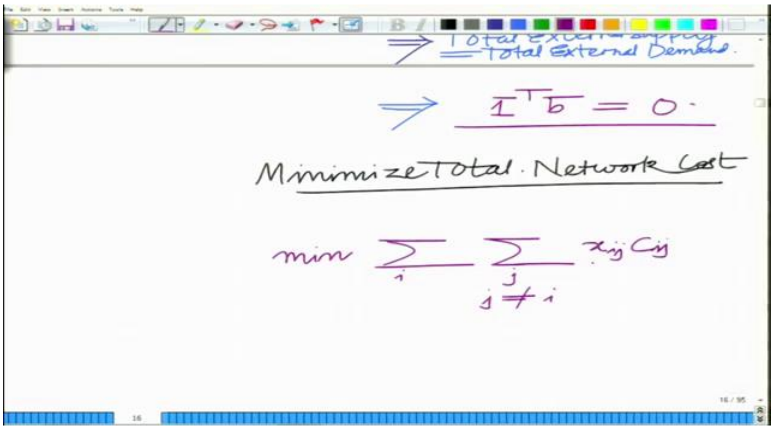

So we want to formulate this network flow problem which is minimize the total cost of the network and it is very simple.

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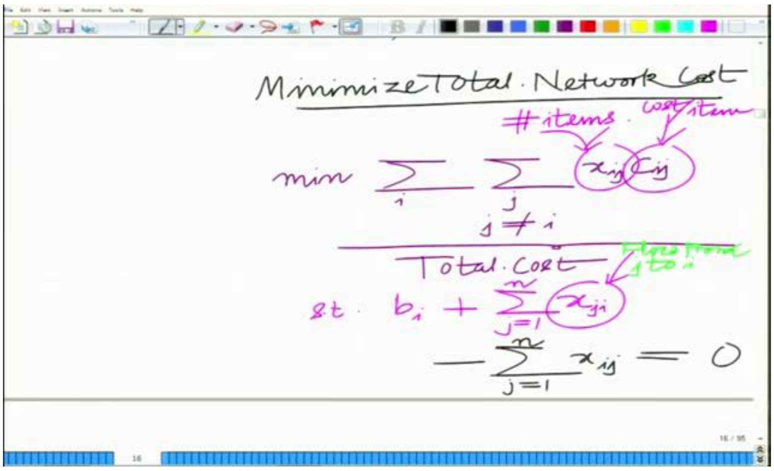

$$\min \sum_{i} \sum_{\substack{j \\ j \neq 1}} x_{ij} C_{ij}$$

So we have $s.t\ b_i + \sum_{j=1}^{n} x_{ji} - \sum_{j=1}^{n} x_{ij} = 0$. So this basically says that the total external supplies

$$l_{ij} \leq x_{ij} \leq u_{ij}$$

at each hub and the total flow from all the other hubs to a particular node i must be equal to the total flow of commodities or goods from node i to the other nodes. So this must hold for all particular loads or all particular hubs in your distribution unit.

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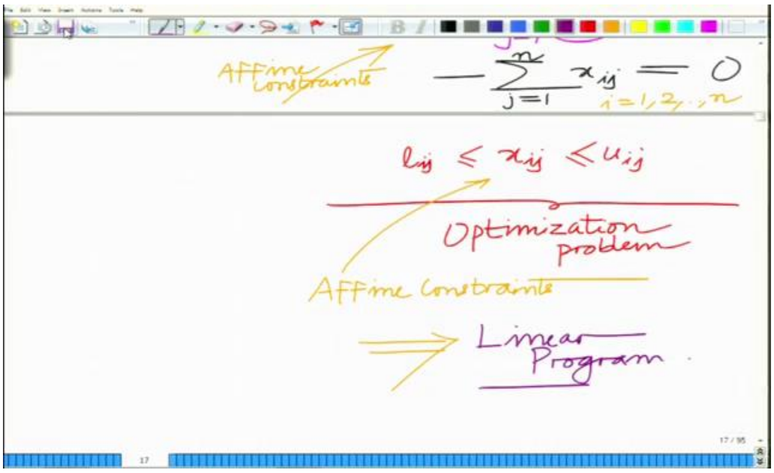

So this is your optimization problem.

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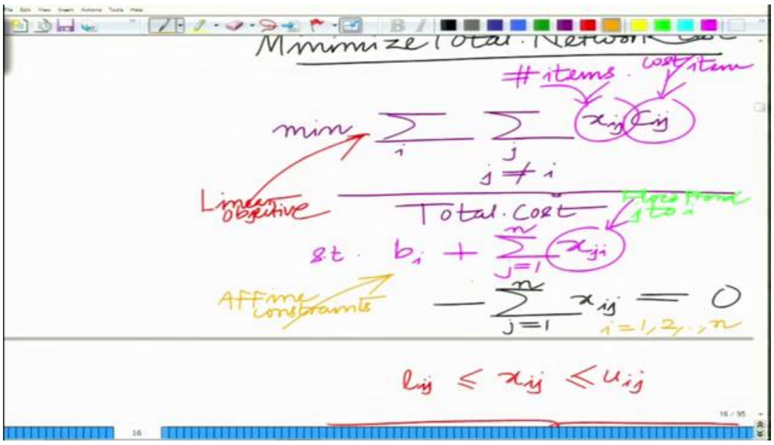

This is a linear objective and we have affine constraints. So you can have a large distribution network with tens or thousands of hubs as long as the total external supplies is equal to the total external demand and this is is a linear program. So this is a very practical problem and there are several such problems which has significant practical relevance. So we will stop here and continue in the subsequent modules. Thank you very much.