Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 36
Practical Application: Multi-antenna Beamforming with Interfering User

Hello, welcome to another module in this massive open online course. So, we are looking at practical applications of optimization. In particular we have discussed about beamforming that how to focus the beam in the direction of a particular user. Let us extend this paradigm to include the interference.

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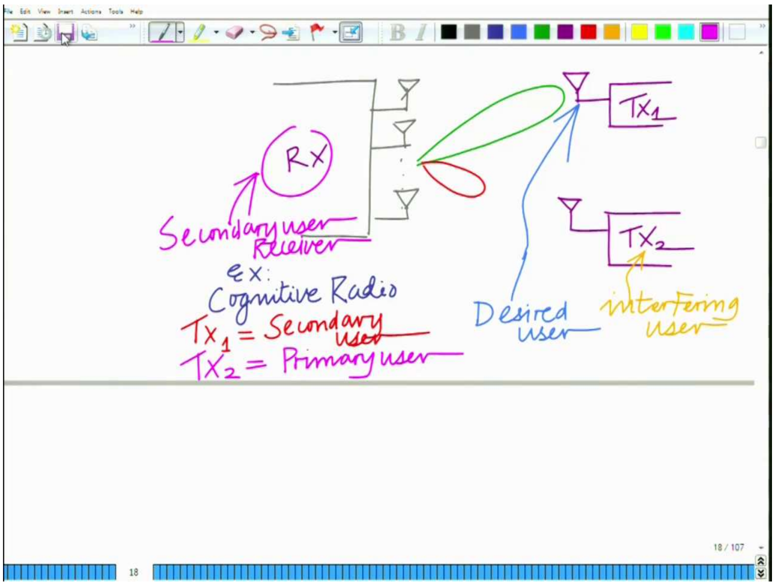

So, for beamforming in the presence of interfering users, consider a scenario of two transmitter system among which one is the desired user while another is the interfering user. This is again a multiple antenna array system. Here the signals from the desired user needed to be received while the signals from the interfering user must be rejected.

This can occur in several scenarios. For instance, in cognitive radio scenario, the interference for secondary user (say Tx_1) is occurred from the primary user (say Tx_2). Thus this can be looked as an interesting application in the evolving paradigm of cognitive radio. Again in this scenario, the primary user is the licensed user so it has priority for the transmission. So, when there is an on-going primary transmission, it is difficult to make sure that this primary user does not impact the signal reception at the secondary user.

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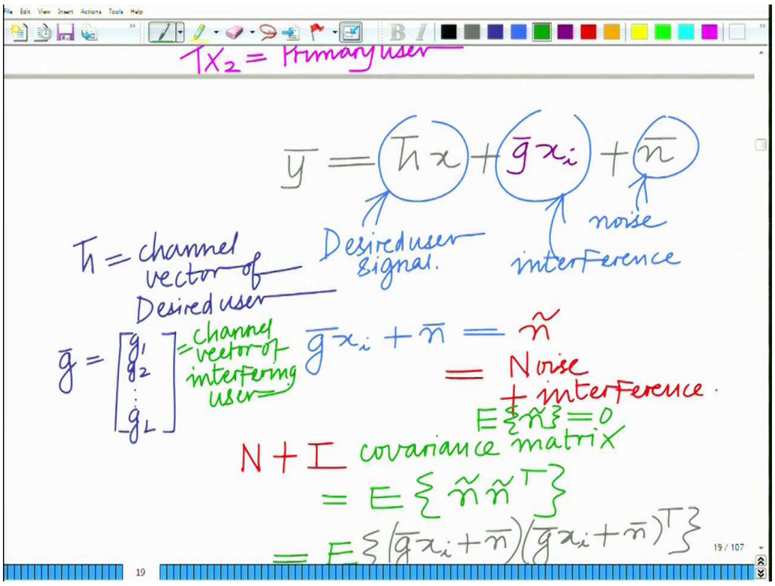

Now, the received signal model will be as follows.

$$\bar{y} = \bar{h}x + \bar{g}x_i + \bar{n}$$

Where $\bar{y}$ is the received signal, $\bar{h}$ is the channel vector of the desired user, x is the transmitted signal and $\bar{n}$ is the additive white Gaussian noise. The additional term $\bar{g}x_i$ defines the interference caused by interferer such that $\bar{g}$ is the channel vector of interfering user. Hence this $\bar{g}x_i$ contributes in the noise. Therefore the new noise $\tilde{n}$, which symbolizes the collective effect of noise and interference, can be defined as follows.

$$\tilde{n} = \bar{g}x_i + \bar{n}$$

This interference, unlike the inter symbol interference, is the multiuser interference as it is arising from the other users.

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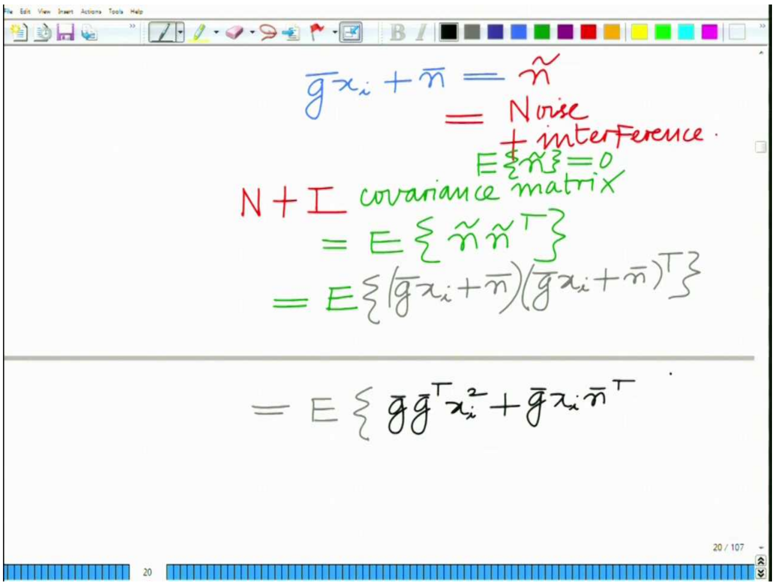

Therefore the covariance of this noise plus interference term is

$$
\begin{aligned}
E\left\{\tilde{n}\tilde{n}^T\right\} &= E\left\{\left(\bar{g}x_i + \bar{n}\right)\left(\bar{g}x_i + \bar{n}\right)^T\right\} \\
&= E\left\{\bar{g}\bar{g}^T x_i^2 + \bar{g}x_i\bar{n}^T + \bar{n}\bar{g}^T x_i + \bar{n}\bar{n}^T\right\} \\
&= \bar{g}\bar{g}^T E\left\{x_i^2\right\} + \bar{g}E\left\{x_i\bar{n}^T\right\} + \bar{g}^T E\left\{\bar{n}x_i\right\} + E\left\{\bar{n}\bar{n}^T\right\}
\end{aligned}
$$

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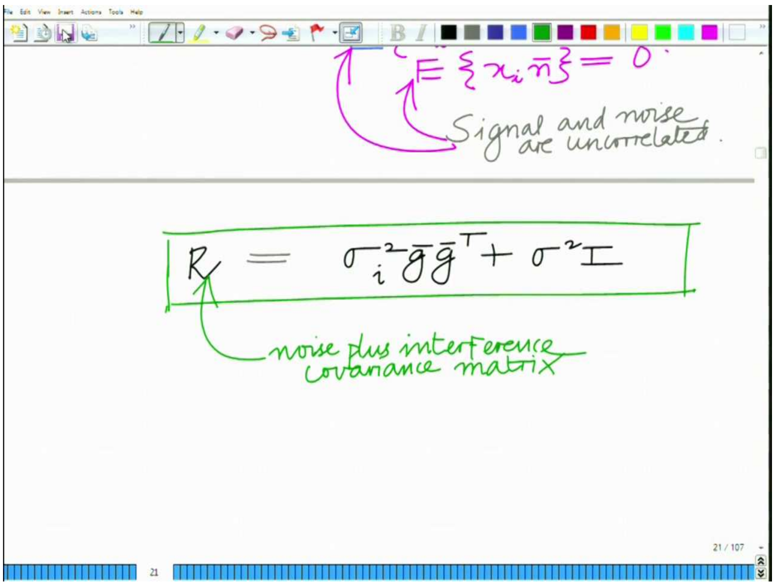

Typically the signal and the noise are uncorrelated because the noise arises from the system and the signal arises from the source. So, the signal and noise are typically uncorrelated or in fact they are independent. Therefore

$$E\{x_i\bar{n}^T\}=0$$
$$E\{\bar{n}x_i\}=0$$

Also, it is assumed that the signal power or the interference power is σ_i^2. Therefore,

$$E\{x_i^2\}=\sigma_i^2$$

And the noise samples are Independent and Identically Distributed (IID) therefore noise covariance is

$$E\{\bar{n}\bar{n}^T\}=\sigma^2 I$$

Hence the covariance of the noise plus interference term is

$$R=E\{\tilde{n}\tilde{n}^T\}$$
$$=\sigma_i^2\bar{g}\bar{g}^T+\sigma^2 I$$

So, this is the noise plus interference covariance matrix corresponding to the above scenario of multiuser beamforming with interference.

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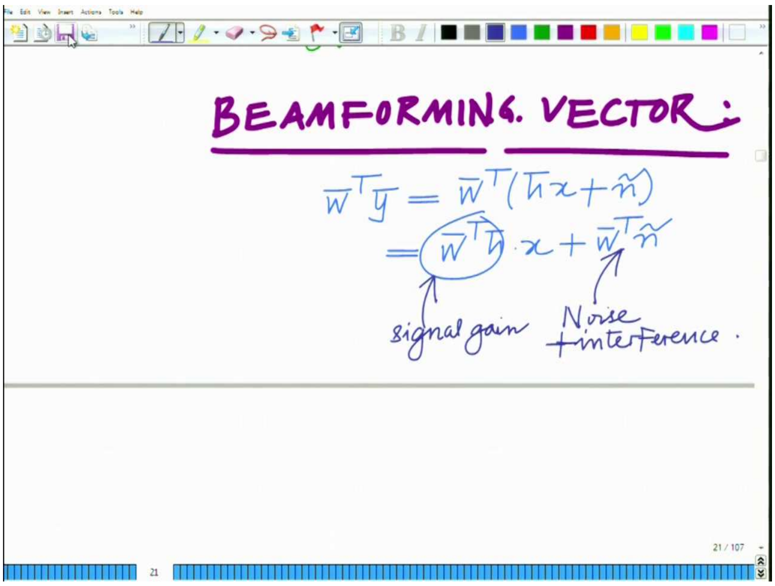
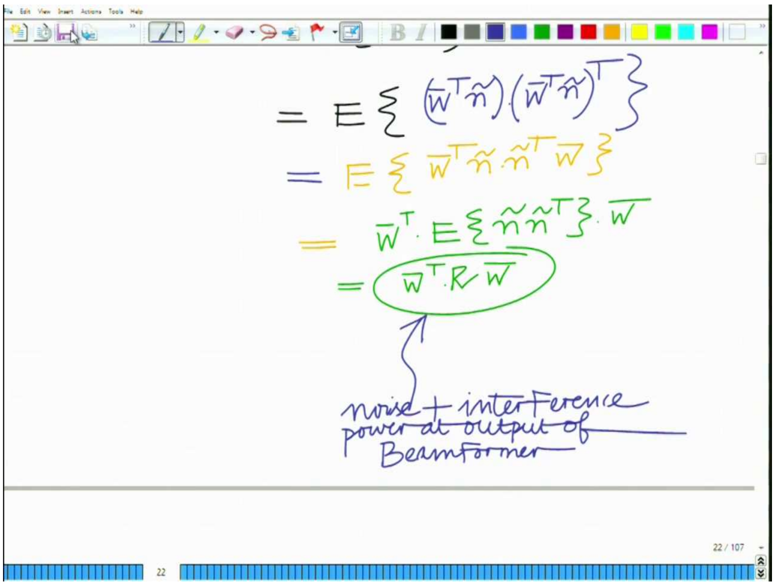


Let us now move to the beamforming vector $\bar{w}$. So, for electronic steering, linearly combine the samples of the received signal with weights.

$$
\begin{aligned}
\bar{w}^T \bar{y} &= \bar{w}^T \left(\bar{h}x + \tilde{n}\right) \\
&= \bar{w}^T \bar{h}x + \bar{w}^T \tilde{n}
\end{aligned}
$$

Here, $\bar{w}^T \bar{h}$ is the signal gain and $\bar{w}^T \tilde{n}$ is the weighted noise plus interference term. (Refer Slide Time: 16:22)



To minimize the effect of this noise plus interference at the output, let us calculate its power.

$$
\begin{aligned}
E\left\{\left(\bar{w}^T \tilde{n}\right)\left(\bar{w}^T \tilde{n}\right)^T\right\} &= E\left\{\bar{w}^T \tilde{n}\tilde{n}^T \bar{w}\right\} \\
&= \bar{w}^T E\left\{\tilde{n}\tilde{n}^T\right\}\bar{w} \\
&= \bar{w}^T R\bar{w}
\end{aligned}
$$

So, this is the noise plus interference power at the output of the beam former. Consider the optimization problem to evaluate the optimal value of the beamforming vector $\bar{w}$ by simply minimizing this noise plus interference term while restricting it in such a way that that the signal gain is unity.

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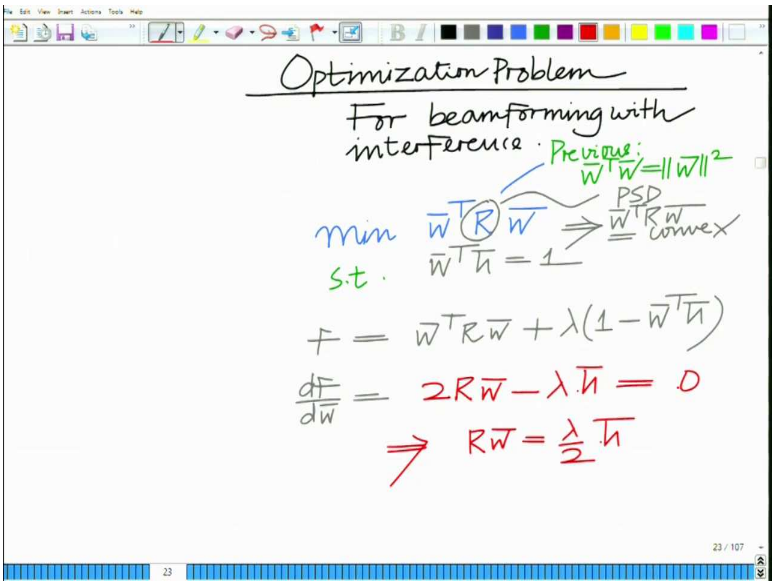

Therefore the optimization problem for beam forming with interference is such that minimize the noise plus interference power such that the signal gain is unity.

$$\min \ \bar{w}^T R \bar{w}$$
$$\text{such that } \ \bar{w}^T \bar{h} = 1$$

Thus the Lagrangian F for this optimization problem is as follows.

$$F = \bar{w}^T R \bar{w} + \lambda\left(1 - \bar{w}^T \bar{h}\right)$$

For minimum value of $\bar{w}$; put the first derivative of this Lagrangian equal to zero.

$$\frac{dF}{d\bar{w}} = 0$$
$$2R\bar{w} + \lambda\bar{h} = 0$$
$$\bar{w} = \frac{\lambda}{2} R^{-1} \bar{h}$$

Again the covariance matrix is the positive semi definite matrix. This implies that $\bar{w}^T R \bar{w}$ is convex. Therefore this is a convex optimization problem because the objective function is convex.

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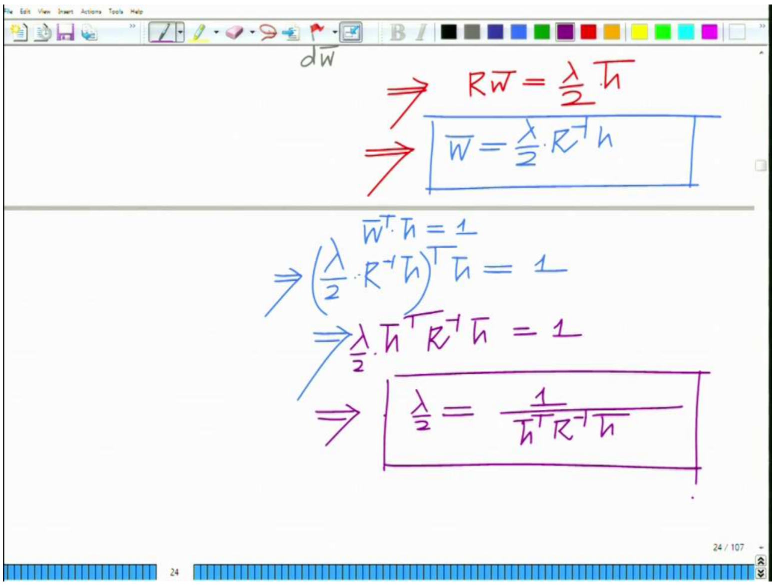

So,

$$\bar{w}^T \bar{h} = 1$$

$$\left(\frac{\lambda}{2} R^{-1}\bar{h}\right)^T \bar{h} = 1$$

$$\frac{\lambda}{2}\bar{h}^T R^{-1}\bar{h} = 1$$

$$\lambda = \frac{2}{\bar{h}^T R^{-1}\bar{h}}$$

Hence the optimal beamformer vector with interference is

$$\bar{w}^* = \frac{R^{-1}\bar{h}}{\bar{h}^T R^{-1}\bar{h}}$$

This is the optimal Beamformer that maximizes the signal power in the desired direction while minimizing the noise plus interference effect on the output.

So it is clear that slightly modifying the objective function can make a general problem more comprehensive. Also, this interference can arise from a variety of reasons such as the presence of different users in a cellular scenario. The interference can also be used due to a malicious user trying to interfere with the base station. In other case, this interference can be in a cognitive radius scenario where there is an ongoing transmission of the primary user and this causes interference at the secondary user. So, this interference has several practical applications.