Applied Optimization for Wireless, Machine Learning, Big Data

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Lecture - 28
Jensen's Inequality application: Relation between Average BER of Wireless and Wired system and Principle of Diversity

Hello. Welcome to another module in this massive open online course. Let us to look at a comparison of the performance of a wireless communication system with that of a conventional digital or a wire line communication system.

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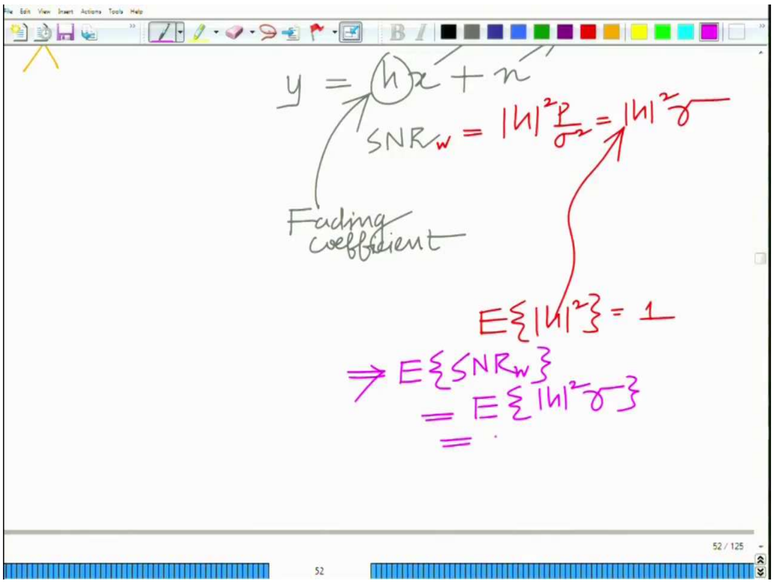

So the wireless channel model is

$$y = hx + n$$

Where x is the transmitted symbol, y is the received symbol and n is the white Gaussian noise with zero mean and σ^2 variance. The extra term h is the fading channel coefficient which is a random variable. And the SNR of this wireless channel is

$$SNR = \frac{P}{\sigma^2}|h|^2 = \gamma|h|^2$$

Here γ is the SNR of wire line channel.

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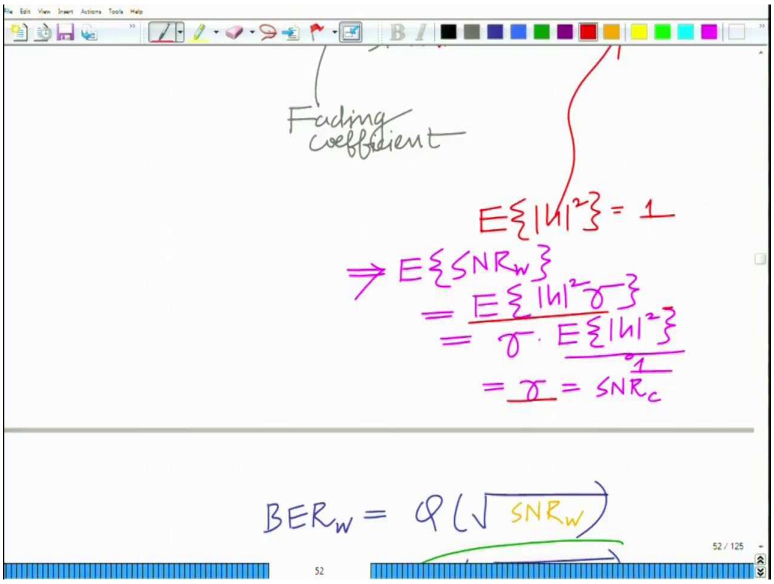

To perform a fair comparison between wireless and wired communication systems, set the average value of $|h|^2$ to unity.

$$E\left\{|h|^2\right\} = 1$$

So the average of the SNR of the wireless communication system would be

$$\begin{aligned} E\{\mathrm{SNR}\} &= E\left\{|h|^2 \gamma\right\} \\ &= \gamma E\left\{|h|^2\right\} \\ &= \gamma \end{aligned}$$

This implies that the average of the SNR of the wireless communication system is equal to the SNR of the conventional wire line communication system.

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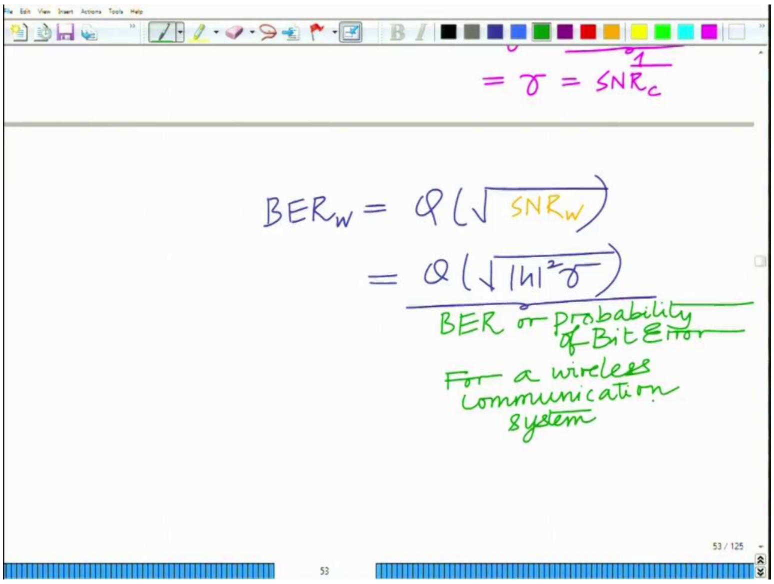

Let us denote the SNR of wireless system as SNR_w and the SNR of conventional wireline system as SNR_c. Now, the probability of bit error (BER_w) for a wireless communication system would be

$$\text{BER}_w = Q\left(\sqrt{\text{SNR}_w}\right) = E\left\{\sqrt{|h|^2 \gamma}\right\}$$

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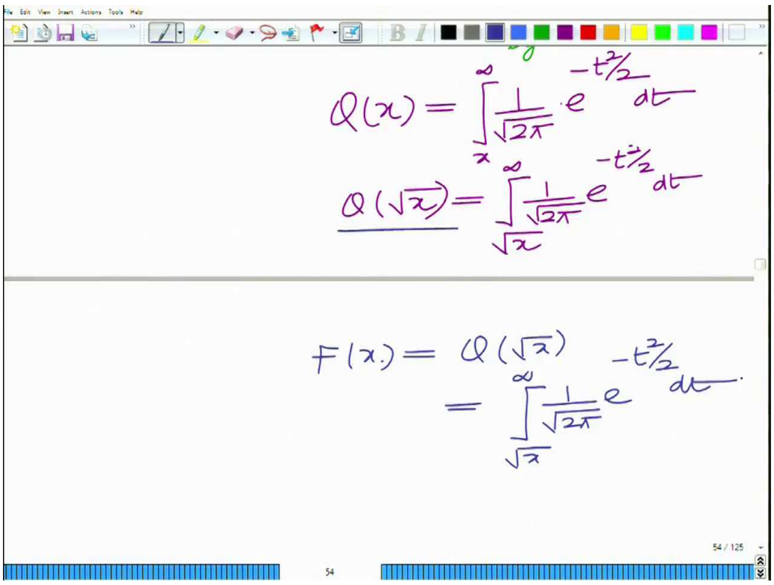

Also the Q function of some random variable x is the CCDF of the standard Gaussian random variable and is given as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Therefore,

$$Q(\sqrt{x}) = \int_{\sqrt{x}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Let us denote the above function as $Q(\sqrt{x}) = F(x)$.

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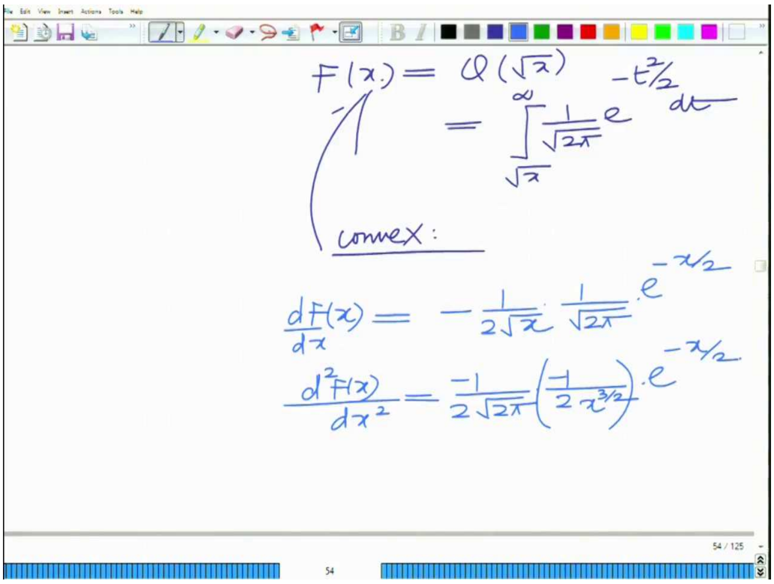

Let us check the convexity of this function. Therefore first gradient of this function is

$$\frac{d}{dx} F(x) = -\frac{1}{\sqrt{2\pi}} \frac{1}{2\sqrt{x}} e^{-\frac{x}{2}}$$

Also the second derivative of this function is

$$\begin{aligned} \frac{d^2}{dx^2} F(x) &= -\frac{1}{2\sqrt{2\pi}} \left(\frac{-1}{2x^{\frac{3}{2}}} \right) e^{-\frac{x}{2}} \\ &= \frac{1}{4\sqrt{2\pi}} \frac{1}{x^{\frac{3}{2}}} e^{-\frac{x}{2}} + \frac{1}{4\sqrt{2\pi}} \frac{1}{x^{\frac{1}{2}}} e^{-\frac{x}{2}} \end{aligned}$$

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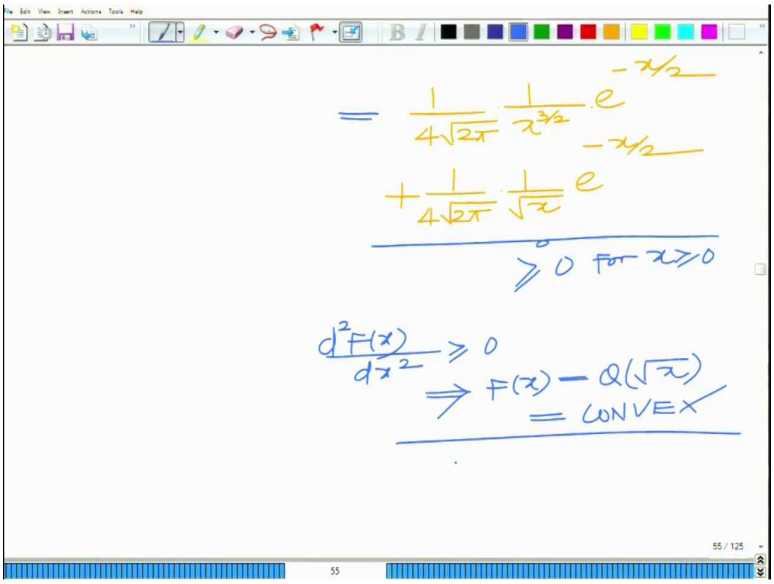

As for $x \geq 0$,

$$\frac{d^2}{dx^2} F(x) \geq 0$$

Therefore $F(x) = Q\left(\sqrt{x}\right)$ is the convex function.

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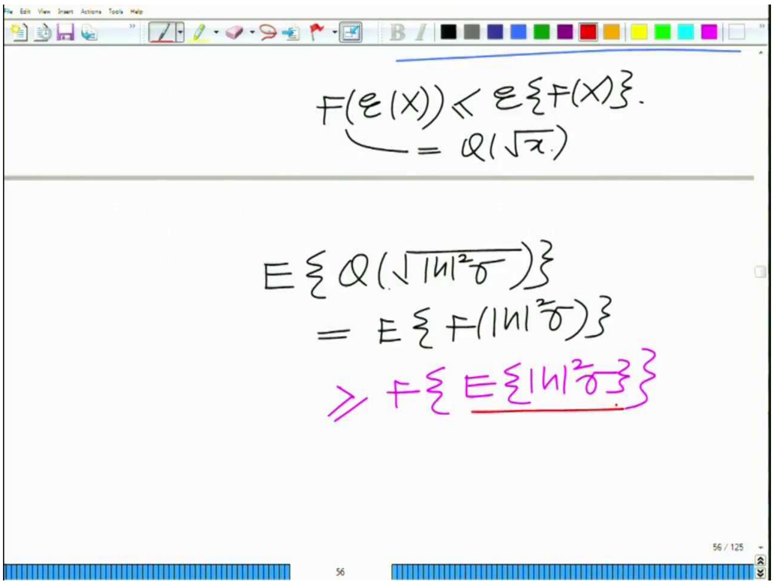

So the Q-function of wireless systems can be shown in the terms of this function $F(x) = Q(\sqrt{x})$ as follows.

$$Q\left(\sqrt{|h|^2 \gamma}\right) = F\left(|h|^2 \gamma\right)$$

Now the average BER of wireless system is $E\left\{Q\left(\sqrt{|h|^2 \gamma}\right)\right\}$. Therefore, using the Jensen's inequality

$$\begin{aligned} E\left\{Q\left(\sqrt{|h|^2 \gamma}\right)\right\} &= E\left\{F\left(|h|^2 \gamma\right)\right\} \\ &\geq F\left(E\left\{|h|^2 \gamma\right\}\right) \end{aligned}$$

And also

$$\begin{aligned} F\left(E\left\{|h|^2 \gamma\right\}\right) &= F(\gamma) \\ &= Q\left(\sqrt{\gamma}\right) \\ &= \mathrm{BER}_c \end{aligned}$$

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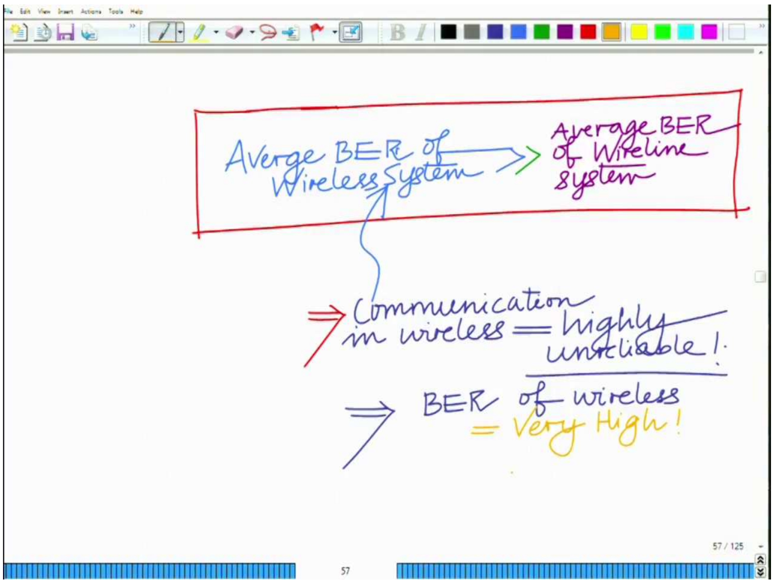

This implies that considering both the systems have same average SNR; the average BER of wireless system is greater than or equal to the average BER of wire line system.

This means signal transmission in wireless system is highly unreliable which arises due to the fading nature of wireless channel.

Therefore, fading has a significant impact on the performance of communication system and it leads to a severe degradation in the performance of a wireless communication system which makes it a big challenge.

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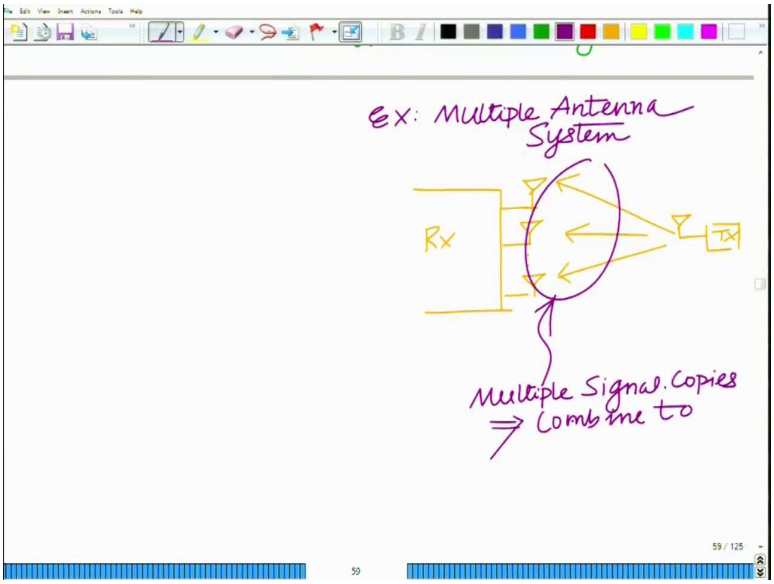

One of the most important technologies to overcome the fading nature is termed as diversity. Diversity means combining of multiple signals received at the receiver to enhance the signal to noise power ratio as well as the reliability of a wireless communication.

For example, consider a multiple antenna system with multiple receiving antennas. It receives multiple copies from the transmitter which are combined to enhance its SNR.

Hence, the diversity is a key principle in a wireless communication system and a very important technology innovation.

Hence, Jensen's inequality has a lot of applications. It can also be used to derive and prove several results in the context of information theory.