Applied Optimization for Wireless, Machine Learning, Big data
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Lecture - 51
Practical Application: Multiple Input Multiple Output (MIMO) Beamformer Design

Keywords: *MIMO Beamformer Design, Transmit beamformer, Receive beamformer, Principal Eigenvector*

Hello welcome to another module in this massive open online course, so we are looking at MIMO beam forming, how to design the optimal transmit and receive Beamformers in a multiple input multiple output wireless communication system.

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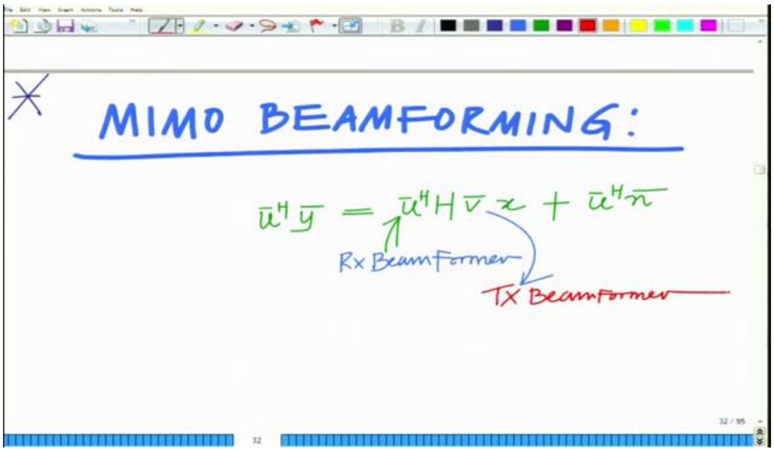

So at receive beamformer we have $\bar{u}^H \bar{y} = \bar{u}^H H \bar{v} x + \bar{u}^H \bar{n}$ where $\bar{u}$ is the receive beamformer and $\bar{v}$ is the transmit beam former and we want to jointly design the transmit and the receive beamformers to maximize the SNR. Now we set $H\bar{v} = \bar{h}$.

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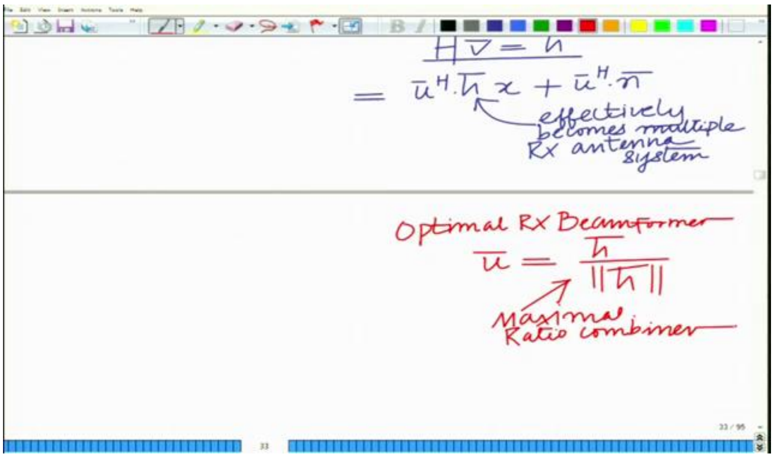

So this becomes $\bar{u}^H \bar{h} x + \bar{u}^H \bar{n}$. Now this effectively becomes a single input multiple output system or simply a multiple receiver antenna system for which we already know the optimal beamformer, $\bar{u} = \frac{\bar{h}}{\|\bar{h}\|}$ which is the maximal ratio combiner.

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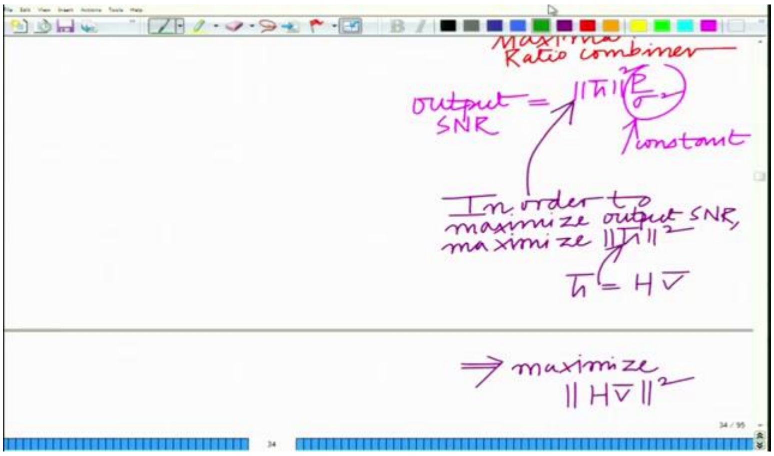

And the output SNR of the maximal ratio combiner is given as $\|\bar{h}\|^2 \frac{P}{\sigma^2}$ where P is the transmit power and σ^2 is the noise power. So to maximize the SNR we have to maximize $\|\bar{h}\|^2$. But $\bar{h} = H\bar{v}$. So substituting this we have to maximize $\|H\bar{v}\|^2$. Now $\|H\bar{v}\|^2 = \bar{v}^H H^H H \bar{v} = \bar{v}^H G \bar{v}$ where $G = H^H H$.

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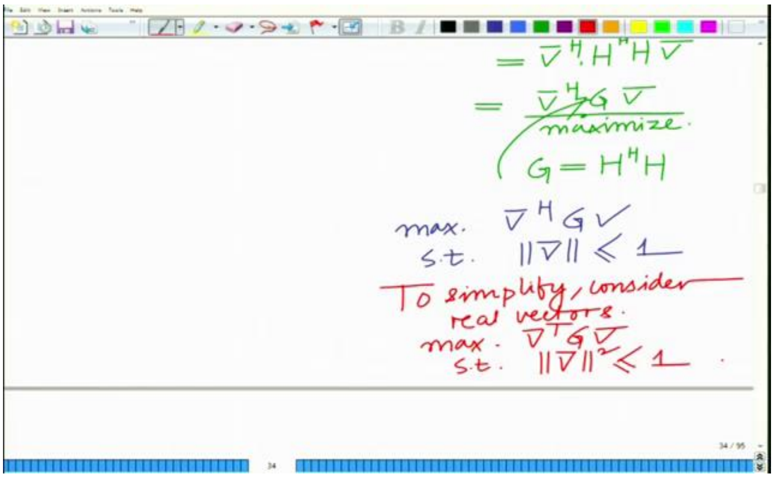

So the resulting problem for optimal beam forming becomes $\begin{matrix} \max \bar{v}^H G \bar{v} \\ s.t \|\bar{v}\| \leq 1 \end{matrix}$. And just to simplify it we are going to assume real vectors and thus it becomes $\begin{matrix} \max \bar{v}^T G \bar{v} \\ s.t \|\bar{v}\|^2 \leq 1 \end{matrix}$ and this is a non-convex problem.

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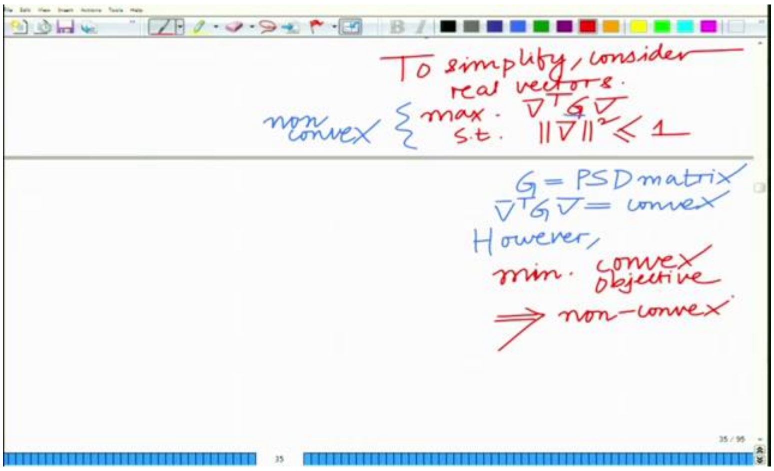

In case of a standard form convex optimisation problem, we have a convex objective, but we are minimising it, here we are maximizing it and hence it becomes non-convex. In fact you are minimising over a convex objective function implies that this is non-convex and therefore we will form the Lagrangian in the same way that we have done before.

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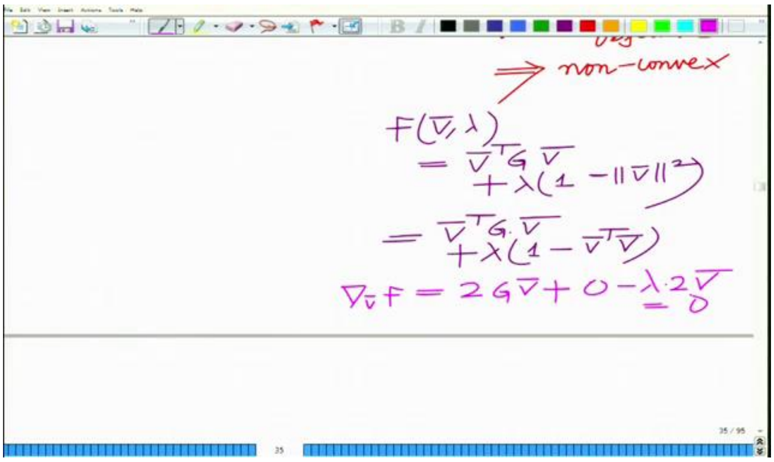

So this is $f(\bar{v},\lambda) = \bar{v}^T G \bar{v} + \lambda\left(1 - \left\|\bar{v}\right\|^2\right)$ and this can be solved as shown in slide.

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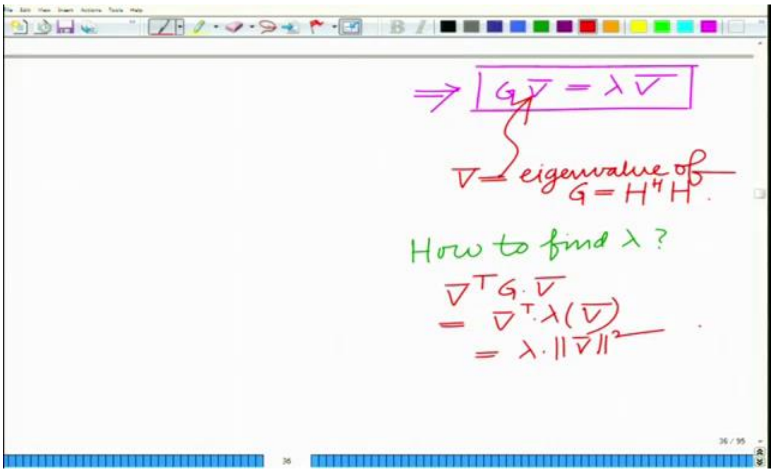

Now we get $G\bar{v} = \lambda\bar{v}$ and this equation is nothing but the definition of the eigenvector of G. So this has many eigenvalues and we need to find the Lagrange multiplier as shown.

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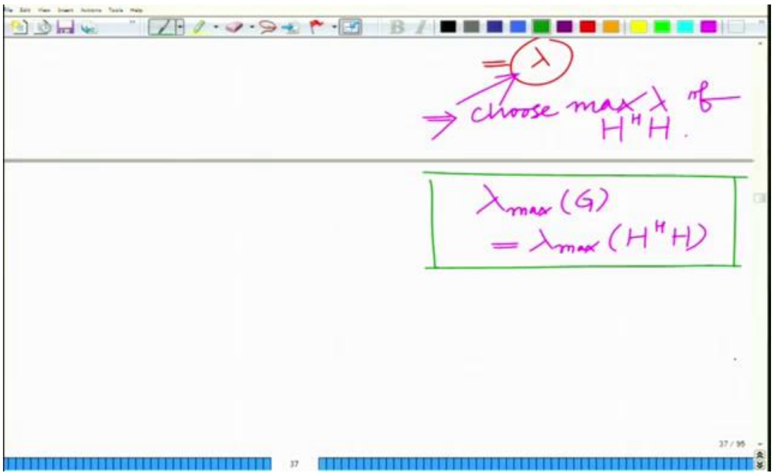

So we want to maximize this, implies choose the eigenvector corresponding to maximum eigenvalue of $H^H H$. So the transmit beamformer corresponds to the maximum eigenvalue of $H^H H$ and this maximizes the SNR at the receiver.

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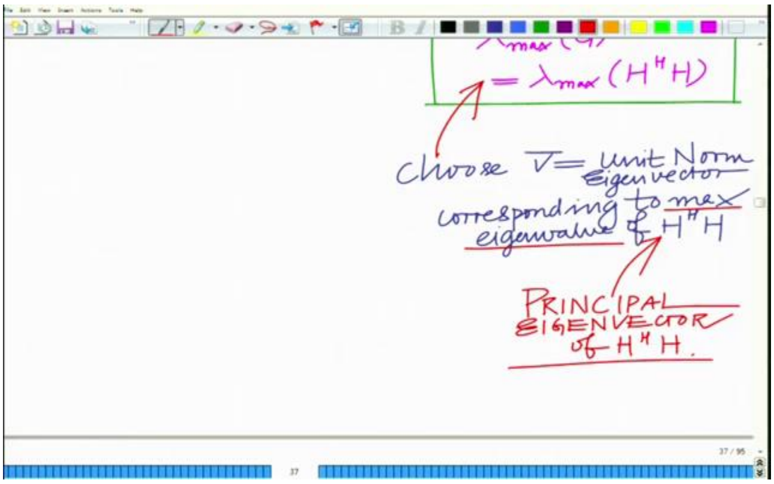

So this Eigen vector corresponding to the largest eigenvalue is termed as the principal eigenvector.

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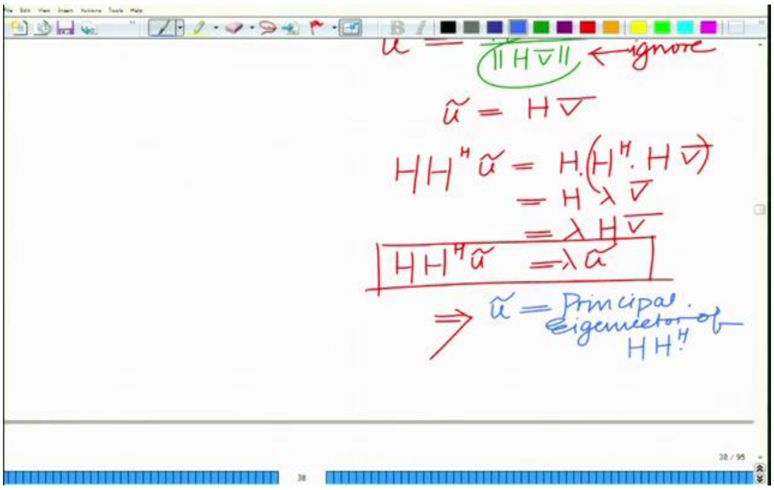

Now the receive beam former is also found as shown in slide and we can see that the receive beam former is the eigenvector corresponding to largest eigenvalue of HH^H .

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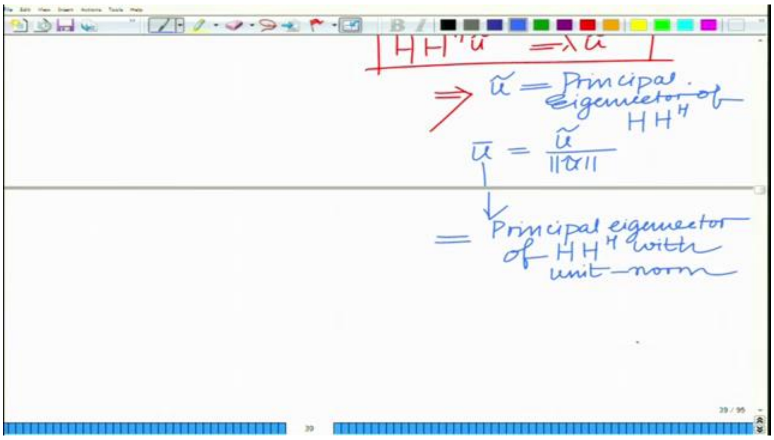

And that basically gives us both the transmit and receive beamformers.

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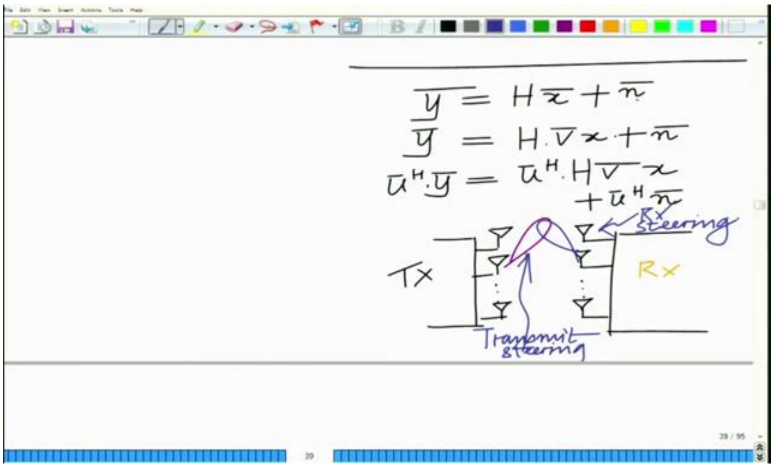

And we have to perform beamforming at both the ends in the MIMO system. So you have the transmitter, you have the receiver, you are transmitting from the transmitter in a particular direction, at the receiver you are also collecting or processing the signal, you are steering the receiver antenna array in a particular direction. All these are electronic steering, so you do not need to physically steer.

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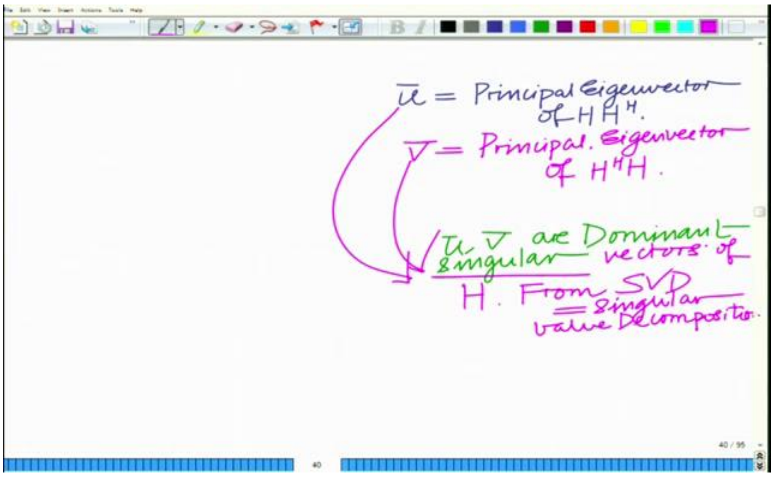

So $\bar{u}$ is the principal eigenvector of HH^H and $\bar{v}$ is the principal eigenvector of H^HH. Now later when we the singular value decomposition of the channel matrix it will turn out that $\bar{u}$ and $\bar{v}$ are in fact the dominant left and right singular vectors corresponding to the larger singular value.

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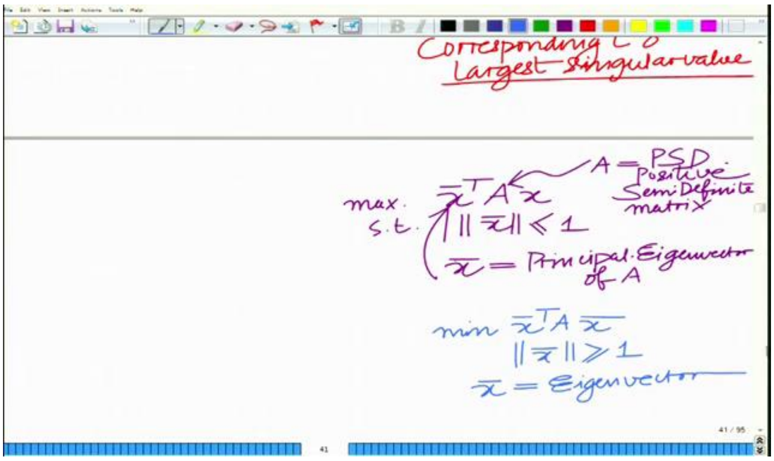

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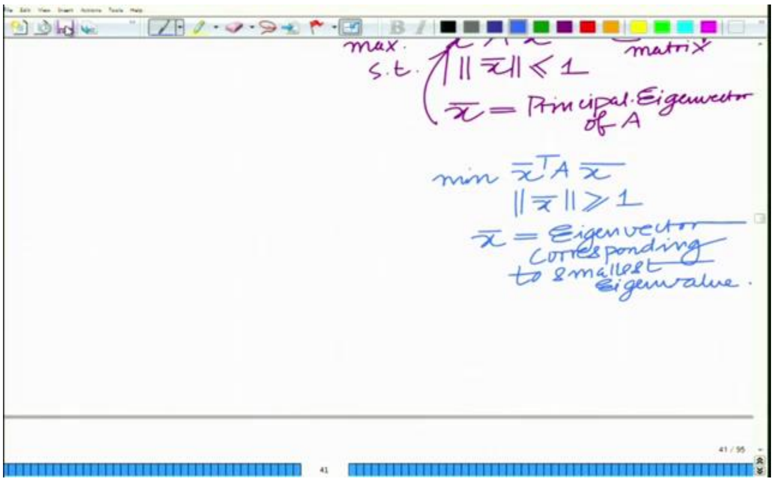

So this is a very interesting application with respect to beam forming in MIMO systems. So we will stop here and continue in the subsequent modules. Thank you very much.