

1. The pseudo-inverse of \mathbf{X} is $(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

Ans c

2. Since $\bar{\mathbf{x}} \geq \mathbf{0}$, it follows that $\bar{\mathbf{c}}^T \bar{\mathbf{x}} \geq \min\{\mathbf{c}_i\} \sum_{i=1}^n x_i = \min\{\mathbf{c}_i\}$. This can be achieved by setting x_i corresponding to index i for which c_i is minimum to 1 and rest to 0

Ans d

3. The constraint for the beamformer $\bar{\mathbf{w}}$, with estimate of the nominal CSI denoted by $\bar{\mathbf{h}}_e$, for a suitable matrix \mathbf{P} , is $\|\mathbf{P}^T \bar{\mathbf{w}}\| \leq \bar{\mathbf{w}}^T \bar{\mathbf{h}}_e - 1$

Ans d

4. Given the base station cooperation problem with K base stations, M users, $P_{i,j}, h_{i,j}$ denoting the power and channel coefficient from base station i to user j , respectively. As shown in lectures, the minmax optimization problem is

$$\begin{aligned} \min. & t \\ \text{s. t.} & \sum_j P_{i,j} \leq t, 1 \leq i \leq K \\ & \sum_i P_{i,j} |h_{i,j}|^2 \geq \tilde{P}_j, 1 \leq j \leq M \end{aligned}$$

Ans c

5. The channel estimate is

$$\begin{aligned} & (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \\ & = \left(\begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix} \\ & = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \end{aligned}$$

Ans c

6. The pseudo-inverse of \mathbf{X} is $(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T = \frac{1}{4} \begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

Ans a

7. The optimal vector $\bar{\mathbf{x}}$ that minimizes the regularized least-squares cost function $\min. \|\mathbf{A}\bar{\mathbf{x}} - \bar{\mathbf{b}}\|^2 + \lambda \|\bar{\mathbf{x}}\|^2$ is $(\mathbf{A}^T \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^T \bar{\mathbf{b}}$

Ans b

8. Given the full column rank matrix \mathbf{A} . The projection matrix for the column space of \mathbf{A} is $\mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$

Ans d

9. Given the least-squares problem $\min. \|\bar{\mathbf{y}} - \mathbf{A}\bar{\mathbf{x}}\|^2$, with $\bar{\mathbf{y}} - \mathbf{A}\bar{\mathbf{x}} = \bar{\mathbf{e}}$. For the optimal solution $\hat{\mathbf{x}}$, the corresponding error vector $\bar{\mathbf{a}}$ is perpendicular to each column of \mathbf{A}

Ans c

10. The channel estimate is

$$(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Ans a