

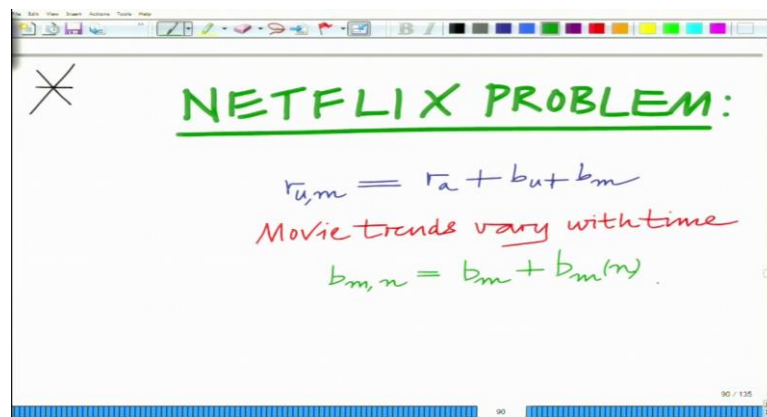
Applied Optimisation for Wireless, Machine Learning, Big Data.
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 79
Matrix Completion Problem in Big Data: Netflix-II

Keywords: *Matrix Completion Problem*

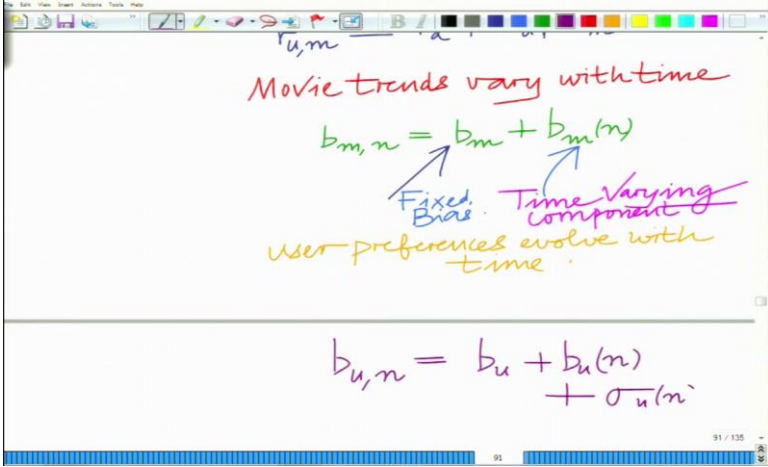
Hello, welcome to another module in this massive open online course.

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So we are looking at the Netflix problem that is how to predict the user ratings for movies which he or she has not seen. So this is basically a specific case of matrix completion problems. And we have seen that you can express the rating of each movie as shown in slide. Now let us refine this a little bit because the movie trends change with time. So a more refined model can be expressed as a fixed bias along with a time varying bias as shown in slide.

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Movie trends vary with time

$$b_{m,n} = b_m + b_m(n)$$

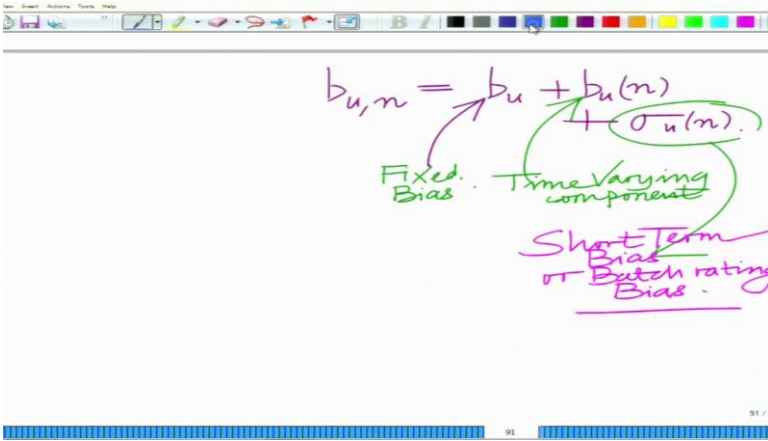
Fixed Bias: Time Varying Component

user preferences evolve with time.

$$b_{u,n} = b_u + b_u(n) + \sigma_u(n)$$

So user preferences are also varying with time.

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$$b_{u,n} = b_u + b_u(n) + \sigma_u(n)$$

Fixed Bias: Time Varying component

Short Term Bias or Batch rating Bias.

So we have $b_{u,n} = b_u + b_u(n) + \sigma_u(n)$. For any rating system, the same account can be used by multiple users. So one set of ratings can be given by a male member of the family, another set of ratings can be given by a child, another set of ratings can be given by a female member of the family and so on, different users of the same account have different preferences. So this accounts for that short term bias resulting from different users giving ratings in a batch.

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	A	B	C
User ₃	2 Jan	5 Jan	4 Jan

or Batch rating Bias

$$2 = r_a + b_a + b_a(\text{Jan}) + b_3 + \sigma_3(1) + b_3(\text{Jan})$$

Let us look at for instance a simple example lets refine our previous model itself comprising of 3 movies A B C with a person's ratings if you remember 2 5 4. Now let us say he or she rated those movies in January. And let us say this corresponds to batch 1, so these are the batches. So some person who is using the same account as user 3 rated movies A and C in January in the first batch while possibly another person rated movie B in the same month, but in another batch. So now you can develop the model again as shown in slide.

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	A	B	C
User ₃	2 Jan	5 Jan	4 Jan

Batch

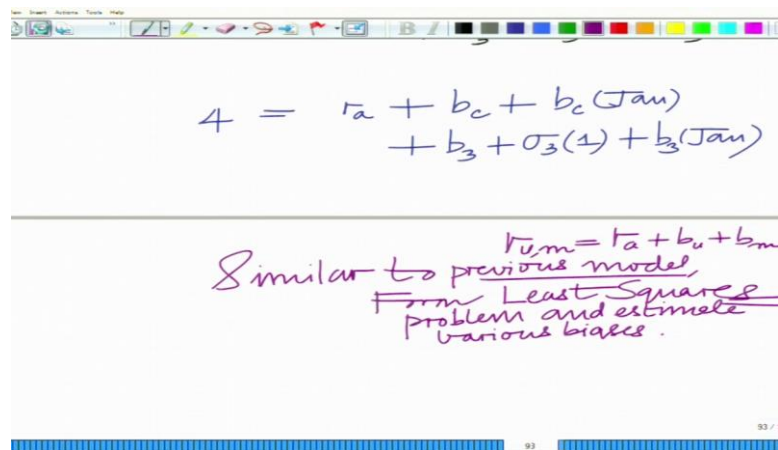
$$2 = r_a + b_a + b_a(\text{Jan}) + b_3 + \sigma_3(1) + b_3(\text{Jan})$$

Batch Bias for batch = 1

$$5 = r_a + b_b + b_b(\text{Jan}) + b_3 + \sigma_3(2) + b_3(\text{Jan})$$

So this is done as shown in slides below.

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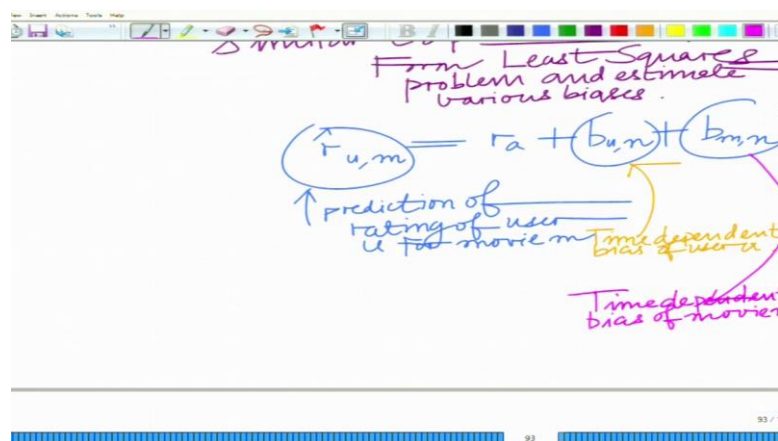
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$$4 = r_a + b_c + b_c(\text{Jan}) + b_3 + \sigma_3(4) + b_3(\text{Jan})$$

Similar to previous model, $r_{u,m} = r_a + b_u + b_m$
 Form Least Squares problem and estimate various biases.

So now you have many more biases, so you can form a least squares again similar to the previous one and solve for the biases.

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Situation: Form Least Squares problem and estimate various biases.

$$\hat{r}_{u,m} = r_a + (b_{u,n}) + (b_{m,n})$$

Annotations:

- $\hat{r}_{u,m}$: prediction of rating of user u for movie m
- $b_{u,n}$: time dependent bias of user u
- $b_{m,n}$: time dependent bias of movie m

Now, at this stage you can once again predict substituting the biases and the missing prediction $q_{u,m} = r_{u,m} - r_a - b_{u,n} - b_{u,m}$.

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To make better
Subtract biases.

$$\hat{q}_{u,m} = r_{u,m} - r_u - b_{u,m}$$

unbiased rating
or innovation

Now, we can refine this model slightly further by subtracting the biases. So after removing the biases we can call this as the unbiased rating or the innovation, something that we have not been able to predict.

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	A	B	C
1	q_{1A}	q_{1B}	q_{1C}
2	q_{2A}	?	q_{2C}
3	q_{3A}	q_{3B}	q_{3C}

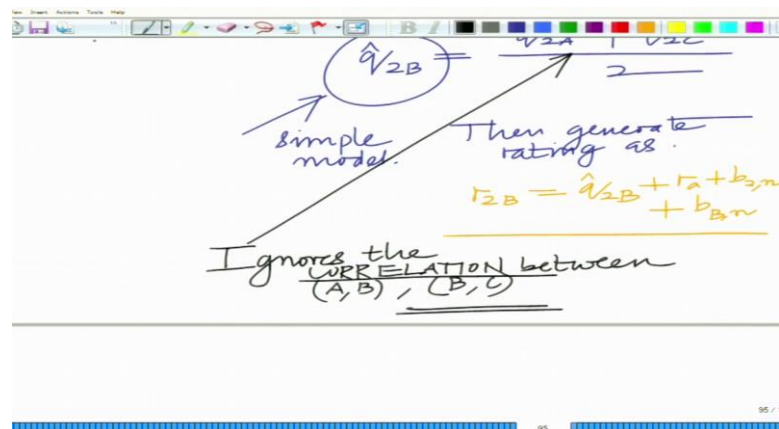
missing innovation
for user 2 movie B

$$\hat{q}_{2B} = \frac{q_{2A} + q_{2C}}{2}$$

And now you can form the innovations corresponding to the different ratings that are available as shown in slide and compute the missing innovation. So we

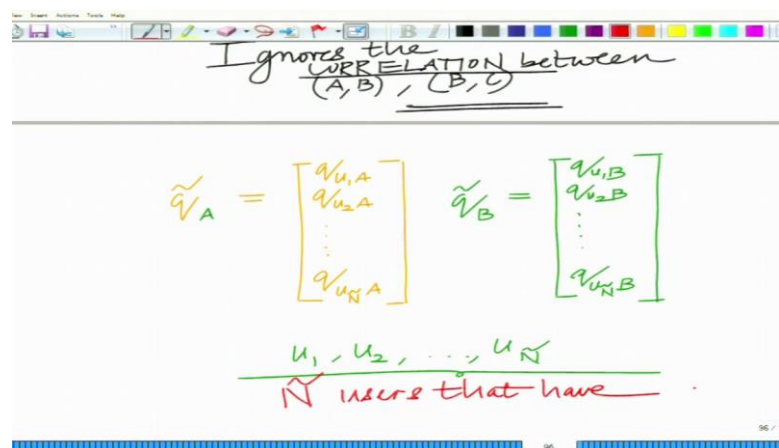
have $q_{2B} = \frac{q_{2A} + q_{2C}}{2}$.

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This is a simple model, then once you do this you generate the rating as $r_{2B} = q_{2B} + r_a + b_{2,n} + b_{B,n}$ and this is your prediction model. However, this model ignores the correlation between movies A, B and movies B, C. Because if A and B are very similar then you have to give higher weightage to A, on the other hand B and C are very similar then you have to give a higher weightage to B. So one has to give adequate weightage depending on the correlation.

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Let us consider now the innovation vectors as shown in slide.

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$$\tilde{q}_A = \begin{bmatrix} q_{u_1 A} \\ q_{u_2 A} \\ \vdots \\ q_{u_N A} \end{bmatrix} \quad \tilde{q}_B = \begin{bmatrix} q_{u_1 B} \\ q_{u_2 B} \\ \vdots \\ q_{u_N B} \end{bmatrix}$$

u_1, u_2, \dots, u_N

N users that have rated both movies A, B.

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Correlation between movies A, B

$$\triangleq \frac{\sum_{i=1}^N q_{u_i A} q_{u_i B}}{\|\tilde{q}_A\| \|\tilde{q}_B\|}$$

$$d_{BA} = \frac{\tilde{q}_A^T \cdot \tilde{q}_B}{\|\tilde{q}_A\| \|\tilde{q}_B\|}$$

Then the correlation between these two can be defined as $\frac{\sum_{i=1}^N q_{u_i A} q_{u_i B}}{\|\tilde{q}_A\| \|\tilde{q}_B\|}$. So we call this as

the similarity coefficient or measure of similarity, so what we are doing is we are taking the ratings of users who have rated both A and B, computing the inner product between them and dividing them by their norms of these two vectors.

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The slide shows a handwritten formula for the similarity coefficient d_{BA} enclosed in a green box:

$$d_{BA} = \frac{\tilde{q}_A^T \cdot \tilde{q}_B}{\|\tilde{q}_A\| \|\tilde{q}_B\|}$$

Below the formula, there are two annotations:

- An arrow points from the text "index of similarity between B, A." to the d_{BA} term.
- An arrow points from the text "Correlation coefficient or measure of similarity between movies A, B." to the entire formula.

Below the text, a vector diagram shows two vectors, \tilde{q}_A and \tilde{q}_B , originating from the same point. The angle between them is labeled θ . A vertical blue arrow points upwards from the origin.

In fact, this is exactly the cosine of the angle between the two vectors.

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The slide shows a vector diagram with two vectors, \tilde{q}_A and \tilde{q}_B , originating from the same point. The angle between them is labeled θ . Below the diagram, the following text is written:

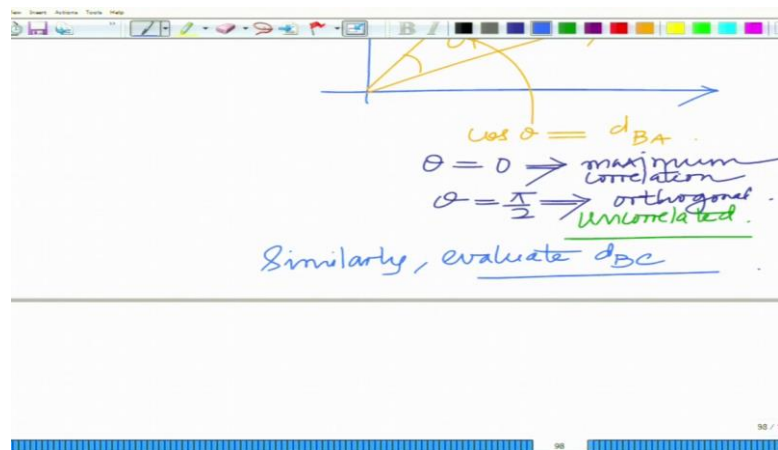
$\cos \theta = d_{BA}$

$\theta = 0 \Rightarrow \text{maximum correlation}$

$\theta = \frac{\pi}{2} \Rightarrow \text{orthogonal}$

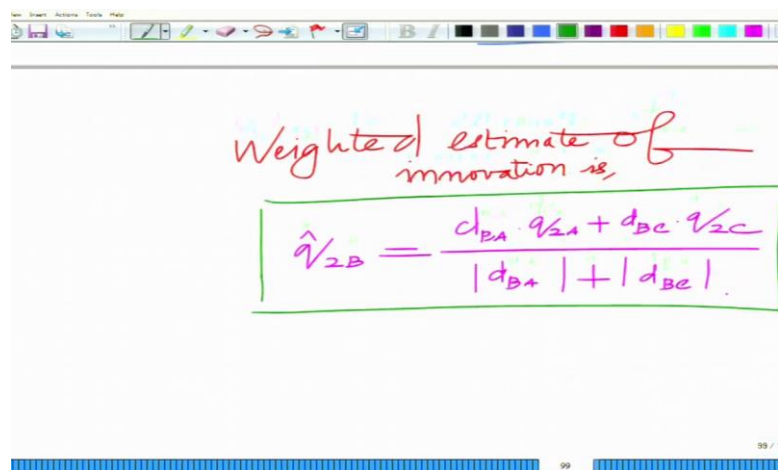
So cosine of the angle between these two vectors is nothing but what we have defined as the similarity coefficient. And you can clearly see that if $\theta = 0$ the vectors are aligned, movies A and B are similar, as θ increases movies A and B are dissimilar. If $\theta = \frac{\pi}{2}$ they are perpendicular in fact, A has no bearing on B implies they are orthogonal or no correlation, implies uncorrelated, the innovations are uncorrelated.

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And similarly now evaluate $d_{B,C}$, this is the correlation or measure of similarity.

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Now, the weighted innovation or weighted estimate is $q_{2B} = \frac{d_{BA} q_{2A} + d_{BC} q_{2C}}{|d_{BA}| + |d_{BC}|}$ and this is the final step.

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Weight of movie innovation is, 0

$$\hat{q}_{2B} = \frac{(d_{BA} \cdot q_{2A} + d_{BC} \cdot q_{2C})}{|d_{BA}| + |d_{BC}|}$$

Weighted.

weighting movie by indices of similarity.

$$\hat{r}_{2B} = \hat{q}_{2B} + r_a + b_{2,n} + b_{B,n}$$

Final Prediction of rating of user 2 for movie B.

And now once you compute this innovation estimate you can add it to the biases as $\hat{r}_{2B} = q_{2B} + r_a + b_{2,n} + b_{B,n}$ and that is basically the final step in this procedure. So this is your final prediction of rating of user 2 for movie B. So this brings across various ideas in both linear algebra as well as optimization. So we will stop here and continue in the subsequent modules. Thank you very much.