Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 69
Example problem on Optimal MIMO Power allocation (Waterfilling)

***Keywords**: Optimal MIMO Power allocation, Waterfilling Algorithm, Singular Value Decomposition(SVD)*

Hello, welcome to another module in this massive open online course. So we are looking at Optimal MIMO Power Allocation. Now let us do an example to understand this better.

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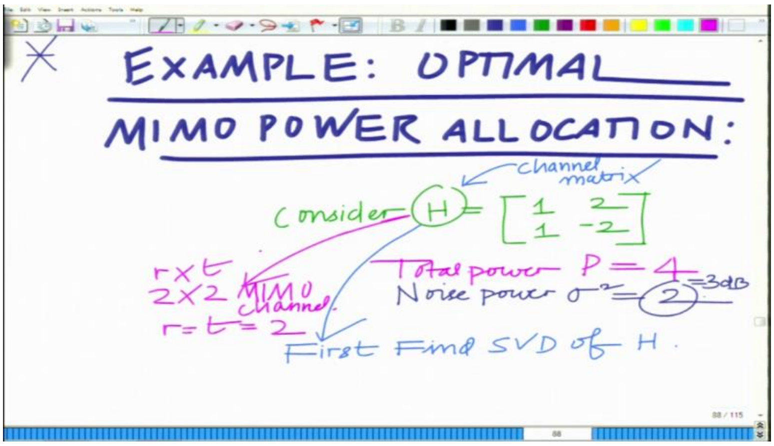

So now consider the MIMO channel matrix $H = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$.

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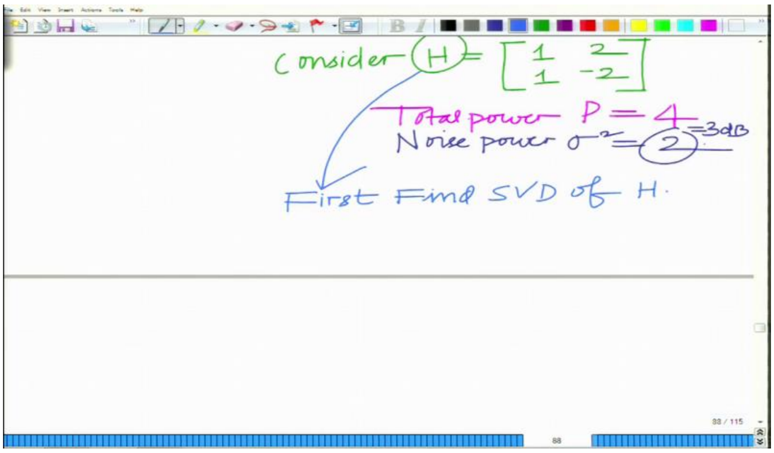

We have a total power P = 4 and noise power $\sigma^2 = 3dB = 2$. And now we have to optimally allocate this total power. So for optimal power allocation which maximizes the sum rate we have to first start with the singular value decomposition.

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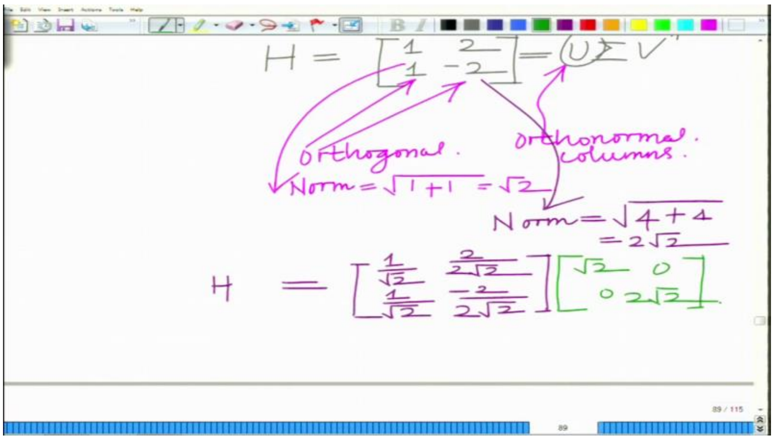

Now this can be written as $H = U\Sigma V^H$ where U contains orthonormal columns and these columns are orthogonal to each other. So all we have to do is we have to simply normalize them and this is as shown in slide. So we will get

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{2\sqrt{2}} \\ \frac{2}{\sqrt{2}} & \frac{-2}{2\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{bmatrix}.$$

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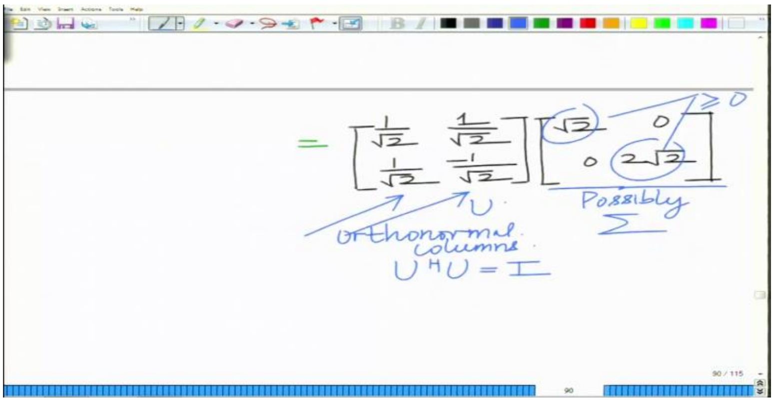

Now we have orthonormal columns and this satisfies the property of U. And in fact this matrix can be possibly Σ because this is a diagonal matrix and these are non-negative, so these are the possible singular values. And now we need the V matrix which is a unitary matrix. So I can simply use the identity matrix in this case as unitary matrix.

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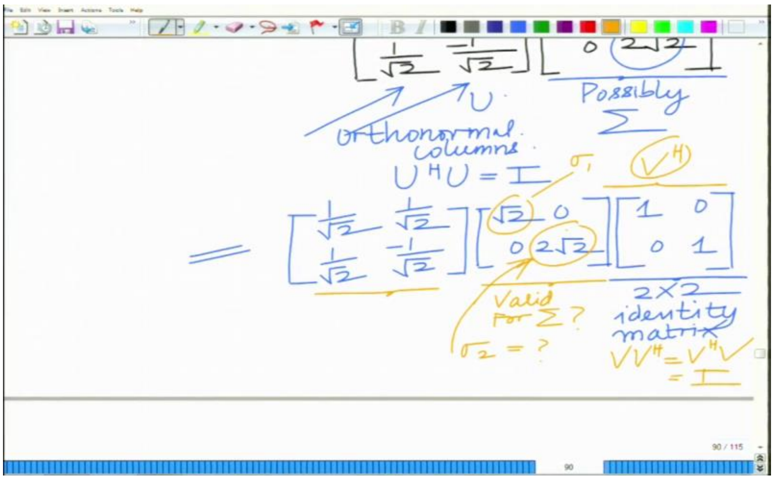

Now the only problem is the that the singular values should be arranged in decreasing order which is not possible in this obtained matrix.

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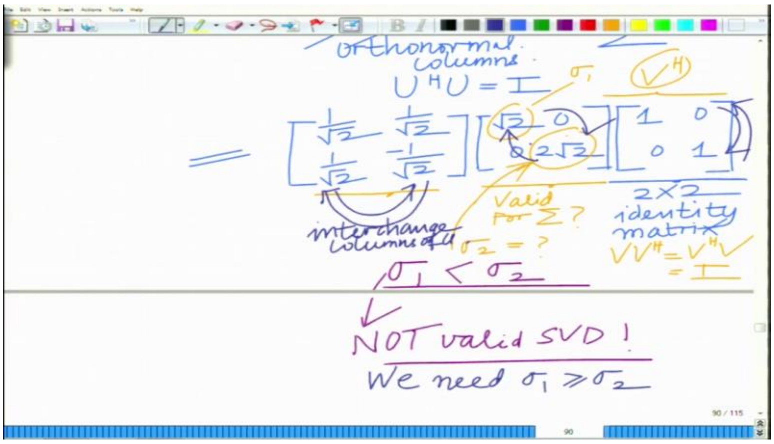

So this is not a valid SVD. So we need the singular values to be ordered in decreasing order. So we have to somehow switch these values and this is as shown in slide. So this is possible if I basically interchange the columns of U and the rows of V^H and then I can flip the singular values.

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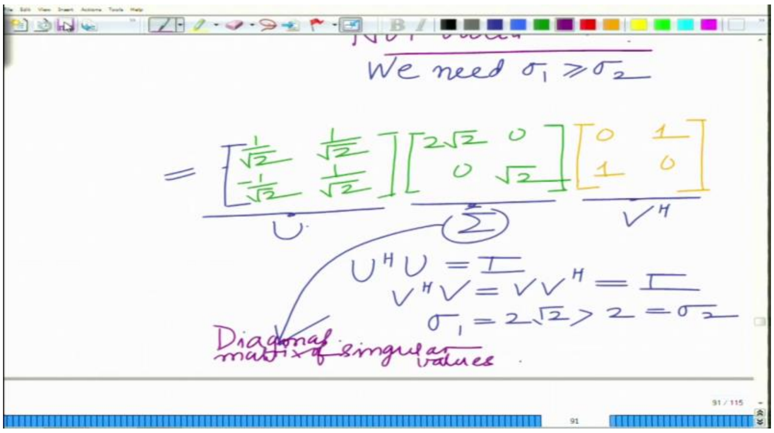

So finally we have $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and now we have to do optimal power allocation that is you can decompose this using pre coding as the combination of two parallel channels.

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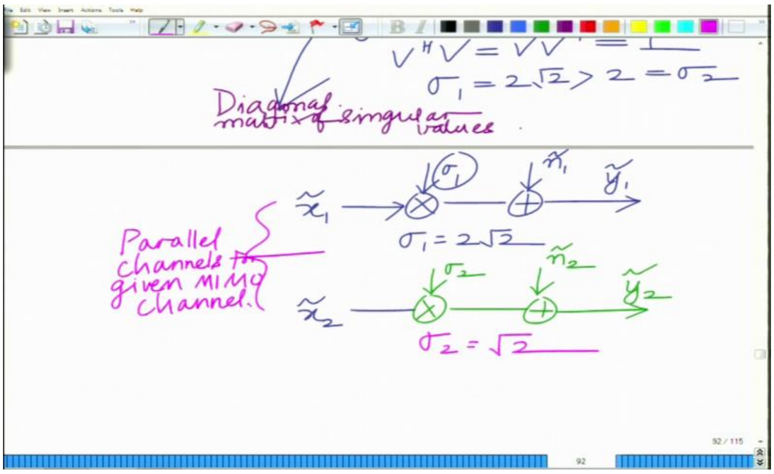

So you are transmitting x_1 through the first channel that has gain σ_1 and noise n_1 to give y_1. And similarly x_2 through the second channel that has gain σ_2 and noise n_2 to give y_2, so these are the parallel channels for the given MIMO channel. So this is a 2×2 MIMO channel.

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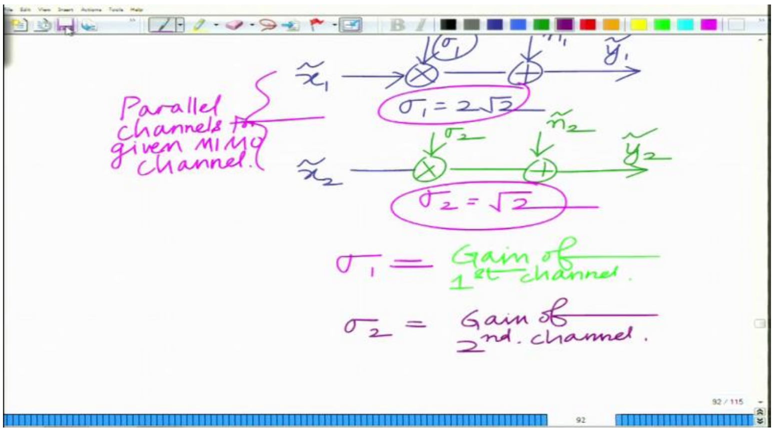

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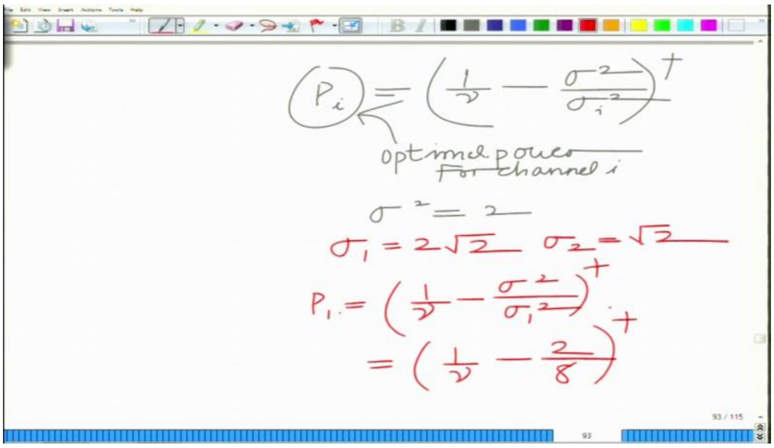

Now for the optimal power allocation we substitute the required quantities as shown in slide.

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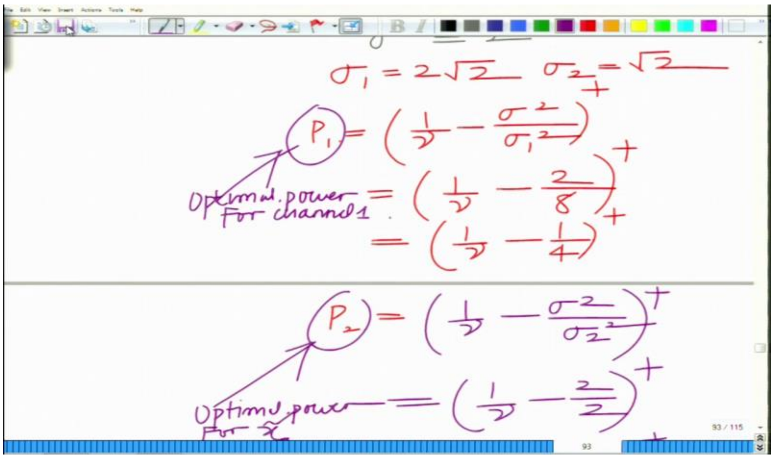

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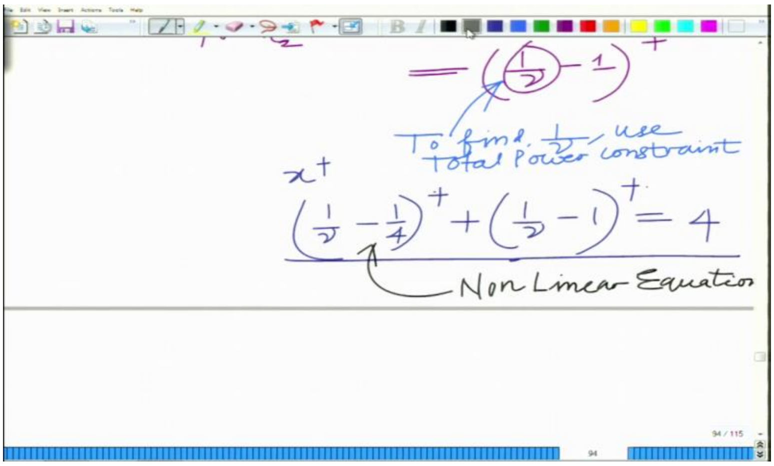

So finally to find ν we have to use the total power constraint. So this is basically a non-linear equation and this is proceeded as shown in slide.

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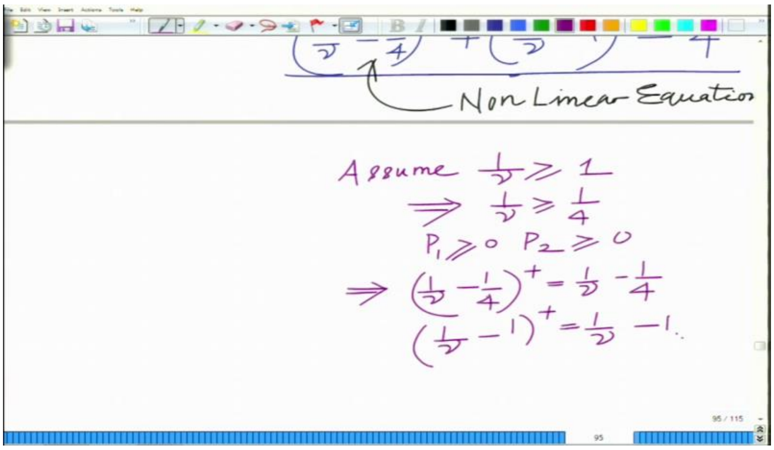

So start with the assumption, $\frac{1}{\nu} \geq 1$.

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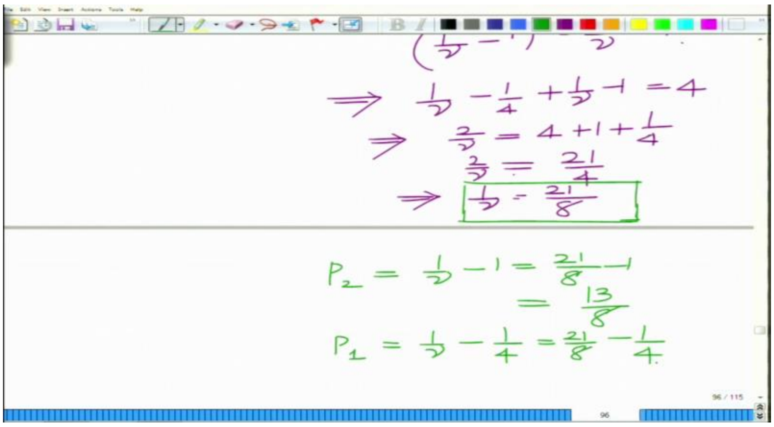

So finally we get $P_1 = \frac{19}{8} > 0, P_2 = \frac{13}{8} > 0$.

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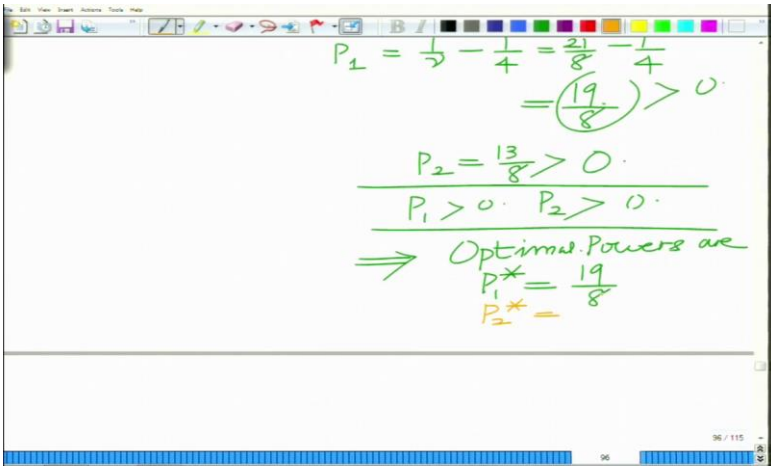

So the original assumption holds and we get the optimal powers.

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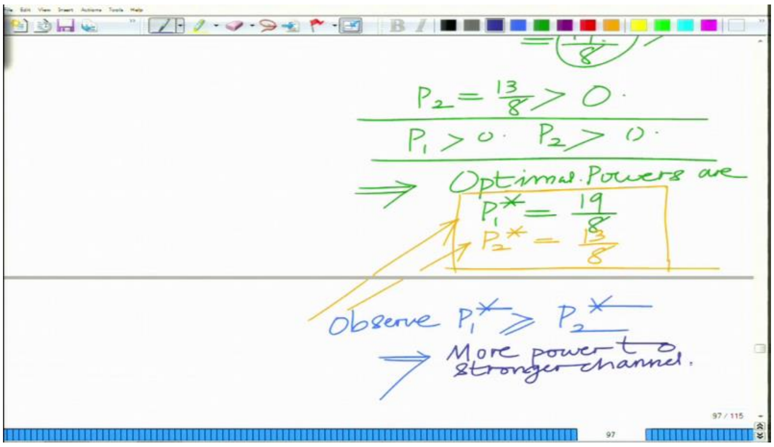

Now, if one of the powers would have been negative that implies our original assumption is incorrect. So the power is negative implies that that corresponding channel is above the water level. So power is not allocated. So in the corresponding channel the power has to be set to 0 and the problem has to be repeated with the total power constraint. So this is the procedure alright and now you also observe that more power is allocated to the stronger channel.

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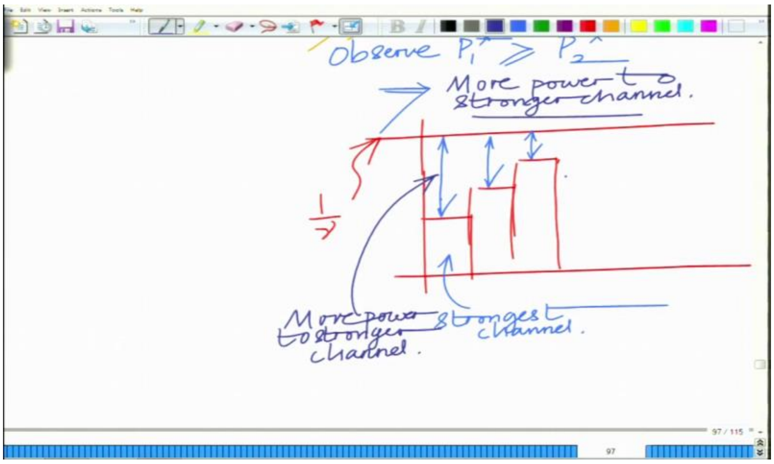

So what this shows is that to maximize capacity more power is allocated to the stronger channel that is the one with the largest singular value.. So we will stop here and continue in the subsequent modules. Thank you very much.