

1. Given an additive white Gaussian noise (AWGN) channel with  $\text{SNR} = \frac{P}{\sigma^2} = \gamma$ . The BER corresponding to the transmission of BPSK modulated symbols is  $Q(\sqrt{\gamma})$

Ans c

2. The Hessian of  $f(\bar{\mathbf{x}}) = \frac{x_1^2}{x_2}$  is  $\frac{2}{x_2^3} \begin{bmatrix} x_2 & \\ & -x_1 \end{bmatrix}$

Ans a

3. The function  $\sqrt{|x|}$  is Quasi-convex

Ans d

4. The  $l_\infty$  norm of vector  $\bar{\mathbf{x}} = [x_1 \ x_2 \ \dots \ x_n]^T$  is given as  $\max\{|x_i|, 1 \leq i \leq n\}$

Ans b

5. Given function  $f(\bar{\mathbf{x}}) = \sum_{i=1}^n x_i \ln x_i$  equals the negative entropy. Since entropy is concave, negative entropy is convex. This can also be seen as follows

$$(x_i \ln x_i)'' = (\ln x_i + 1)' = \frac{1}{x_i} \geq 0 \Rightarrow \text{convex}$$

Ans b

6. The function  $f(x) = h(g(x))$  is convex if  $g$  is convex, and  $h$  is convex non-decreasing

Ans a

7. Given the function  $f(\bar{\mathbf{x}}) = \log \sum_{k=1}^n e^{x_k}$  and  $z_k = e^{x_k}$ . The Hessian of  $f(\bar{\mathbf{x}})$  is

$$\frac{\text{diag}(\bar{\mathbf{z}})}{\bar{\mathbf{1}}^T \bar{\mathbf{z}}} - \frac{\bar{\mathbf{z}} \bar{\mathbf{z}}^T}{(\bar{\mathbf{1}}^T \bar{\mathbf{z}})^2}$$

Ans d

8. The function  $f(x) = h(g(x))$  is convex if  $g$  is concave, and  $h$  is convex non-increasing

Ans c

9. Given  $f(x_1, x_2) = x_1 x_2$ , for  $x_1, x_2 > 0$ . The Hessian is  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ . Its eigenvalues are  $\pm 1$ .

Hence, it is not PSD and therefore, not convex. Similarly, it is not concave. It can be seen that the level sets  $-x_1 x_2 \leq a$  or  $x_1 x_2 \geq a$  are convex. Hence,  $-x_1 x_2$  is quasi-convex, which implies  $x_1 x_2$  is quasi-concave

Ans c

10. The function  $\sum_{i=1}^m \log g_i(x)$  is concave, for  $g_i(x) \geq 0$ , when  $g_i(x)$  are concave. This can be seen as follows.

$$(\log g_i(x))'' = \left( \frac{g_i'(x)}{g_i(x)} \right)' = \frac{g_i''(x)g_i(x) - (g_i'(x))^2}{g_i^2(x)}$$

It can be seen that  $(\log g_i(x))'' \leq 0$  if  $g_i(x)$  is concave.

Ans b