Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 48
Linear Program Practical Application: Base Station Co – operation

Keywords*: Linear Program, Base Station Co – operation*

Hello, welcome to another module in this massive open online course. So we are looking at convex optimization problems and in this module, let us look at an important subclass of convex optimization problems which is basically linear programs also referred to as LPs.

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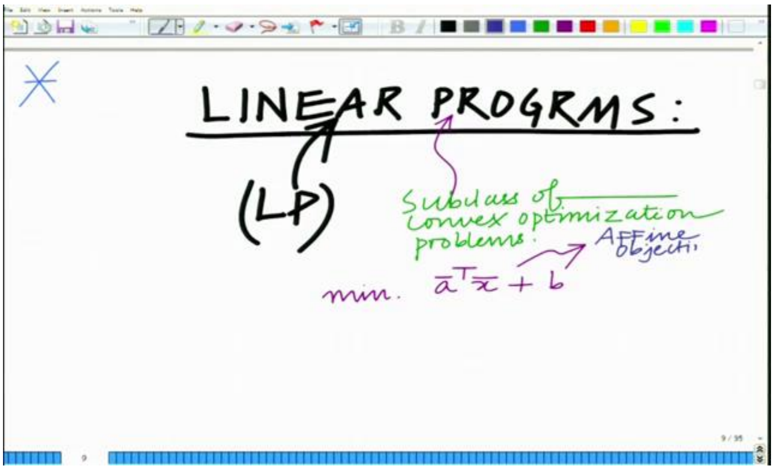

This forms a subclass of convex optimization problems. A linear program can be expressed as follows

$$\begin{aligned} &\min \ \bar{a}^T \bar{x} + b \\ &\text{s.t} \ \ \bar{c}_i^T \bar{x} \le d_i \\ &\qquad i = 1, 2, .., l \\ &\qquad c_j^T \bar{x} = d_j \\ &\qquad j = 1, 2, .., m \end{aligned}$$

, that is any convex optimization has an objective here it has a linear objective or an affine objective function, subject to the constraints which are also affine.

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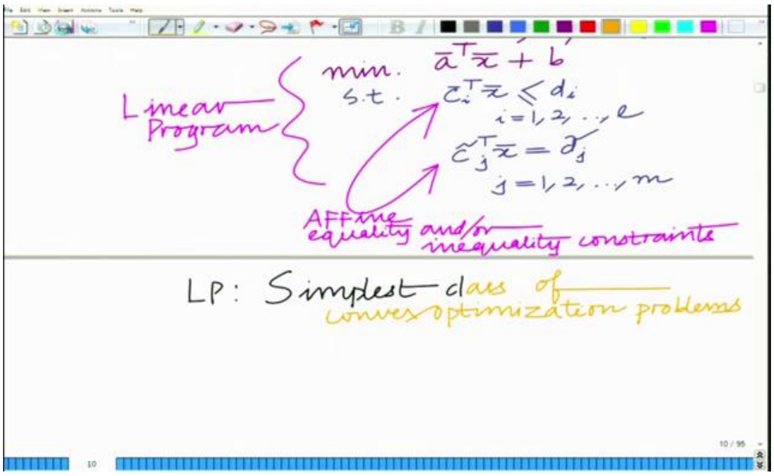

So linear program is a subclass of convex optimization problems in which the objective function as well as the constraints, equality as well as inequality constraints are all affine in nature. This is the simplest class or category of convex optimization problems.

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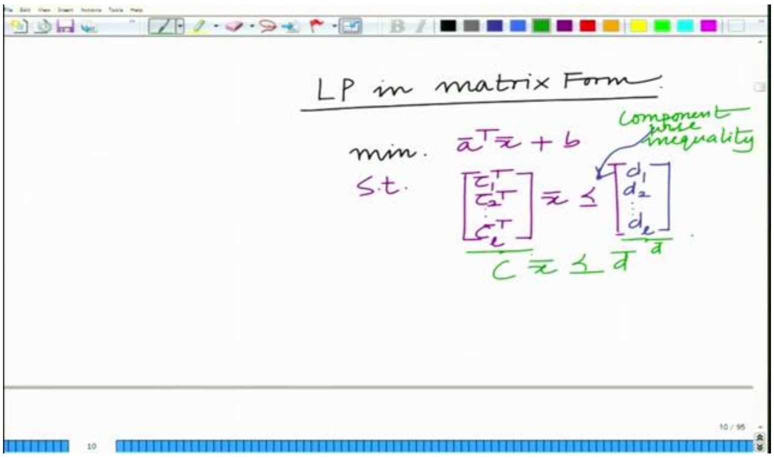

Now we can write this in matrix form as shown in slide. Now, here each component of the vector on the left has to be less than each component of the vector on the right. So this is a component wise inequality.

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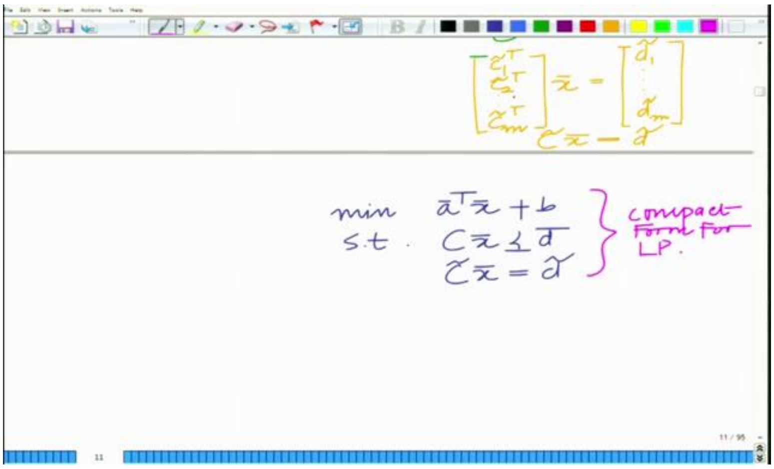

So $\begin{array}{l} \min\ \bar{a}^{T}\bar{x}+b \\ s.t\ \ \bar{c}\bar{x}\le \bar{d} \\ \qquad \tilde{c}\bar{x}=d \end{array}$, this basically expresses the linear program in a very compact using vectors and matrices.

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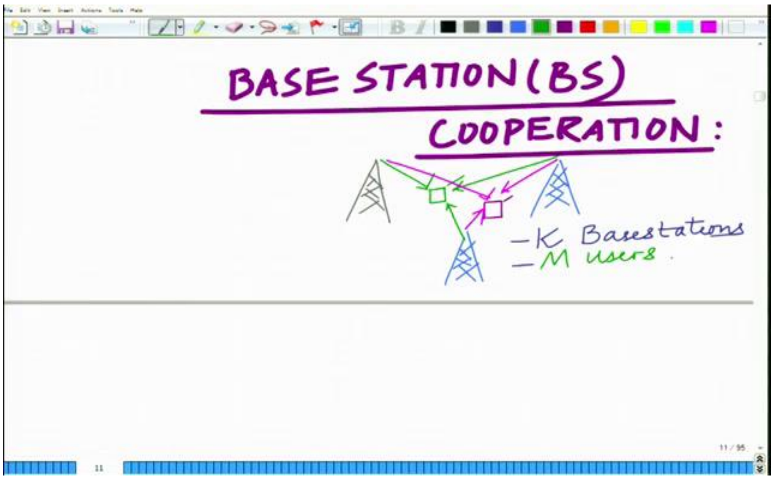

Let us look at an example for this linear program which is the base station cooperation where there are several base stations in a cellular communication scenario where user at the edge of several cells can be served by several base stations. So in base station cooperation, we have a group of cells that are cooperating to transmit to one or many users. Normally, we have a single base station serving any particular user, but in this particular scenario base stations can cooperate to serve the various users thereby enhance the SNR, enhance the reliability of communication in a wireless communication scenario. So we have K base stations and M users.

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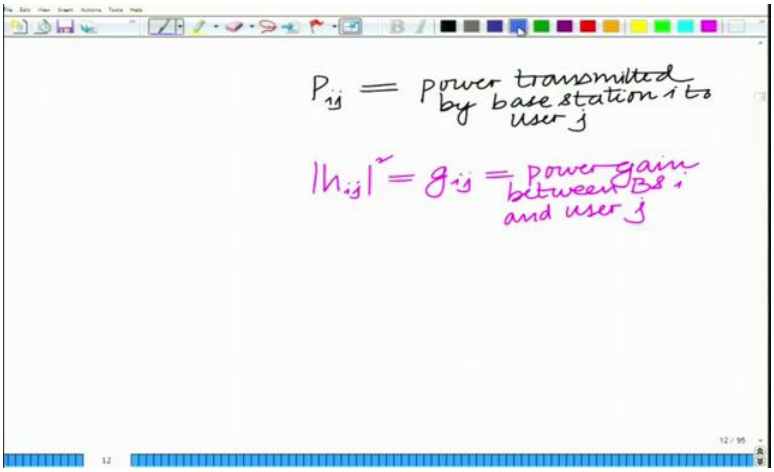

And in this scenario, let P_{ij} denote the power transmitted by base station i to user j and h_{ij} is the fading channel coefficient. So $|h_{ij}|^2 = g_{ij}$ represents the power gain between base station i and user j. Now if you look at the power that is received by each user i, it is the sum of the powers received from all the base stations.

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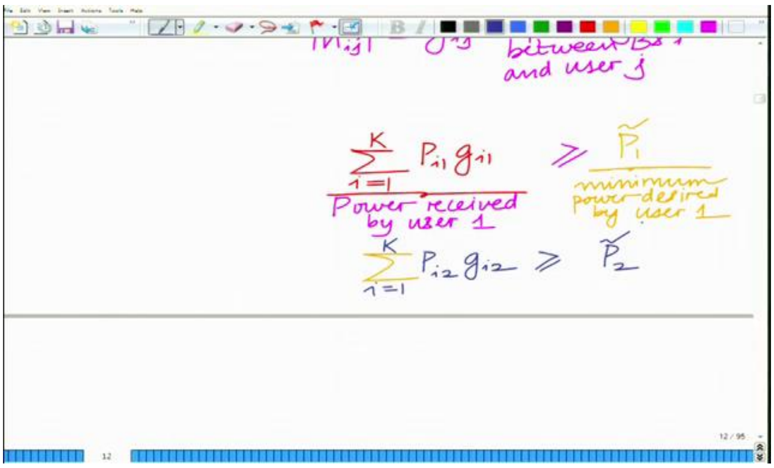

So the of the powers received from all base stations has to be greater than or equal to P_1. This has to also hold for the other users. So as shown in slide these are the affine constraints.

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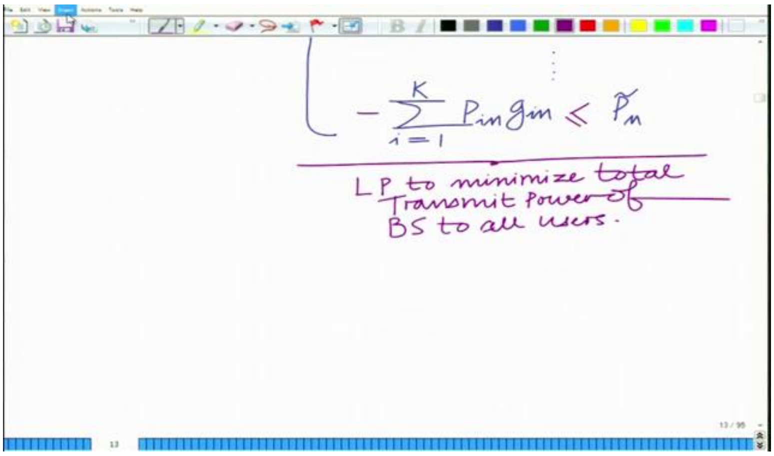

And now we want to meet the desired power level at each user, but simultaneously we also want to transmit the minimum amount of power. So the objective function can be to minimize the total power transmitted by all the base stations to all the users. So we have $\sum_{i=1}^{K} \sum_{j=1}^{M} P_{ij}$ is the total power by all base stations to all users. So the optimization problem is to minimize the total power of all the base stations to all the users subject to these constraints as shown in slide and this is a linear program. So this cooperative base station transmission can be formulated as a linear program.

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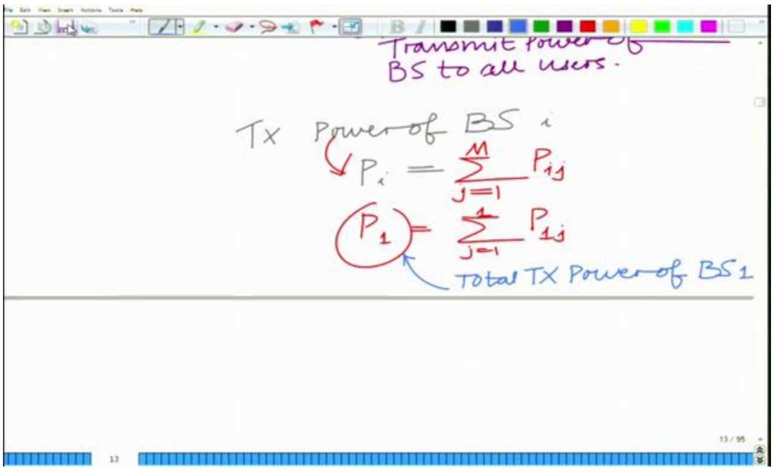

Now if you look at the transmit power of any single base station i, that will be represented as $P_i = \sum_{j=1}^{M} P_{ij}$.

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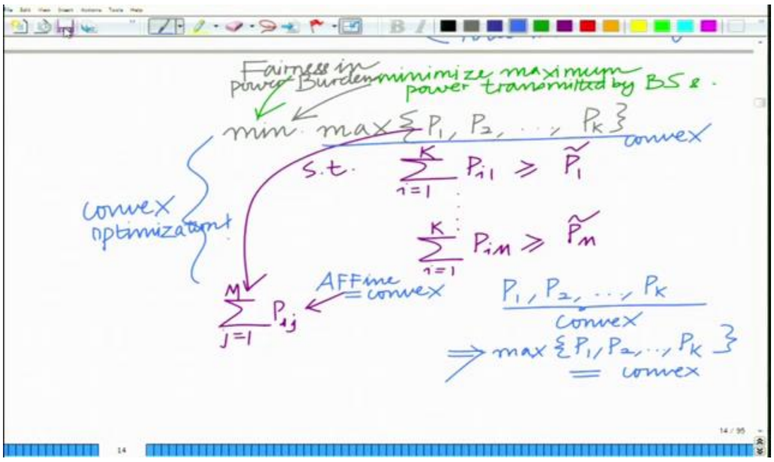

And now we want to consider an interesting optimization objective function in which we want to minimize the maximum of the powers transmitted by the various base stations. Now, typically in this cooperative cellular scenario, there are several base stations and when you minimize the total power transmitted by all the base stations together that might result in an undue burden on a single base station. So one particular base station which probably has good channel conditions can be over-burdened, so this does not ensure that the transmitter power burden is not uniformly levied on all the base stations. There might be different base stations which are levied more in comparison to others. But, when you are minimizing the maximum transmit power, it sort of ensures that this power burden the different users are fairly distributed on all the base stations. So you can say the min max criterion basically ensures fairness in the power burden.

So we have

$$\begin{aligned} &\min \max \{P_1, P_2, \ldots, P_K\} \\ &\quad \sum_{i=1}^{K} P_{i1} \geq P_1 \\ &\quad \vdots \\ s.t \\ &\quad \sum_{i=1}^{K} P_{iM} \geq P_M \end{aligned}$$

. And therefore this is the convex optimization problem. Now at present it seems unrelated to a linear program, but we use the epigraph trick to show how it can be written as a linear program.

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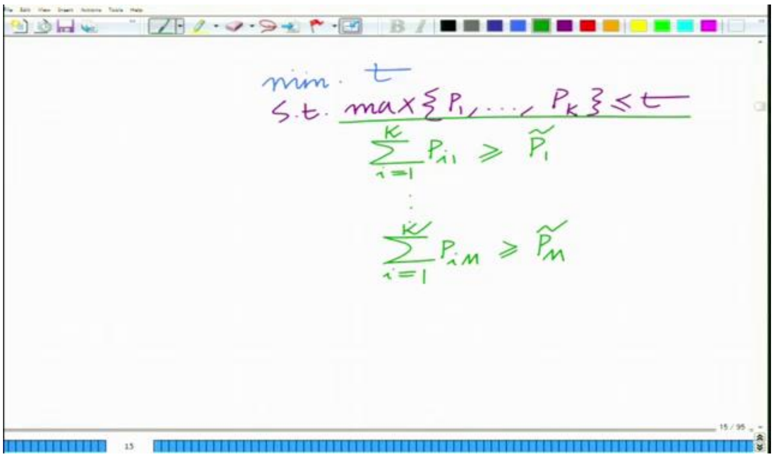

So this is written as shown in slides.

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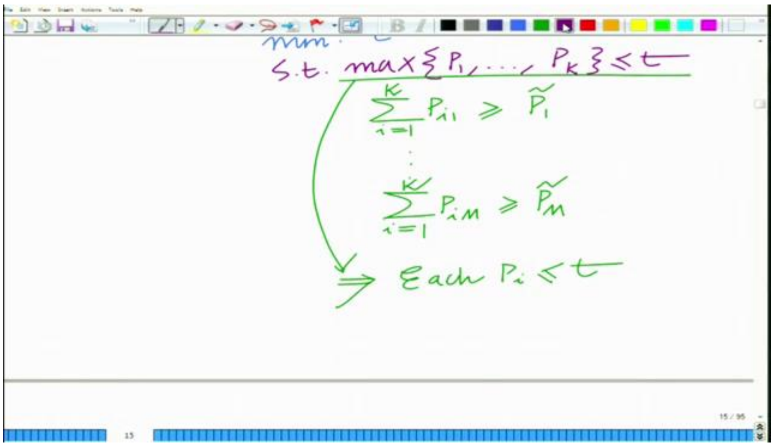

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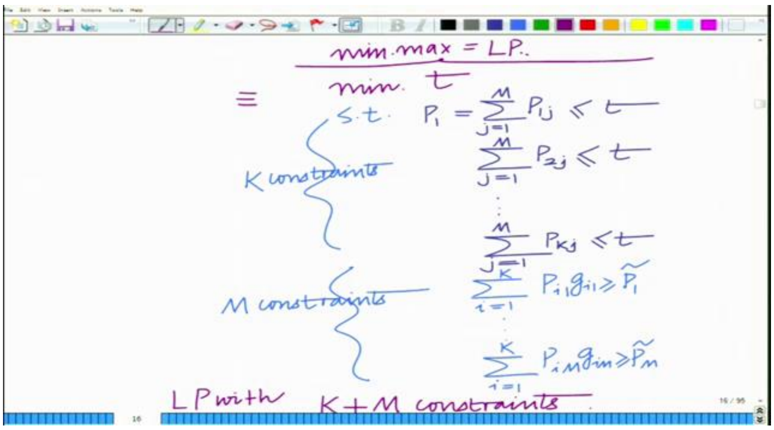

Therefore, the linear program can be written in various forms and not only can it be used to minimize the total transmit power, but also it can be used to minimize the maximum power transmitted by any of these base stations, thereby ensuring that this transmitted power burden is fairly distributed among all the cooperating base stations.

So that basically introduces the linear program and demonstrates its application in a practical wireless scenario for base station corporation. We will stop here and continue in the subsequent modules. Thank you very much.