1. The optimal beamformer, i.e., the maximal ratio combiner, with signal gain $= 1$ is given as

$$\frac{\bar{\mathbf{h}}}{\|\bar{\mathbf{h}}\|^2}$$

Ans d

2. The noise variance at the output of the beamformer is

$$\sigma^2\|\bar{\mathbf{w}}\|^2$$

Ans a

3. The noise plus interference at the output of the beamformer is

$$\bar{\mathbf{w}}^T\mathbf{R}\bar{\mathbf{w}}$$

Ans c

4. Given beamforming with interference where channel vector of desired user is $\bar{\mathbf{h}}$ and noise plus interference covariance is $\mathbf{R}$. The optimal beamforming vector $\bar{\mathbf{w}}$ for this scenario is $\frac{\mathbf{R}^{-1}\bar{\mathbf{h}}}{\bar{\mathbf{h}}^T\mathbf{R}^{-1}\bar{\mathbf{h}}}$

Ans b

5. The ZF beamformer for this scenario is obtained as

$$\mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\begin{bmatrix}1\\0\end{bmatrix}$$

Ans d

6. As shown in lectures, the matrix $\mathbf{C}^T = \begin{bmatrix}1 & 1 & 1 & 1\\1 & 2 & 3 & 4\end{bmatrix}$. The ZF beamformer is

$$\mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{e}_1 = \begin{bmatrix}1 & 1\\1 & 2\\1 & 3\\1 & 4\end{bmatrix}\begin{bmatrix}4 & 10\\10 & 30\end{bmatrix}^{-1}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 2\\1 & 3\\1 & 4\end{bmatrix}\frac{1}{20}\begin{bmatrix}30 & -10\\-10 & 4\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\\frac{1}{2}\\0\\-\frac{1}{2}\end{bmatrix}$$

Ans b

7. As shown in lectures, the matrix $\mathbf{C}^T = \begin{bmatrix}1 & -1 & -1 & 1\\4 & 3 & 2 & 1\end{bmatrix}$. The ZF beamformer is

$$\mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{e}_1 = \begin{bmatrix}1 & 4\\-1 & 3\\-1 & 2\\1 & 1\end{bmatrix}\begin{bmatrix}4 & 0\\0 & 30\end{bmatrix}^{-1}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{4}\\-\frac{1}{4}\\-\frac{1}{4}\\\frac{1}{4}\end{bmatrix}$$

Ans a

8. Given the channel vector of desired user is $\bar{\mathbf{h}} = [1 \quad 1 \quad -1 \quad -1]^T$and noise plus interference covariance is $\mathbf{R} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 2\end{bmatrix}$. The optimal beamforming vector $\bar{\mathbf{w}}$ for this scenario is

$$\frac{\mathbf{R}^{-1}\bar{\mathbf{h}}}{\bar{\mathbf{h}}^T\mathbf{R}^{-1}\bar{\mathbf{h}}} = \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & -\frac{1}{6} & -\frac{1}{6}\end{bmatrix}^T$$

Ans d

9. The solution to the least squares problem min.$\|\bar{\mathbf{y}} - \mathbf{A}\bar{\mathbf{x}}\|^2$, where size of matrix $\mathbf{A}$ is $m \times n$, with $m > n$, is $\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\bar{\mathbf{y}}$

   Ans c

10. The solution to the least norm (LN) optimization problem given is $\mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\bar{\mathbf{y}}$

    Ans a