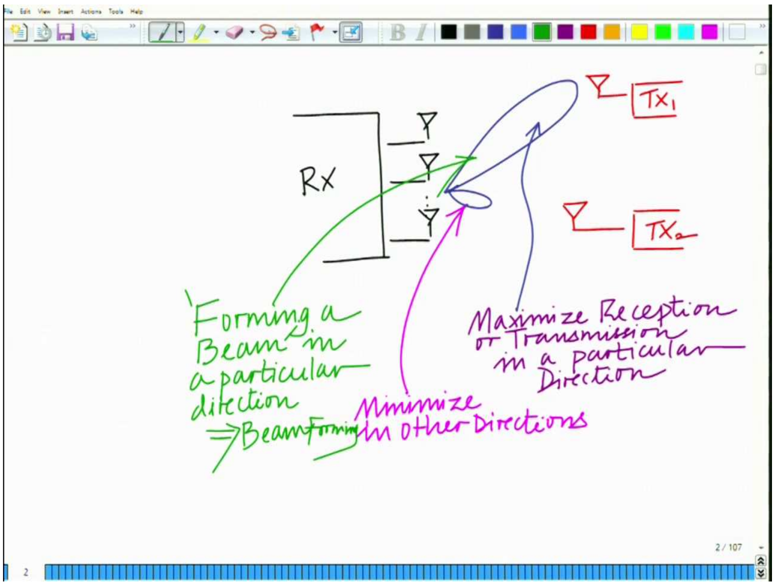
Beamforming in Multi-antenna Wireless Communication
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology Kanpur

Lecture - 34
Problems on Grassed Waterways

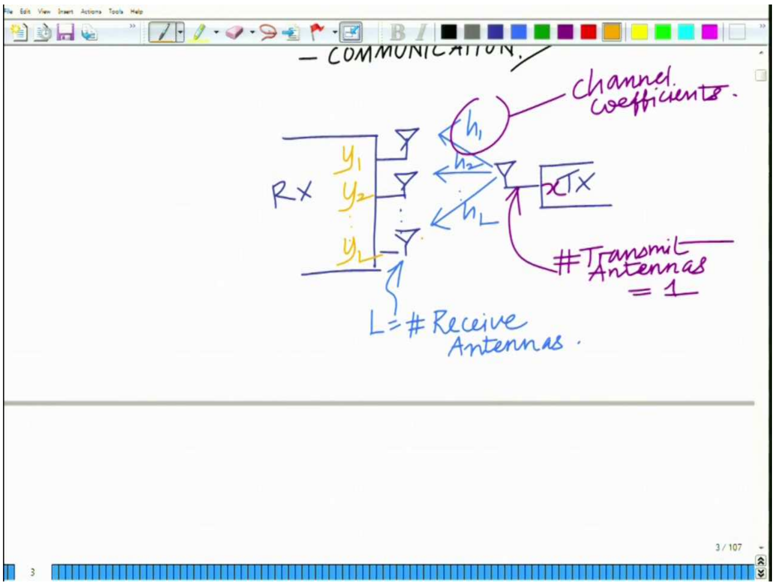
Hello welcome to another module in this Massive Open Online Course. Let us look at it through an example of Beamforming, which is one of the most important techniques in modern wireless communication system. This has a lot of applications in the context of 3G-4G wireless communication systems which have multiple antenna system.

(Refer Slide Time: 02:55)



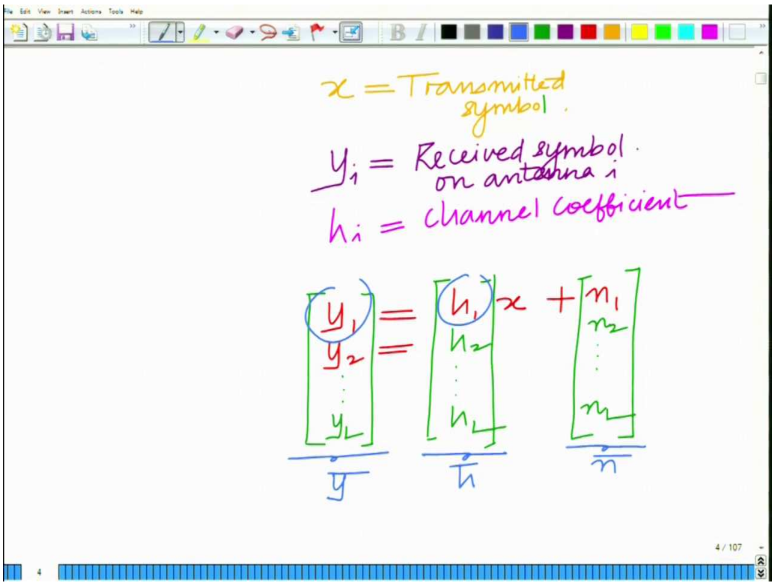
Consider a multiple antenna system where beam forming can be done at the transmitter and receiver. Beamforming is forming a beam in a particular direction to maximize the signal to noise power ratio in that particular direction. For this, the power is suppressed in the direction of the interfering user or the unintended users. This significantly improves the energy efficiency of system and hence it improves the signal to noise ratio (SNR) of the system. In that sense; beamforming is a very important even in radar.

(Refer Slide Time: 08:48)



So consider a single input multiple output (SIMO) system with L receive antennas. This is known as receive antenna diversity. Here the transmitted symbol from the single antenna is *x*, the received symbols at the *L* different antennas are $y_1, y_2, \ldots, y_L$ and the fading channel coefficients are $h_1, h_2, \ldots, h_L$.

(Refer Slide Time: 09:26)



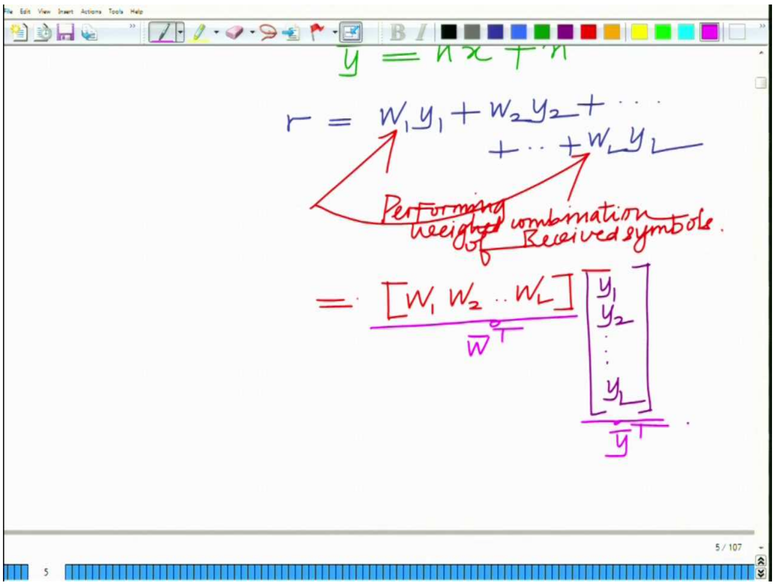
Hence this system model can be expressed as follows.

$$\bar{y} = \bar{h}x + \bar{n}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

Where x is the transmitted symbol, $\bar{y}$ is the received vector, $\bar{h}$ is the channel coefficient vector and $\bar{n}$ is the noise vector containing L noise elements corresponding to each channel.

(Refer Slide Time: 12:21)



So, for beam forming, perform a weighted combination with the received symbols. Therefore this combination can be written as follows.

$$r = w_1 y_1 + w_2 y_2 + \ldots + w_L y_L$$

In vector form,

$$r = \bar{w}^T \bar{y}$$

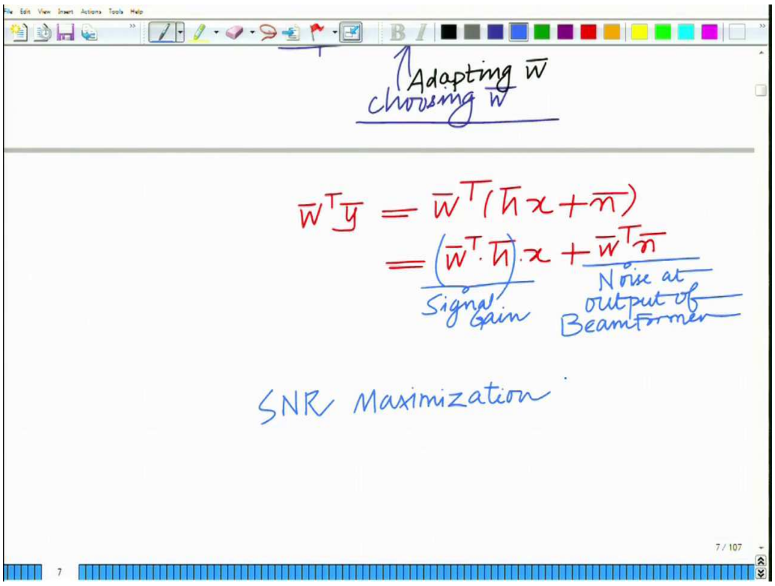
$$r = \begin{bmatrix} w_1 & w_2 & \cdots & w_L \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix}$$

Where $\bar{w}$ is the beam forming vector containing weight elements. This process of multiplying the various receive samples with the weights and then combining them is known as beam forming. These weights are chosen in such a way that the beam focuses in a particular direction while suppressing the interference.

So the Beamforming problem is to choose the optimal vector to maximize the SNR which is known as the optimum and this is an optimization problem. In literature this is known as electronic steering where weigths are electronically chosen.

In Mechanical steering, the array is rotated or tilted in a particular direction to change the direction of the beam. This technique is extremely time consuming, expensive and energy inefficient because array tilting with precision is necessary. The electronic steering has the biggest advantage that it is less complex and more precise.

(Refer Slide Time: 18:53)

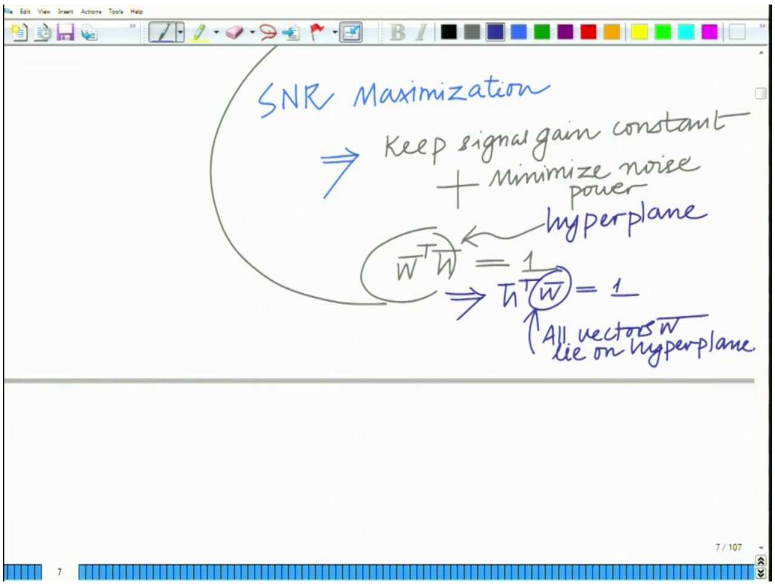


In fact electronic steering is also adaptive. This has a lot of applications including adaptive signal processing. Now on substituting the expression of $\bar{y}$, the beamformer expression becomes

$$r = \bar{w}^T \bar{y}$$
$$r = \bar{w}^T \left(\bar{h}x + \bar{n}\right)$$
$$r = \left(\bar{w}^T \bar{h}\right) x + \left(\bar{w}^T \bar{n}\right)$$

In the above expression, $\bar{w}^T\bar{h}$ is the signal gain and $\bar{w}^T\bar{n}$ is the noise at the output of the beam former. Now, to maximize the signal to noise power ratio, there are two methods. One method is to keep the signal gain constant and minimizing the noise power. Second method is to keep the noise power constant and maximizing the signal power. Both methods result in maximizing signal to noise power ratio.

(Refer Slide Time: 21:23)



Now, constant signal gain means

$$\bar{w}^T\bar{h} = \bar{h}^T\bar{w} = 1$$

This is an affine constraint and basically represents a hyper plane. So, it is a convex function. It means that all vectors $\bar{w}$ lie on the hyper plane. This implies unit gain in signal direction and it is the constraint for the optimization problem of formulation of SNR maximization, which will form the objective function for optimization.