

Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

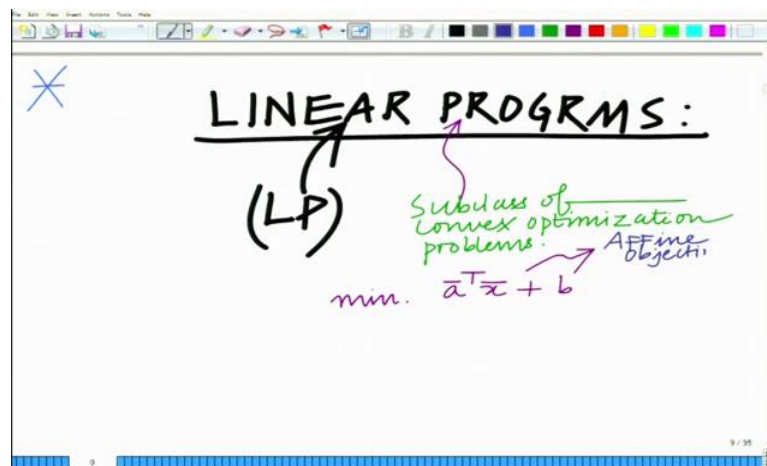
Lecture - 48

Linear Program Practical Application: Base Station Co – operation

Keywords: *Linear Program, Base Station Co – operation*

Hello, welcome to another module in this massive open online course. So we are looking at convex optimization problems and in this module, let us look at an important subclass of convex optimization problems which is basically linear programs also referred to as LPs.

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This forms a subclass of convex optimization problems. A linear program can be

expressed as follows

$$\begin{aligned} \min \quad & \bar{a}^T \bar{x} + b \\ \text{s.t.} \quad & \bar{c}_i^T \bar{x} \leq d_i, \quad i = 1, 2, \dots, l \\ & \bar{c}_j^T \bar{x} = d_j, \quad j = 1, 2, \dots, m \end{aligned}$$

, that is any convex optimization has an objective

here it has a linear objective or an affine objective function, subject to the constraints which are also affine.

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A handwritten slide defining a Linear Program. The title 'Linear Program' is written in purple. To its right, the optimization problem is stated: minimize $\bar{a}^T \bar{x} + b$ subject to $\bar{c}_i^T \bar{x} \leq d_i$ for $i = 1, 2, \dots, \ell$ and $\bar{c}_j^T \bar{x} = d_j$ for $j = 1, 2, \dots, m$. A purple bracket groups the constraints, with a note 'Affine equality and/or inequality constraints' written below it. At the bottom, it states 'LP: Simplest class of convex optimization problems' in yellow.

$$\text{Linear Program} \left\{ \begin{array}{ll} \min. & \bar{a}^T \bar{x} + b \\ \text{s.t.} & \bar{c}_i^T \bar{x} \leq d_i \quad i=1, 2, \dots, \ell \\ & \bar{c}_j^T \bar{x} = d_j \quad j=1, 2, \dots, m \end{array} \right.$$

Affine equality and/or inequality constraints

LP: Simplest class of convex optimization problems

So linear program is a subclass of convex optimization problems in which the objective function as well as the constraints, equality as well as inequality constraints are all affine in nature. This is the simplest class or category of convex optimization problems.

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A handwritten slide titled 'LP in matrix Form'. It shows the optimization problem in matrix notation: minimize $\bar{a}^T \bar{x} + b$ subject to $\begin{bmatrix} \bar{c}_1^T \\ \bar{c}_2^T \\ \vdots \\ \bar{c}_\ell^T \end{bmatrix} \bar{x} \leq \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_\ell \end{bmatrix}$. A green arrow points from the text 'Component wise inequality' to the inequality sign and the vectors. Below the matrix equation, the compact form $C \bar{x} \leq d$ is written in green.

LP in matrix Form

$$\begin{array}{ll} \min. & \bar{a}^T \bar{x} + b \\ \text{s.t.} & \begin{bmatrix} \bar{c}_1^T \\ \bar{c}_2^T \\ \vdots \\ \bar{c}_\ell^T \end{bmatrix} \bar{x} \leq \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_\ell \end{bmatrix} \end{array}$$

Component wise inequality

$$C \bar{x} \leq d$$

Now we can write this in matrix form as shown in slide. Now, here each component of the vector on the left has to be less than each component of the vector on the right. So this is a component wise inequality.

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Handwritten mathematical formulation of a linear program in compact form:

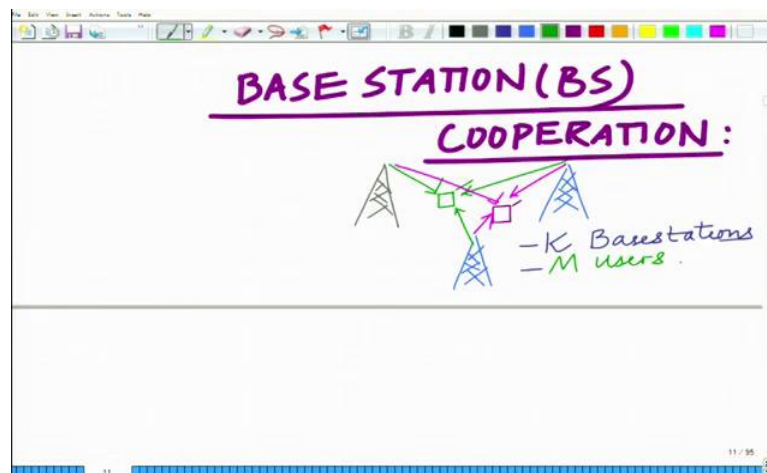
$$\begin{bmatrix} a^T \\ c^T \\ c_m^T \end{bmatrix} \bar{x} = \begin{bmatrix} b \\ d \\ d_m \end{bmatrix}$$

$$\min \quad \bar{a}^T \bar{x} + b$$

$$\text{s.t.} \quad \begin{cases} C \bar{x} \leq d \\ c \bar{x} = d \end{cases} \quad \left. \vphantom{\begin{matrix} C \bar{x} \leq d \\ c \bar{x} = d \end{matrix}} \right\} \text{compact Form for LP.}$$

So $\min \bar{a}^T \bar{x} + b$, this basically expresses the linear program in a very compact using vectors and matrices.

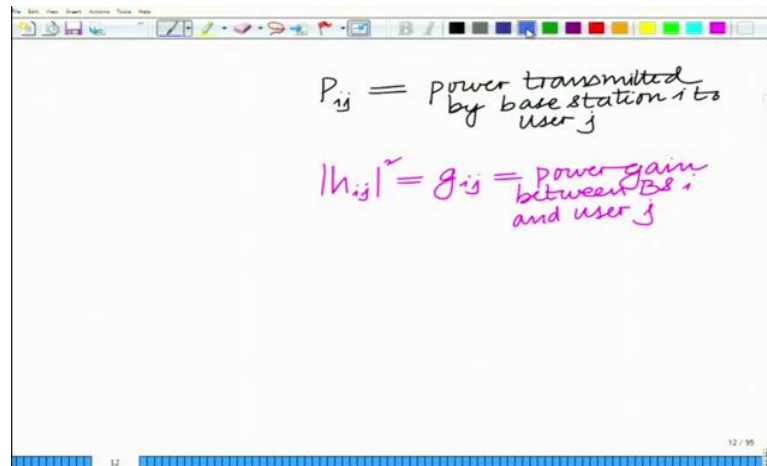
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Let us look at an example for this linear program which is the base station cooperation where there are several base stations in a cellular communication scenario where user at the edge of several cells can be served by several base stations. So in base station cooperation, we have a group of cells that are cooperating to transmit to one or many users. Normally, we have a single base station serving any particular user, but in this particular scenario base stations can cooperate to serve the various users thereby enhance

the SNR, enhance the reliability of communication in a wireless communication scenario. So we have K base stations and M users.

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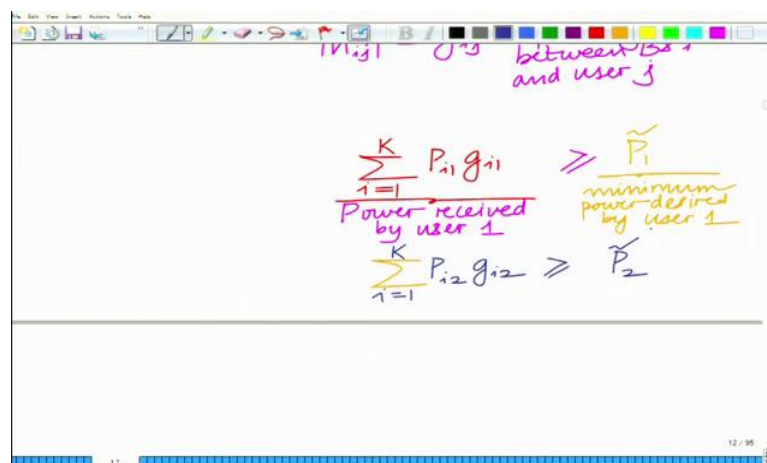
Handwritten definitions on a whiteboard:

$$P_{ij} = \text{power transmitted by base station } i \text{ to user } j$$

$$|h_{ij}|^2 = g_{ij} = \text{power gain between BS } i \text{ and user } j$$

And in this scenario, let P_{ij} denote the power transmitted by base station i to user j and h_{ij} is the fading channel coefficient. So $|h_{ij}|^2 = g_{ij}$ represents the power gain between base station i and user j . Now if you look at the power that is received by each user i , it is the sum of the powers received from all the base stations.

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Handwritten power constraints on a whiteboard:

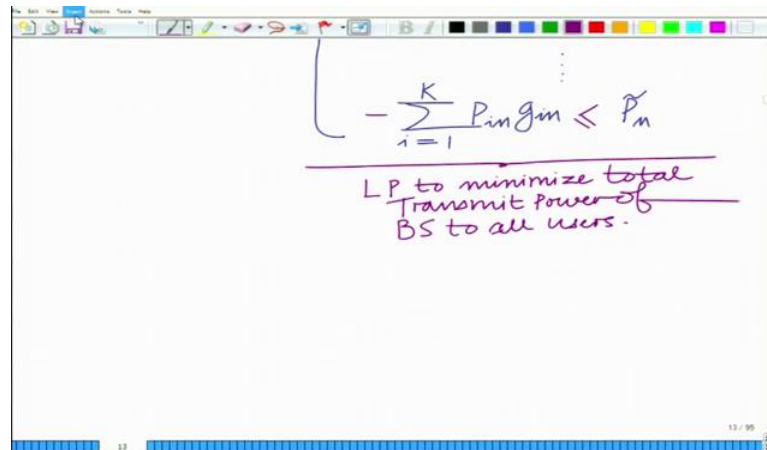
$$\sum_{i=1}^K P_{i1} g_{i1} \geq \tilde{P}_1$$

Power received by user 1 \geq minimum power desired by user 1

$$\sum_{i=1}^K P_{i2} g_{i2} \geq \tilde{P}_2$$

So the of the powers received from all base stations has to be greater than or equal to P_1 . This has to also hold for the other users. So as shown in slide these are the affine constraints.

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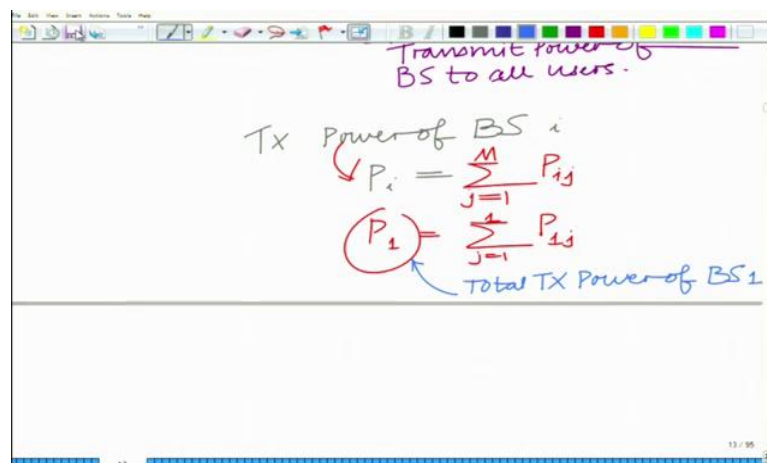
Handwritten slide content:

$$-\sum_{i=1}^K P_{in} g_{im} \leq \tilde{P}_m$$

LP to minimize total Transmit power of BS to all users.

And now we want to meet the desired power level at each user, but simultaneously we also want to transmit the minimum amount of power. So the objective function can be to minimize the total power transmitted by all the base stations to all the users. So we have $\sum_{i=1}^K \sum_{j=1}^M P_{ij}$ is the total power by all base stations to all users. So the optimization problem is to minimize the total power of all the base stations to all the users subject to these constraints as shown in slide and this is a linear program. So this cooperative base station transmission can be formulated as a linear program.

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Handwritten slide content:

Transmit power of BS to all users.

TX Power of BS i

$$P_i = \sum_{j=1}^M P_{ij}$$

$(P_1) = \sum_{j=1}^M P_{1j}$

Total TX Power of BS 1

Now if you look at the transmit power of any single base station i , that will be represented as $P_i = \sum_{j=1}^M P_{ij}$.

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Handwritten notes on a whiteboard showing a min-max optimization problem. The problem is to minimize the maximum power transmitted by BSs, subject to constraints on the total power for each user. The notes include a 'Fairness in power Burden' note, a 'convex optimization' label, and a 'min max' formulation.

$$\begin{aligned} & \text{minimize } \max \{P_1, P_2, \dots, P_K\} \\ & \text{s.t. } \sum_{i=1}^K P_{i1} \geq \tilde{P}_1 \\ & \quad \sum_{i=1}^K P_{iM} \geq \tilde{P}_M \\ & \quad P_1, P_2, \dots, P_K \text{ convex} \end{aligned}$$

Annotations: 'Fairness in power Burden', 'minimize maximum power transmitted by BSs', 'convex optimization', 'AFFINE = convex', 'convex', 'max {P1, P2, ..., PK} = convex'.

And now we want to consider an interesting optimization objective function in which we want to minimize the maximum of the powers transmitted by the various base stations. Now, typically in this cooperative cellular scenario, there are several base stations and when you minimize the total power transmitted by all the base stations together that might result in an undue burden on a single base station. So one particular base station which probably has good channel conditions can be over-burdened, so this does not ensure that the transmitter power burden is not uniformly levied on all the base stations. There might be different base stations which are levied more in comparison to others. But, when you are minimizing the maximum transmit power, it sort of ensures that this power burden the different users are fairly distributed on all the base stations. So you can say the min max criterion basically ensures fairness in the power burden.

$$\min \max \{P_1, P_2, \dots, P_K\}$$

$$\sum_{i=1}^K P_{i1} \geq P_1$$

So we have And therefore this is the convex optimization

$$\text{s.t. } \sum_{i=1}^K P_{iM} \geq P_M$$

problem. Now at present it seems unrelated to a linear program, but we use the epigraph trick to show how it can be written as a linear program.

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Handwritten LP formulation on a digital whiteboard:

$$\begin{aligned} \min. & \quad t \\ \text{s.t.} & \quad \max\{P_1, \dots, P_K\} \leq t \\ & \quad \sum_{i=1}^K P_{i1} \geq \tilde{P}_1 \\ & \quad \vdots \\ & \quad \sum_{i=1}^K P_{iM} \geq \tilde{P}_M \end{aligned}$$

The whiteboard interface shows a toolbar at the top and a status bar at the bottom indicating slide 15 of 95.

So this is written as shown in slides.

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Handwritten LP formulation on a digital whiteboard, including an additional conclusion:

$$\begin{aligned} \min. & \quad t \\ \text{s.t.} & \quad \max\{P_1, \dots, P_K\} \leq t \\ & \quad \sum_{i=1}^K P_{i1} \geq \tilde{P}_1 \\ & \quad \vdots \\ & \quad \sum_{i=1}^K P_{iM} \geq \tilde{P}_M \\ & \quad \Rightarrow \text{Each } P_i \leq t \end{aligned}$$

A green arrow points from the first constraint to the conclusion. The whiteboard interface shows a toolbar at the top and a status bar at the bottom indicating slide 15 of 95.

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Handwritten LP formulation on a digital whiteboard, showing constraints grouped by K and M:

$$\begin{aligned} \min. \max & = \text{LP.} \\ \equiv \min. & \quad t \\ \text{s.t.} & \quad P_i = \sum_{j=1}^M P_{ij} \leq t \quad \text{K constraints} \\ & \quad \sum_{j=1}^M P_{2j} \leq t \\ & \quad \vdots \\ & \quad \sum_{j=1}^M P_{Kj} \leq t \\ & \quad \sum_{i=1}^K P_{i1} \geq \tilde{P}_1 \quad \text{M constraints} \\ & \quad \vdots \\ & \quad \sum_{i=1}^K P_{iM} \geq \tilde{P}_M \end{aligned}$$

Brackets group the first set of constraints as 'K constraints' and the second set as 'M constraints'. The final line states 'LP with K+M constraints'. The whiteboard interface shows a toolbar at the top and a status bar at the bottom indicating slide 16 of 95.

Therefore, the linear program can be written in various forms and not only can it be used to minimize the total transmit power, but also it can be used to minimize the maximum power transmitted by any of these base stations, thereby ensuring that this transmitted power burden is fairly distributed among all the cooperating base stations.

So that basically introduces the linear program and demonstrates its application in a practical wireless scenario for base station corporation. We will stop here and continue in the subsequent modules. Thank you very much.