Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 55
Practical Application : Compressive Sensing

***Keywords**: Compressive Sensing*

Hello, welcome to another module in this massive open online course, so let us start looking at another new, in fact revolutionary and path breaking development or technology and that is of Compressive Sensing.

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So let us start by considering a signal $\bar{x}$ which is unknown.

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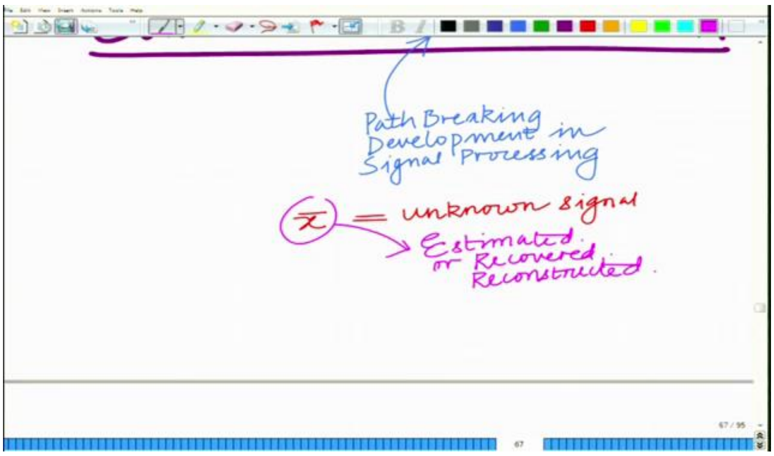

And this has to be therefore either estimated or recovered or reconstructed. If it is an image, then we say that the image has to be reconstructed.

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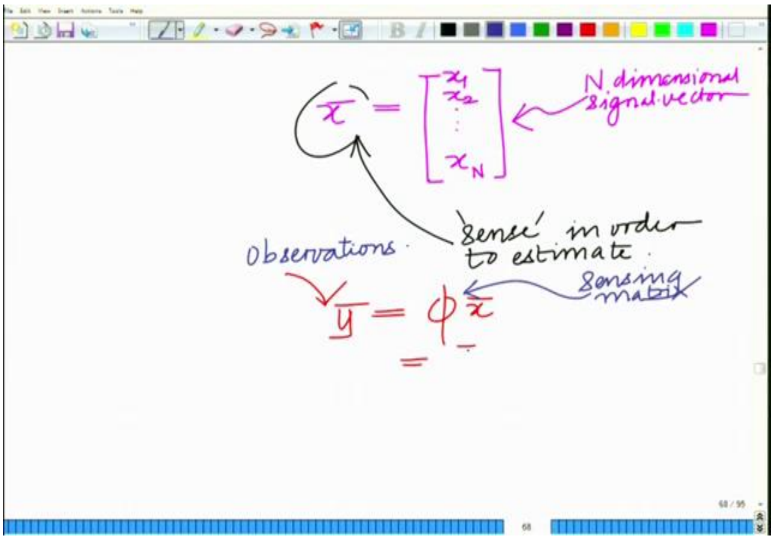

So let us say this $\bar{x}$ is an N dimensional signal vector and we have to make some measurements for this unknown signal vector $\bar{x}$ in order to estimate the signal vector. So we are sensing the signal vector in order to estimate. And therefore, we have $\bar{y} = \phi \bar{x}$. So this $\bar{y}$ is your observation vector and ϕ is your sensing matrix.

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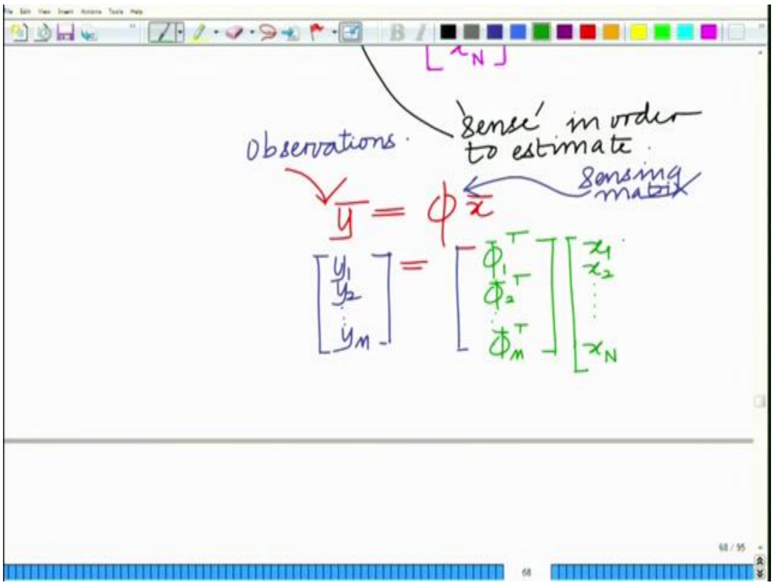

Let us say, we are making M observations as shown in slide.

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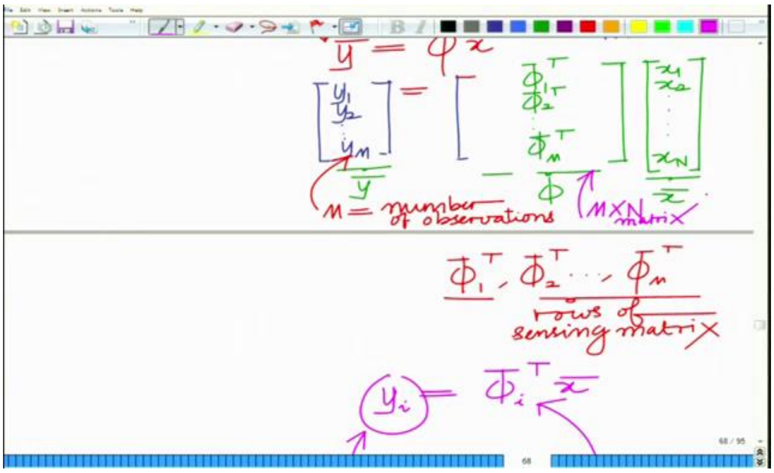

So these are the rows of the sensing matrix we are making these M observations y_1 y_2 y_M through this sensing matrix. So you can think of each observation as a projection of this vector $\bar{x}$ on a row of this sensing matrix ϕ .

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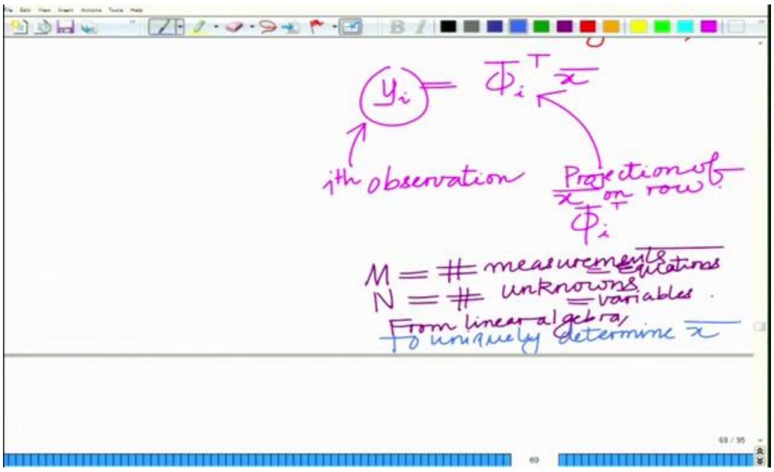

So there are M observations and N unknowns and this matrix ϕ is an $M \times N$ matrix. So this is a system of equations or M is the number of equations and N is the number of unknowns. And from linear algebra we know that in order to recover x which is vector of size N you need at least N equations to uniquely determine x. So to uniquely determine $\bar{x}$ here we need $M \geq N$.

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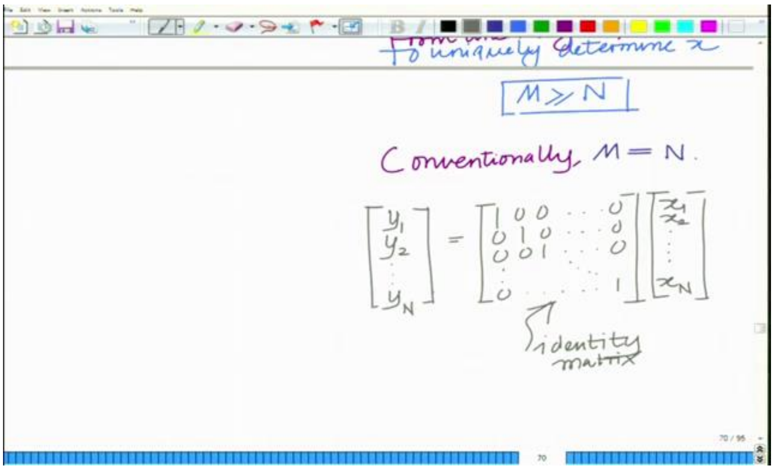

We have these N observations y_1 y_2 up to y_N which is simply the identity matrix. So typically what you have is you are simply sampling the signal at these N different instants, we get N measurements and from those measurements we recover the signal.

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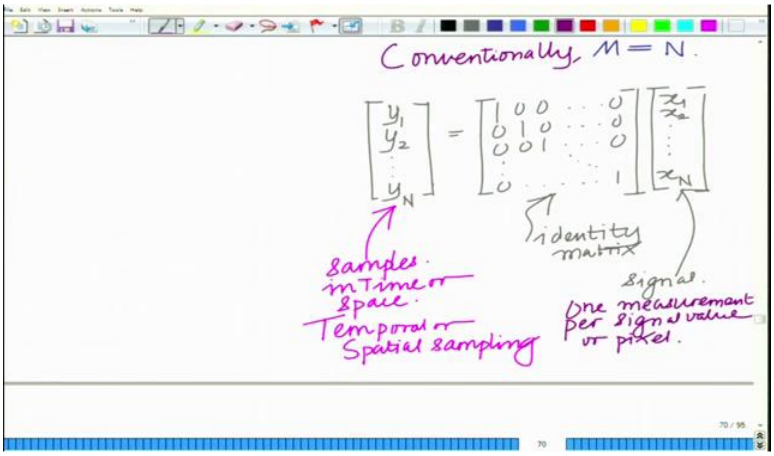

So as in slide these are the samples, so this is temporal or spatial sampling. We are making one measurement per sample or signal value. And therefore, to uniquely determine the signal with N samples we need at least $M \geq N$, we can choose M = N. Now let us take a simple example, consider a typical image for instance.

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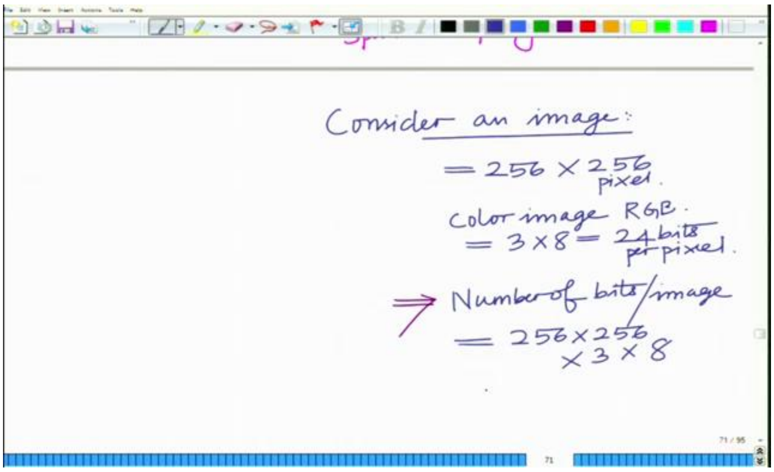

Now, this is a small image it is a 256×256 pixel image and let us say, it is a color image implies for each of these RGB components you need 1 byte for each pixel which means the total number of bits per image is $256 \times 256 \times 3 \times 8 = 1.58 Mb$.

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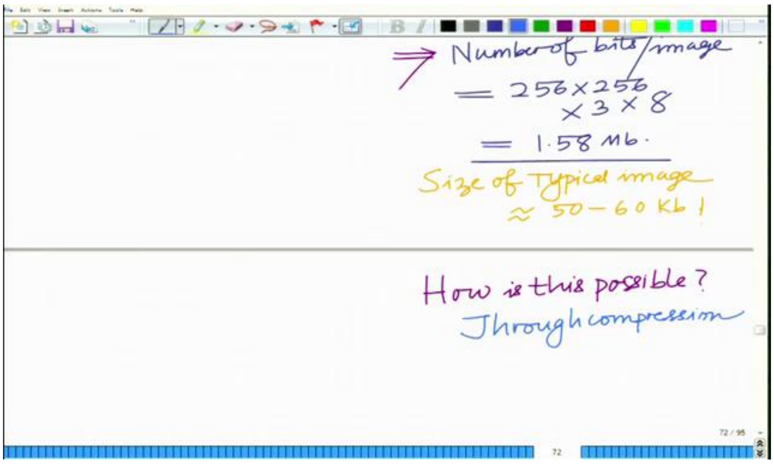

But the size of a typical image is let us say, 50 to 60 Kb only. So how is it that you are able to store an image at such a small size even though the raw image has so many bits, the obvious answer to this is that instead of storing a raw image, this image is being significantly compressed in size in terms of the number of bits.

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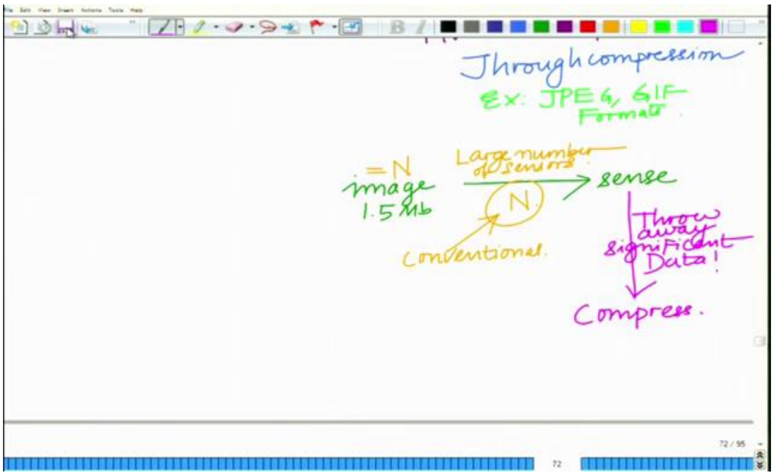

For example, you have JPEG, GIF, these are various formats for compression. So what we are doing is the we are first sensing the image and if it is a large image this implies you require a large number of sensors, the number of sensors required is N, so you are taking one one sensor per sample. However, after sensing you are compressing that is you throw away significant amount of data. You are throwing away a significant amount of data to compress it which means basically you are using a large number of sensors but at the same time you are throwing away a large amount of data, because the compression is coming after the sensing process .

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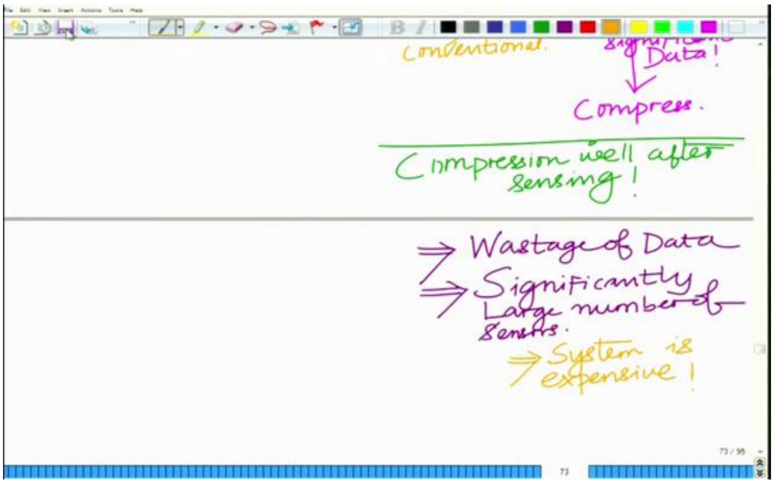

In this conventional paradigm, you have compression well after the sensing process. This leads to a wastage with large number of sensors and hence the resulting system is extremely expensive.

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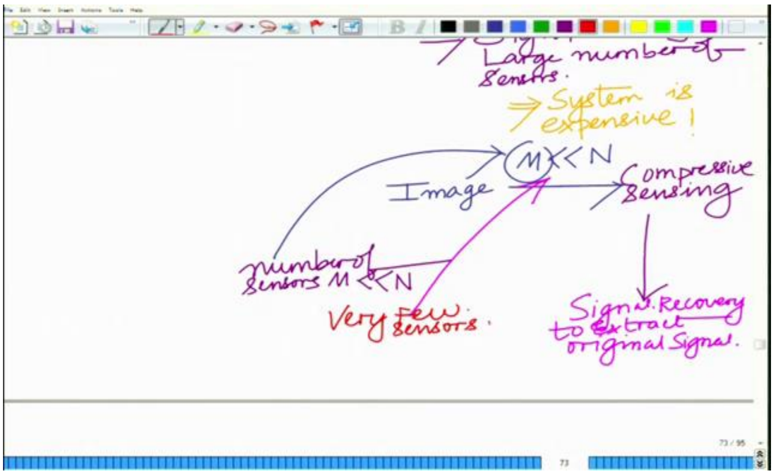

Now instead consider this paradigm where you have an image, you perform M measurements much less than N that is number of sensors M is much less than N, this is termed as compressive sensing. So basically while the sensing process itself you are compressing. So you are compressing while sensing and then you can perform signal recovery to extract the original signal. So this requires very few sensors.

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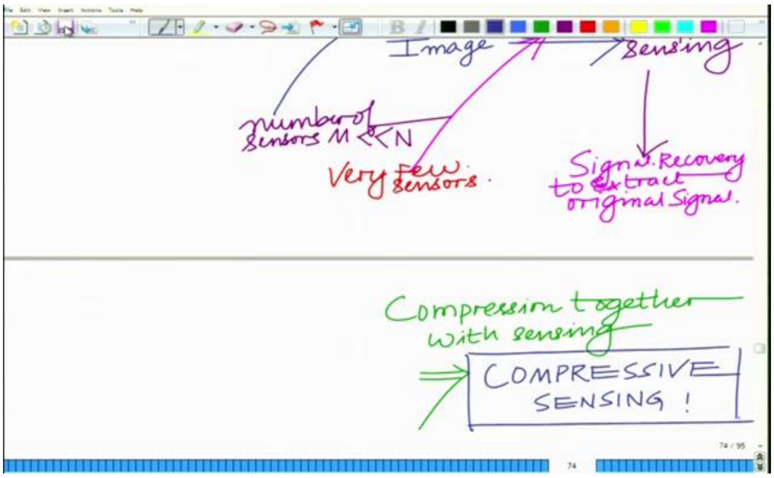

So this results in a significant saving in terms of cost and in terms of the number of sensors, because you are making very few measurements in comparison to the size of the signal. Now since the number of observations is less than the number of signal samples, one cannot uniquely determine the signal vector $\bar{x}$. So therefore, one has to come up with some new techniques to reconstruct the original signal from this compressed or compressively sensed signal $\bar{y}$. So we will stop here. Thank you very much.