

**Applied Optimization for Wireless, Machine Learning, Big Data**  
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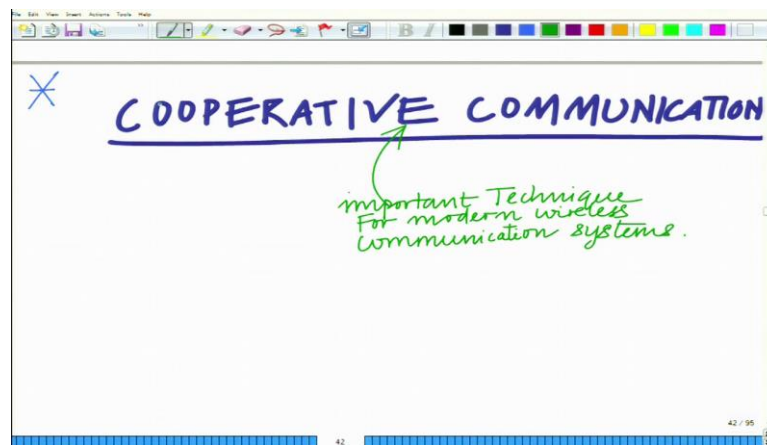
**Lecture - 52**

**Practical Application: Co-operative Communication, Overview and Various  
Protocols Used**

**Keywords:** *Co-operative Communication, Protocols*

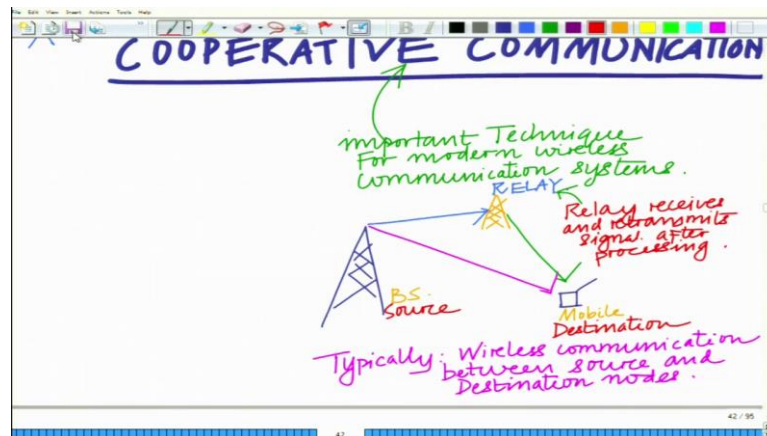
Hello, welcome to another module in this Massive Open Online Course. We are looking at several optimization paradigms, in fact several practical applications of the optimization theory that we have seen so far. Let us we look at yet another interesting and novel application of the optimization framework with respect to cooperative communication.

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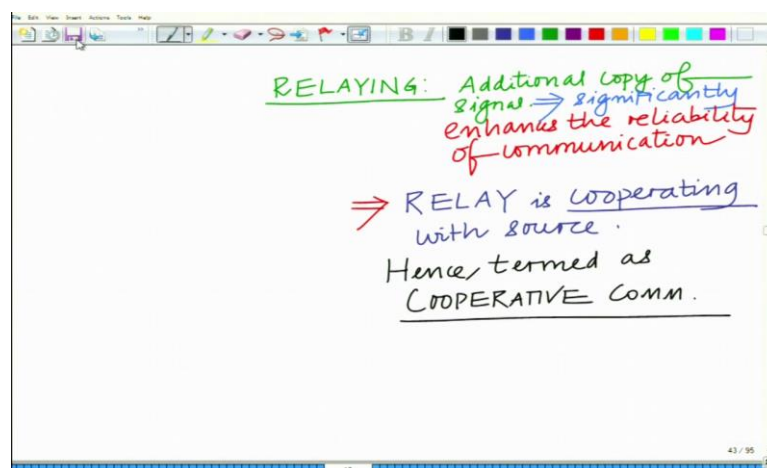
So cooperative communication is in fact a very recent idea and is emerging to be very popular in modern wireless communication systems especially for high data rate wireless communication.

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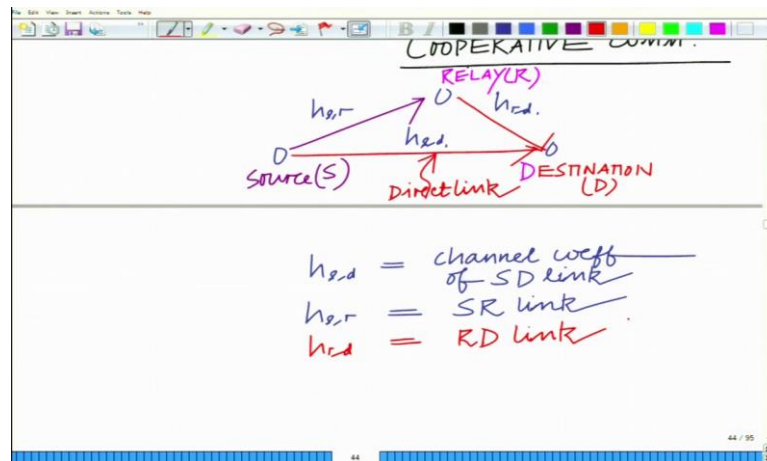
Now, in a typical wireless communication system you have a base station and then you have a mobile which is also termed as the user equipment. The source of the communication signals is  $S$  and there is communication from the source to destination. Now in addition to enhance the quality of communication we use a relay which is basically something that takes the signal and carries it forward. A relay receives the signal from the source, it can perform some signal processing operations on it and then it forwards this signal to the destination. So there can be a single or multiple relays. So relay receives and retransmits the signal after suitable processing. Now this can significantly enhance the reliability.

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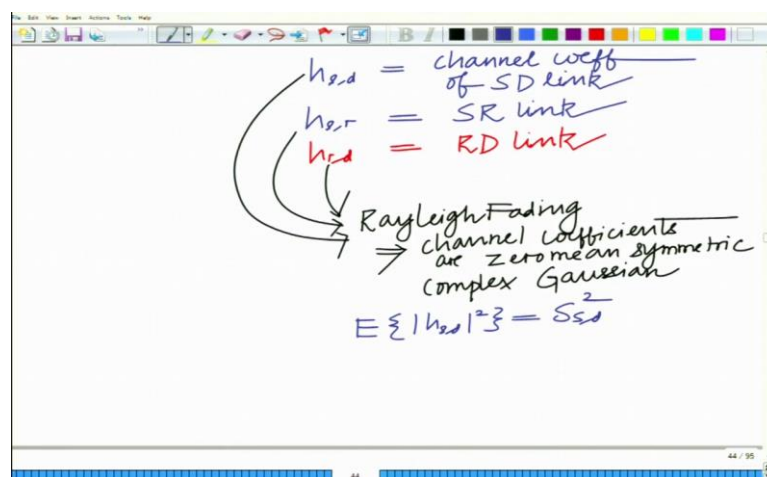
We can see that you have an additional copy and what the relay is doing is termed as cooperation and hence, this paradigm termed as cooperative communication.

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So schematically the cooperative communication system is represented as the combination of these three nodes, the source which is S, destination and relay which is R. So source to destination we have the direct SD link and then you have a link which is the source to relay link and then another from relay to destination. Each of these is a wireless link and therefore each of these will be characterized by a fading wireless channel coefficient. So let us denote these channel coefficients by  $h_{sd}$  which is for SD link,  $h_{sr}$  for SR link and  $h_{rd}$  for RD link.

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Let us say these are Rayleigh fading in nature, which implies that these coefficients are zero mean symmetric complex Gaussian in nature. Now the average powers are given as

$$E\{|h_{sd}|^2\} = \sigma_{sd}^2 \text{ for SD link, } E\{|h_{sr}|^2\} = \sigma_{sr}^2 \text{ for SR link and } E\{|h_{rd}|^2\} = \sigma_{rd}^2 \text{ for RD link.}$$

And these powers can vary depending on the distance between each of these links. So

there can be several scenarios depending on where the relay is located with respect to source and where the destination is located with respect to the source and where the destination is located with respect the relay.

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Handwritten notes on a whiteboard:

$$E\{|h_{sd}|^2\} = \sigma_{sd}^2$$

$$E\{|h_{sr}|^2\} = \sigma_{sr}^2$$

$$E\{|h_{rd}|^2\} = \sigma_{rd}^2$$
  

$$|h_{sd}|^2 = \beta_{sd}$$

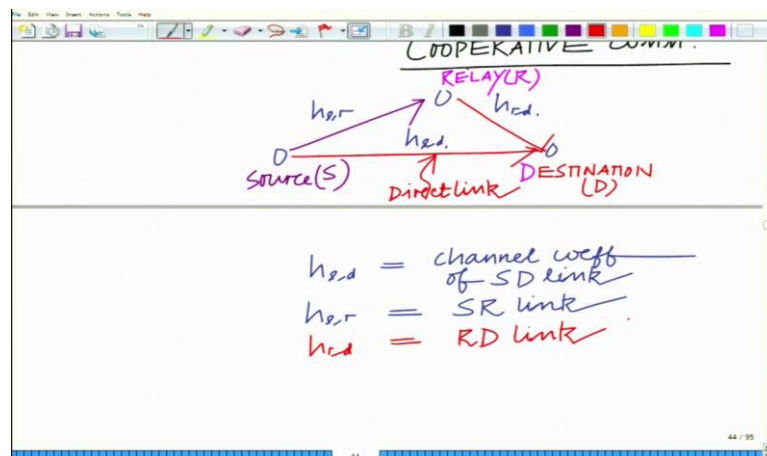
$$|h_{rd}|^2 = \beta_{rd}$$

$$|h_{sr}|^2 = \beta_{sr}$$

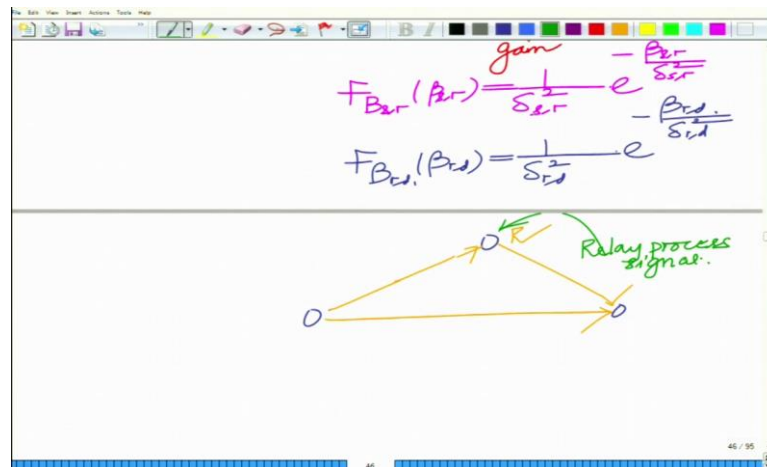
Exponential RVs:

Similarly, you have the gains as  $|h_{sd}|^2 = \beta_{sd}$ ,  $|h_{sr}|^2 = \beta_{sr}$ ,  $|h_{rd}|^2 = \beta_{rd}$  and these are random in nature, because the fading channel coefficient is random and these will have an exponential distribution as shown in slide.

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Handwritten equations and diagram on a slide:

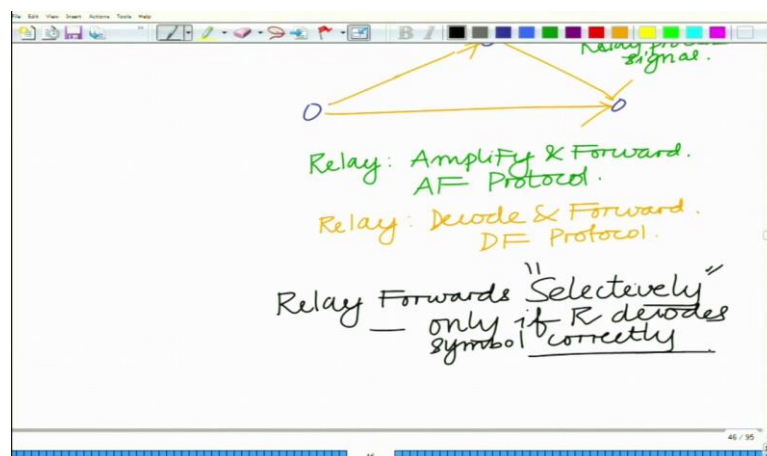
$$F_{B_{sr}}(B_r) = \frac{1}{S_{sr}} e^{-\frac{P_r}{S_{sr}}}$$

$$F_{B_{rd}}(B_{rd}) = \frac{1}{S_{rd}} e^{-\frac{P_{rd}}{S_{rd}}}$$

Diagram: A triangle representing a relay system with nodes S (Source), R (Relay), and D (Destination). Arrows show signal flow from S to R and from R to D. A green arrow points to node R with the text "Relay process signal."

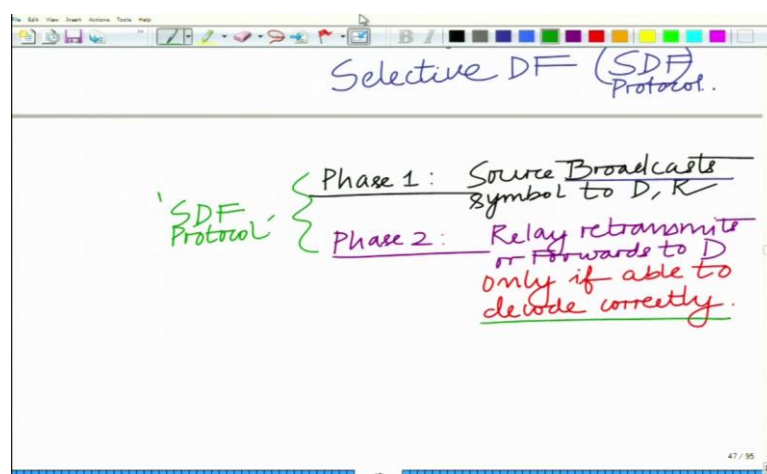
Now, we said that the relay does some processing and depending on the nature of the signal processing you have a different protocol. For instance, relay can simply amplify the signal and forward it that is termed as amplify and forward, the AF protocol. The relay can simply decode the signal and forward it, this is termed as decode and forward, DF protocol.

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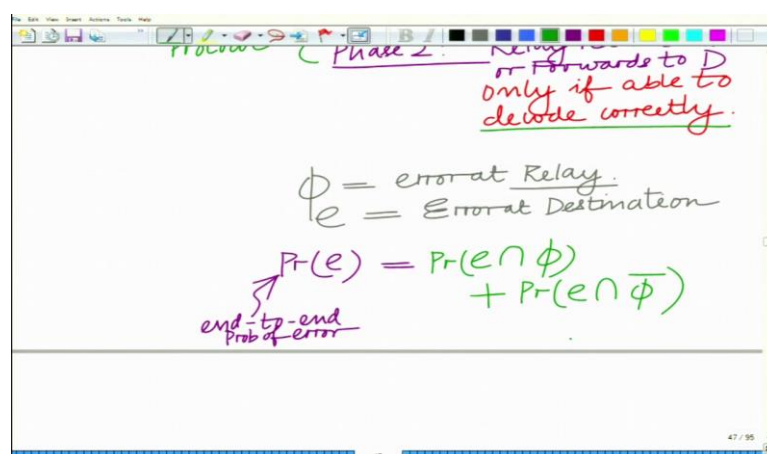
Now we have the selective decode and forward protocol which is an extension of the decode and forward protocol in which the relay receives the signal decodes it and forwards it selectively only if it is able to decode the signal or the symbol correctly.

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So this is known as selective DF or the SDF protocol. So this happens in two phases. So in the SDF phase 1, source transmits the symbol, which is received by destination and relay. In the second phase, phase2, the relay retransmits or forwards to destination only if it is able to decode correctly. So then the destination gets copies from both source and relay.

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So if  $\phi$  denotes the event of an error at relay,  $e$  denotes the error at destination, we have  $\Pr(e)$  as the end to end probability of error. This is given as  $\Pr(e) = \Pr(e \cap \phi) + \Pr(e \cap \bar{\phi})$ . So we have a partition that is mutually exclusive and exhaustive. So  $\phi$  and  $\bar{\phi}$  are mutually exclusive and is an exhaustive partition.

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Handwritten derivation on a whiteboard:

end-to-end Prob of error  $\Rightarrow$

$$= \Pr(e|\phi) \Pr(\phi) + \Pr(e|\bar{\phi}) \Pr(\bar{\phi})$$

Total Probability Rule

$$\Pr(\phi) \approx 0 \text{ at high SNR}$$

$$\Rightarrow 1 - \Pr(\phi) \approx 1 = \Pr(\bar{\phi})$$

And now using the conditional probability we can write this as shown in slide and thus we have  $\Pr(e) \approx \Pr(e|\phi) \Pr(\phi) + \Pr(e|\bar{\phi})$  since  $\Pr(\bar{\phi}) \approx 1$  at high SNR.

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Handwritten derivation on a whiteboard:

$$= \Pr(e|\phi) \Pr(\phi) + \Pr(e|\bar{\phi}) \Pr(\bar{\phi})$$

$$\Pr(\phi) \approx 0 \text{ at high SNR}$$

$$\Rightarrow 1 - \Pr(\phi) \approx 1 = \Pr(\bar{\phi})$$

$$\Rightarrow \Pr(e) \approx \frac{\Pr(e|\phi) \cdot \Pr(\phi)}{1 - \Pr(\phi)}$$

Approximation tight at high SNR

So this is the approximation for the probability of error which is tight at high SNR and this is known as a high SNR approximation. So we will stop here. Thank you very much.