Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 58
Example Problem: Orthogonal Matching Pursuit (OMP) algorithm

Keywords: *Orthogonal Matching Pursuit (OMP) algorithm*

Hello, welcome to another module in this massive open online course. So we are looking at techniques for compressive sensing and we have seen that orthogonal matching pursuit can be used for sparse signal recovery, so let us now look at an example to understand this better.

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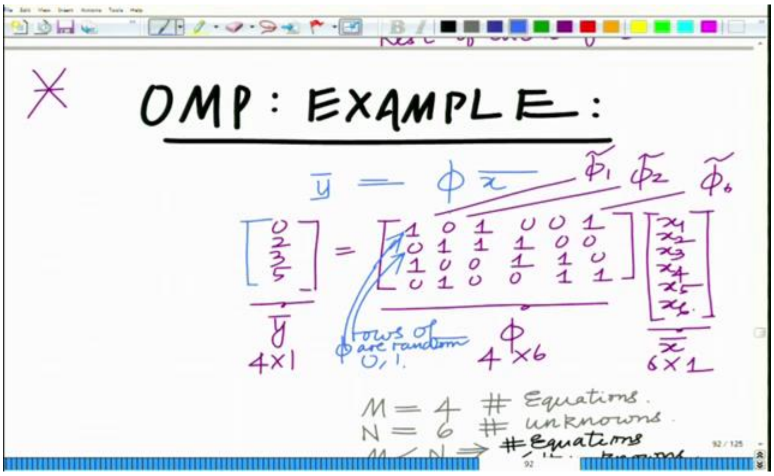

So let us consider the following example, we have $\bar{y} = \phi \bar{x}$ and we have to estimate the vector $\bar{x}$. So let $\bar{y} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 5 \end{bmatrix}$, the matrix $\phi = \begin{bmatrix} 1\,0\,1\,0\,0\,1 \\ 0\,1\,1\,1\,0\,0 \\ 1\,0\,0\,1\,1\,0 \\ 0\,1\,0\,0\,1\,1 \end{bmatrix}$ and $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$.

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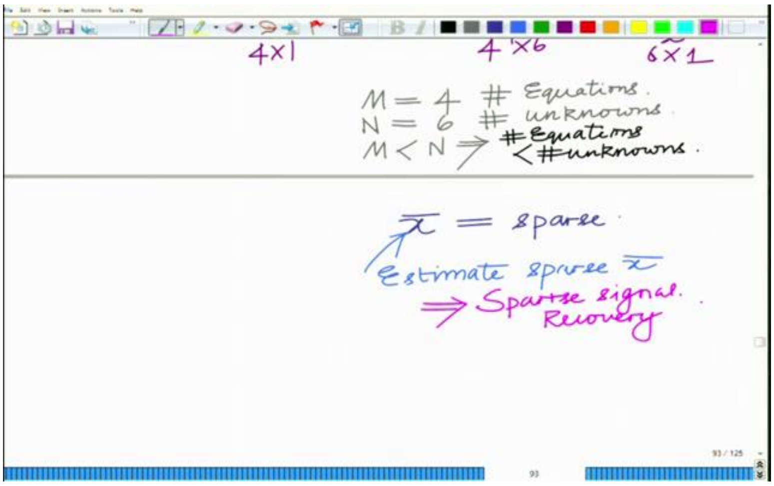

In this problem we have M = 4 which is basically the number of equations and N = 6 which is the number of unknowns and $M < N$ implies number of equations is less than number of unknowns. Therefore, to estimate $\bar{x}$ you cannot use conventional linear algebra, because in linear algebra you need the number of equations at least equal to the number of unknowns to uniquely determine the unknown vector $\bar{x}$. And therefore, one has to enforce sparsity, so we assume that $\bar{x}$ is sparse and then we want to estimate this sparse vector. This is basically termed as sparse signal recovery.

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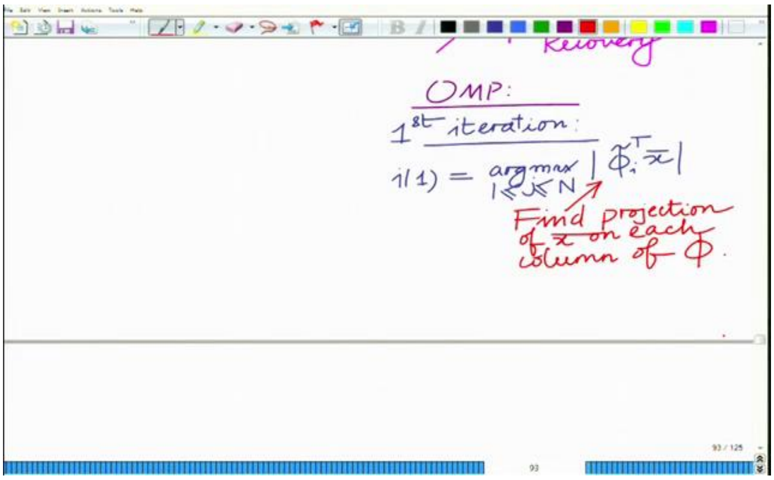

The algorithm for sparse signal recovery is OMP and this can be done as shown in slide.

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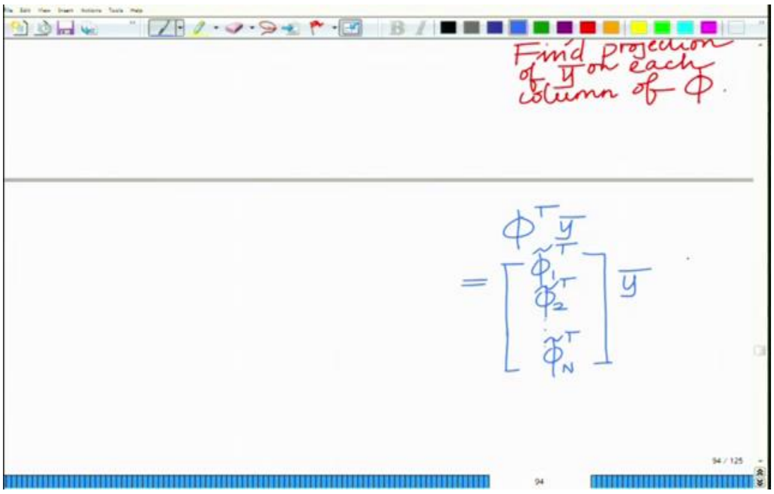

So we perform $\phi^T \bar{y}$ and this is as shown in slide.

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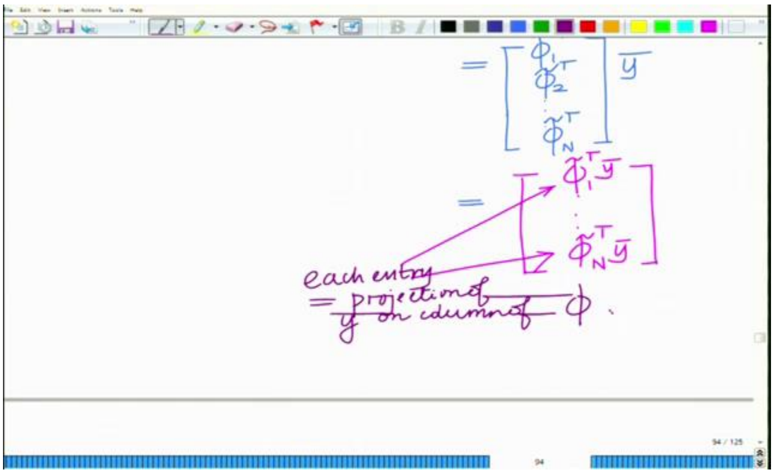

So each of these entries equals the projection of $\bar{y}$ on each column of ϕ. Now the other thing that you must have observed is if you look at these rows, you can see that these rows are random 0's and 1. So these are noise like waveforms. So each measurement is a projection of $\bar{y}$ on this noise like waveform.

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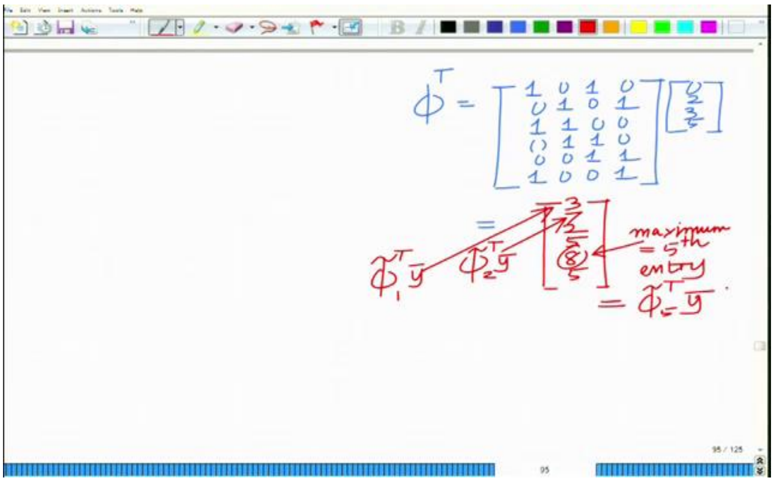

So when we compute this $\phi^T \bar{y}$ we will get the vector $\begin{bmatrix} 3 \\ 7 \\ 2 \\ 5 \\ 8 \\ 5 \end{bmatrix}$ and the maximum is at the 5th entry which is equal to $\phi_5^T \bar{y}$.

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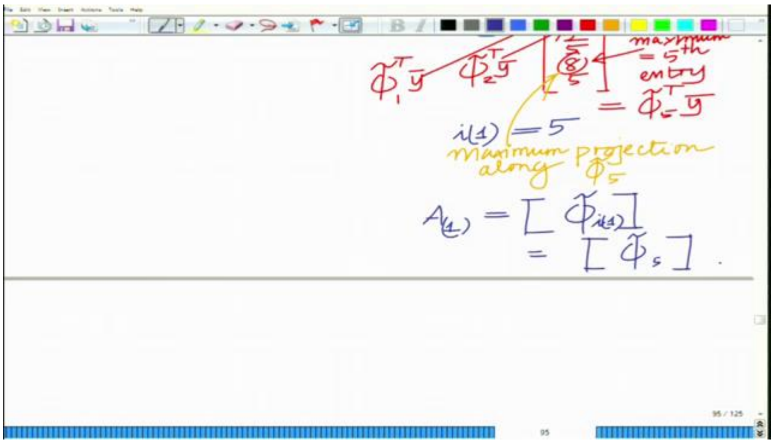

Therefore we form the basis matrix using this column, so we have $A_{(1)} = \left[\phi_5 \right]$.

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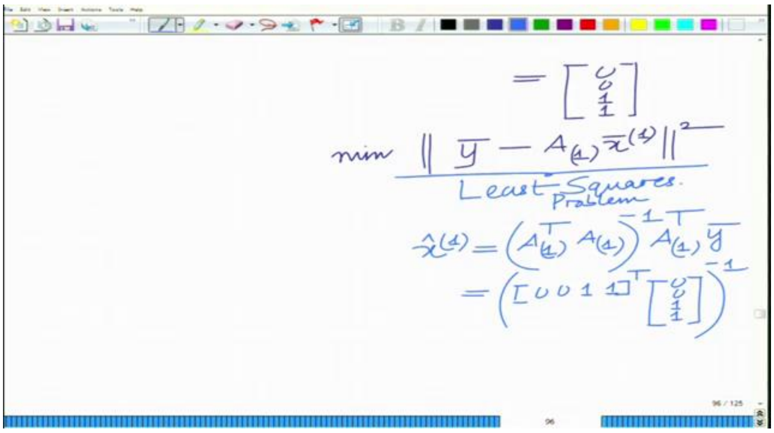

This is nothing but $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ and now you solve the least squares problem $\left\| \bar{y} - A_{(1)} \bar{x}^{(1)} \right\|^2$, this is the first iteration. So the solution to this is $x^{(1)} = \left(A_{(1)}{}^T A_{(1)} \right)^{-1} A_{(1)}{}^T \bar{y}$. So on evaluating this as shown in slide, we get $x^{(1)} = 4$.

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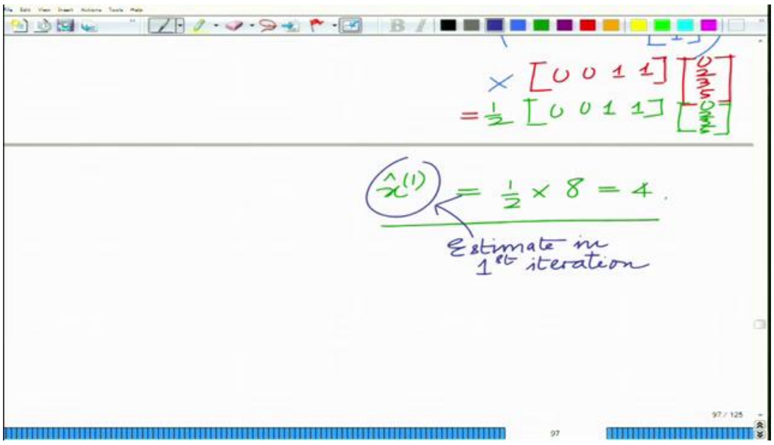

This corresponds to the index of the column that is chosen in the first iteration that is column number 5. So this corresponds to the 5th column or the 5th entry of the vector $\bar{x}$. Now we find the residue for the first iteration.

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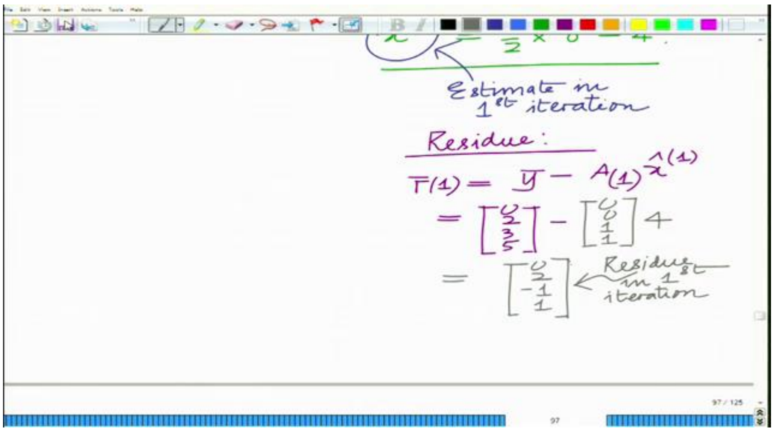

The residue is $\bar{r}(1) = \bar{y} - A_{(1)} x^{(1)}$ which will basically be $\begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ and this is what we carry over to the second iteration. So we subsequently find the projections of the columns of ϕ on this residue, choose the one that has the maximum and perform the least square solution, find the residue and repeat the process.

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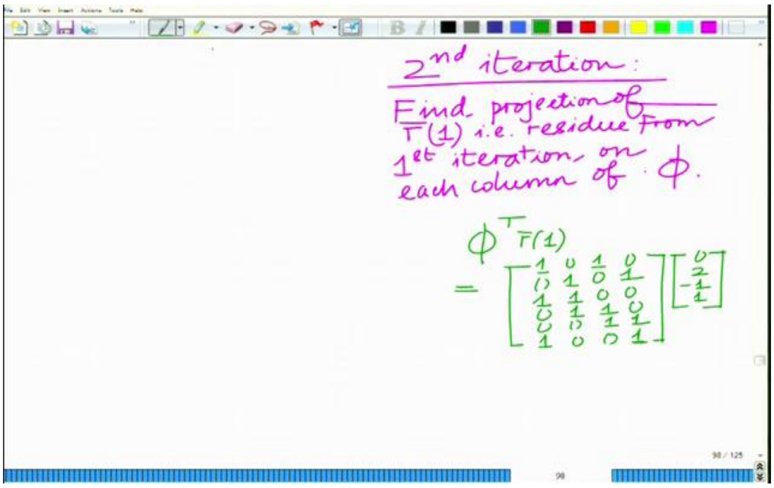

So proceeding the same way, we get the residue as shown in the slides below.

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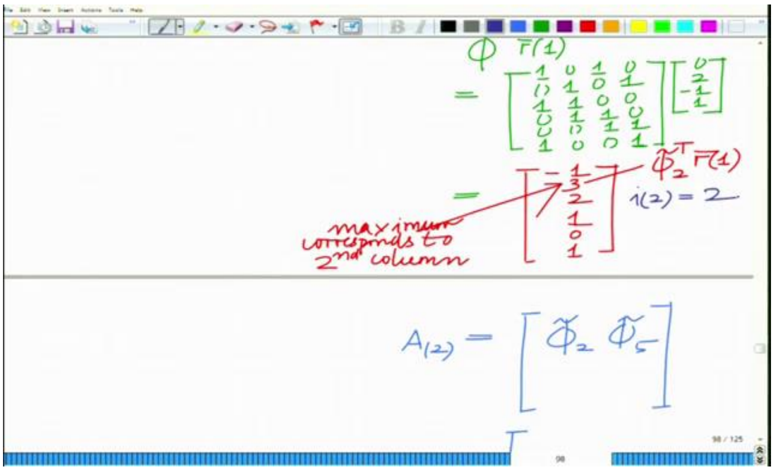

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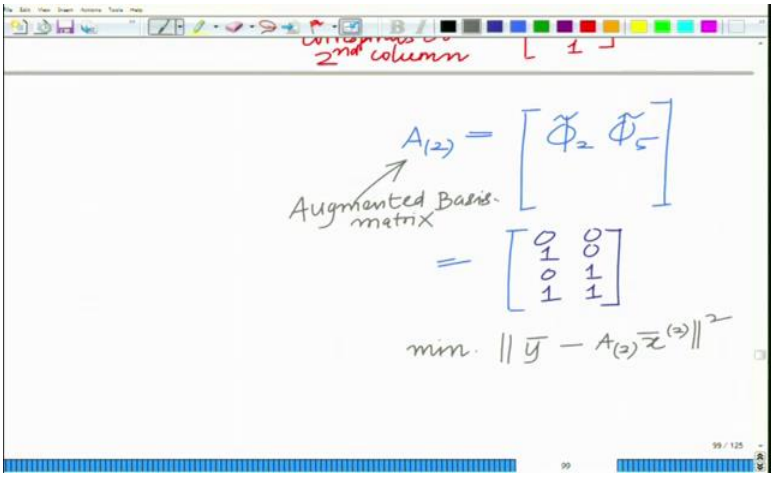

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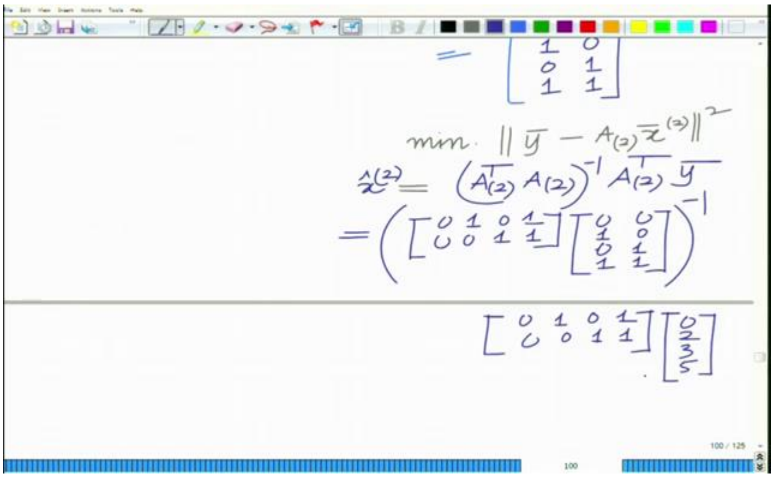

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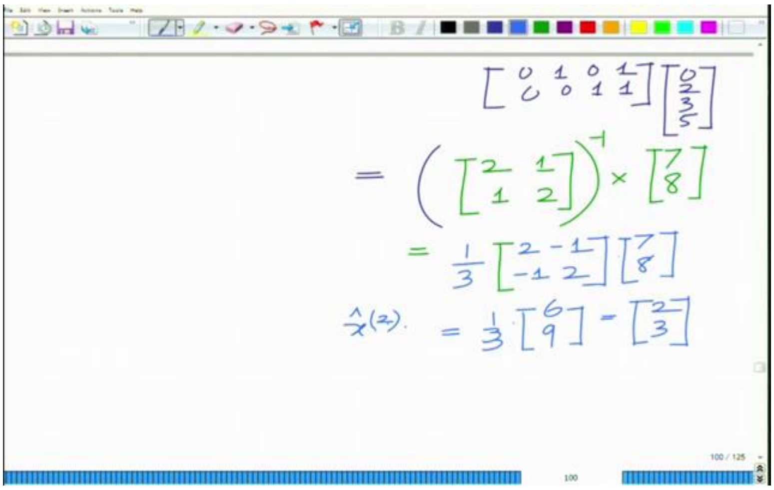

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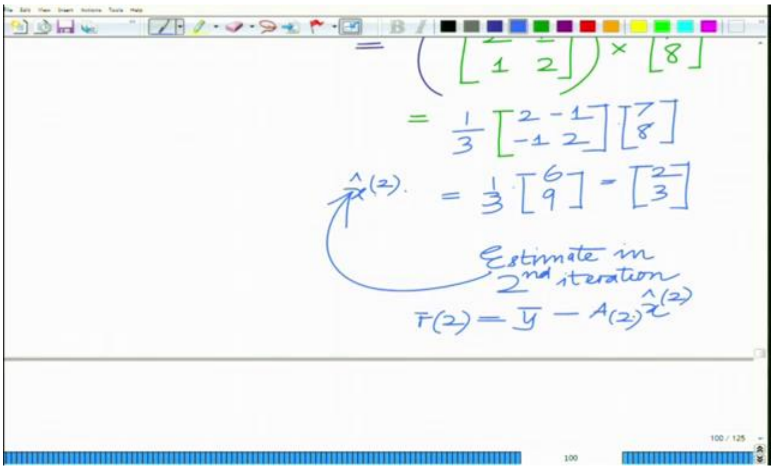

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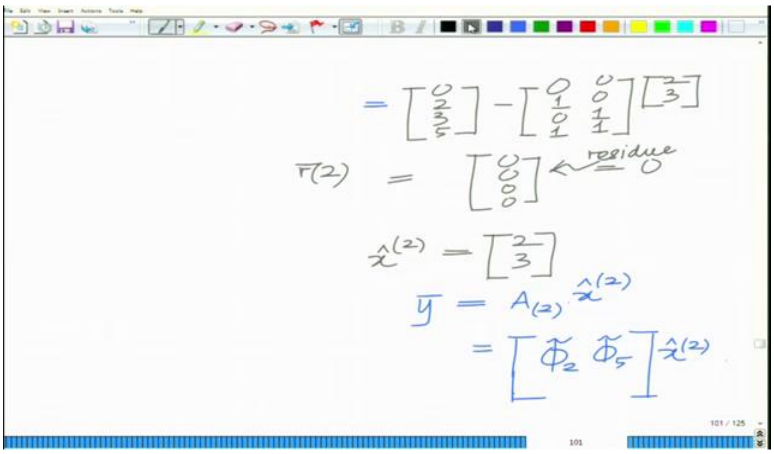

So here the residue is exactly 0 which basically means that you are exactly able to approximate $\bar{y}$ in the second iteration.

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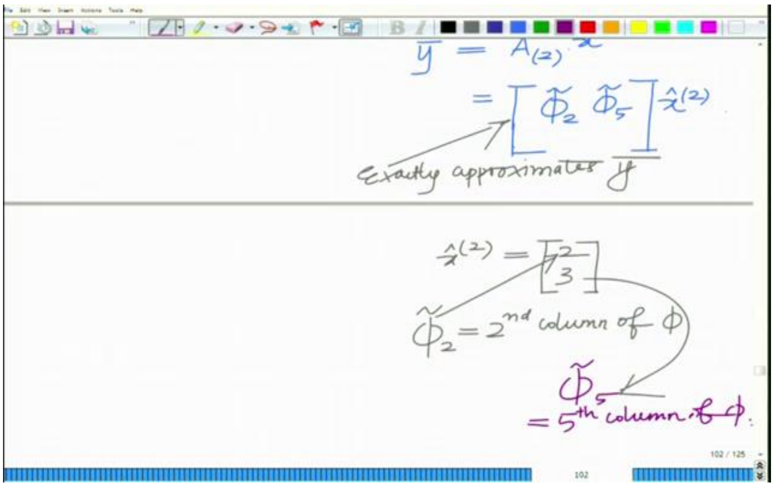

So no further iterations are needed which means $x^{(2)} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and the components 2 and 3 corresponds to ϕ_2 and ϕ_5 which are basically the second and fifth columns of the matrix ϕ respectively.

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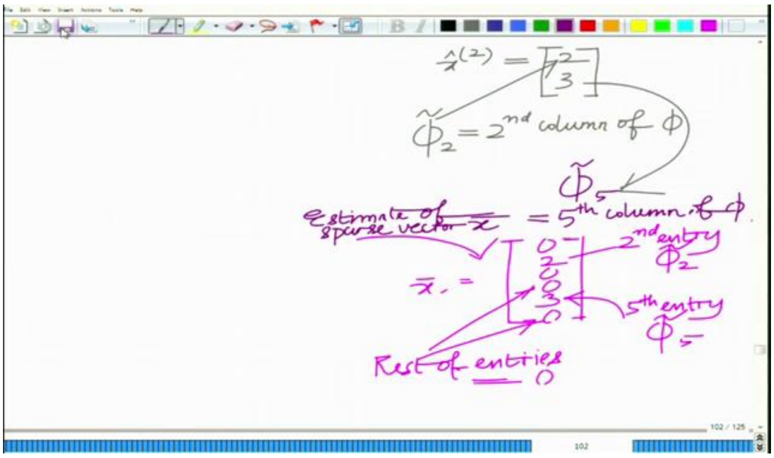

And therefore, now you can reconstruct the sparse vector $\bar{x}$ as follows, only the second entry and the 5th entry will be 2 and 3 respectively and the rest of the entries are 0. This is a simple example, but problems in practice are frequently more complex. But you can use this OMP algorithm for similar scenarios. So let us stop here and continue in the subsequent modules. Thank you very much.