Applied Optimization for Wireless, Machine Learning, Big Data

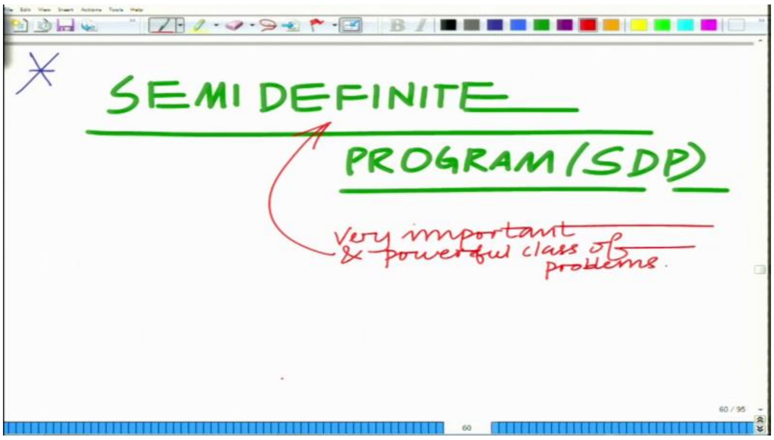
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 75
Semi Definite Program (SDP) and its application: MIMO symbol vector decoding

Keywords: *Semi Definite Program*

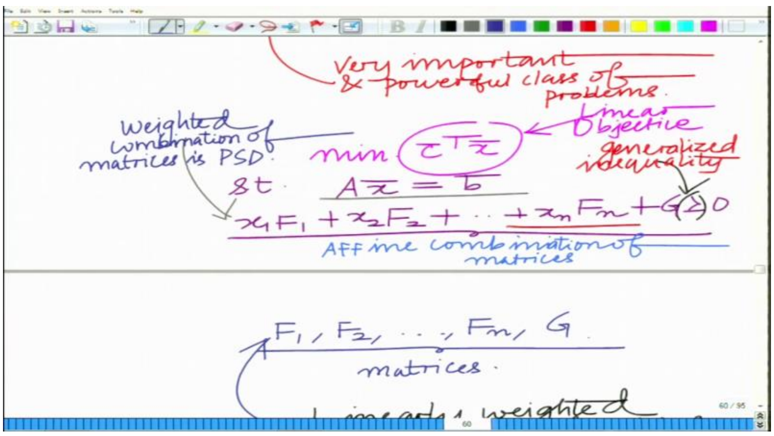
Hello, welcome to another module in this massive open online course. So we are looking at convex optimization problems and their applications. Let us start looking at what is known as Semi Definite Programming.

(Refer Slide Time: 00:43)



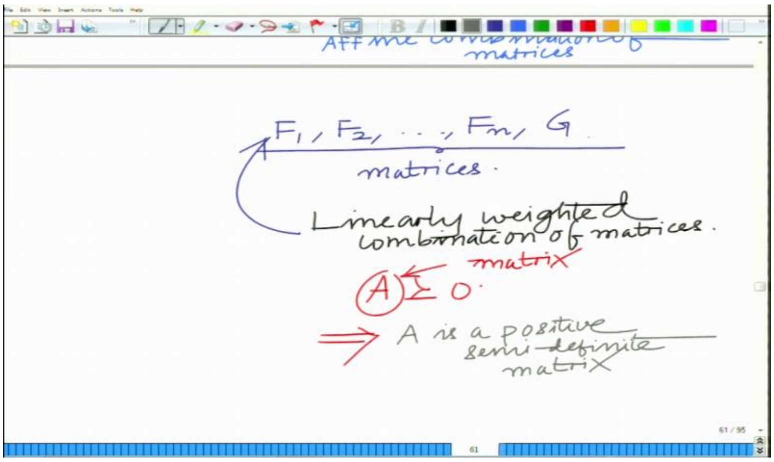
So it is a very important and powerful class of problems and a semi definite program is the following where you are minimizing a seemingly simple objective that is the objective is still a linear objective.

(Refer Slide Time: 01:51)



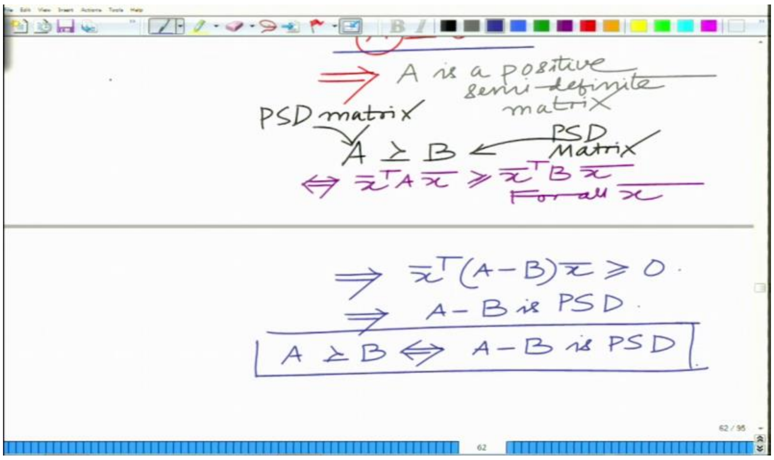
The equality constraint is something that is very similar. But the inequality constraint is a weighted combination of matrices or rather an affine combination.

(Refer Slide Time: 03:25)



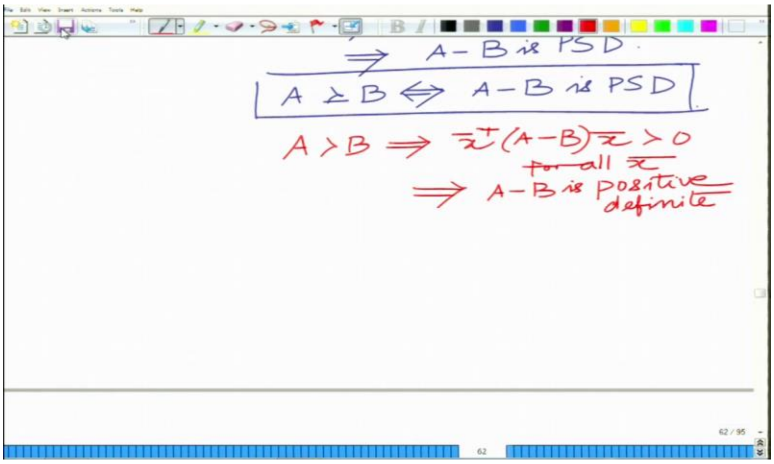
You are saying this matrix has to be greater than equal to 0 which implies that this matrix has to be positive semi definite.

(Refer Slide Time: 06:50)



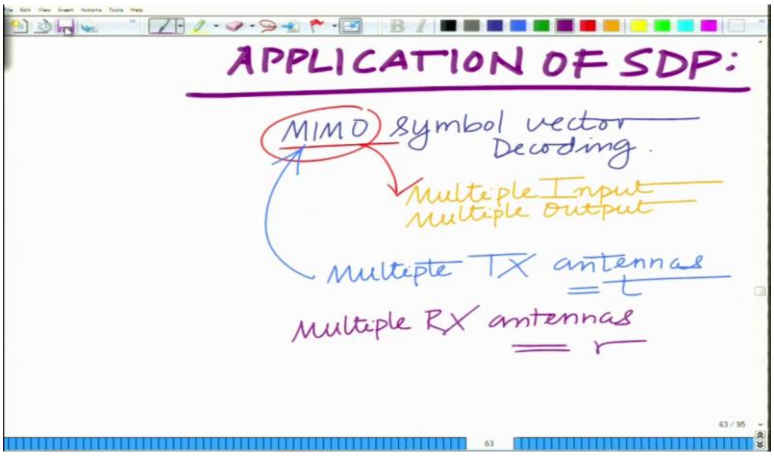
We say two positive semi definite matrices that is $A \geq B$ where A is a positive semi definite matrix, if and only if $\bar{x}^T A \bar{x} \geq \bar{x}^T B \bar{x} \Rightarrow A - B$ is PSD, for all $\bar{x}$.

(Refer Slide Time: 08:45)



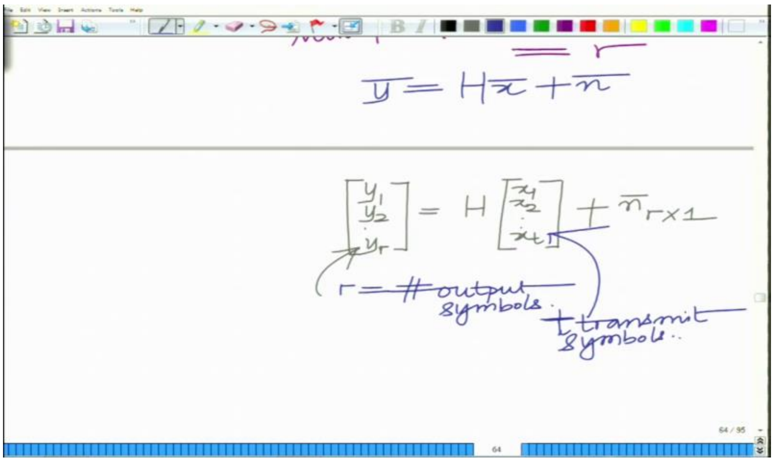
Now, similarly if you have a strict inequality A > B, naturally this implies A - B is positive definite. So this is the notion of this generalized inequality on the set of positive semi definite matrices.

(Refer Slide Time: 09:43)



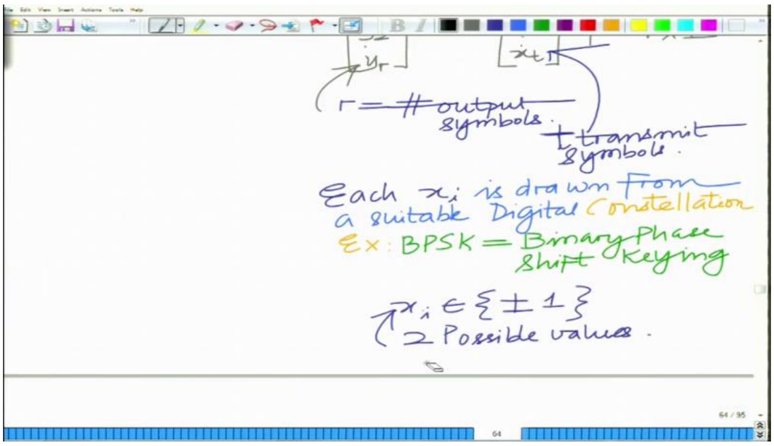
So let us look at an application of SDP and the application is as follows. So let us consider a MIMO system and we want to perform MIMO symbol decoding. So in this MIMO system, you have multiple TX antennas and multiple RX antennas.

(Refer Slide Time: 11:46)



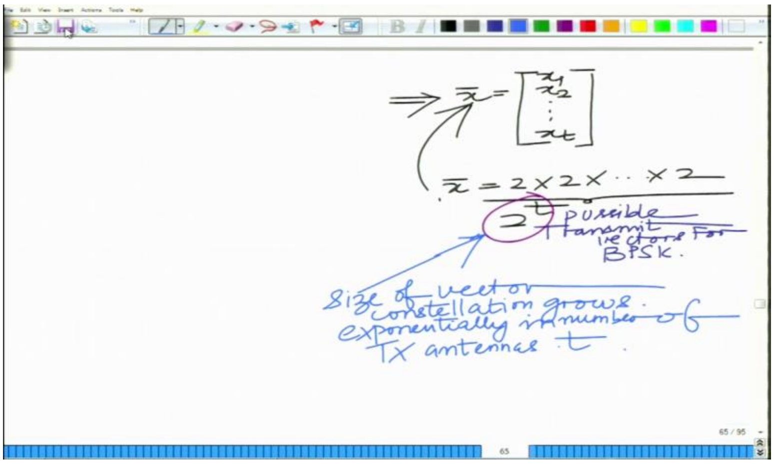
And I can represent this MIMO system model as $\bar{y} = H\bar{x} + \bar{n}$, where you have t transmit symbols and r output symbols.

(Refer Slide Time: 12:49)



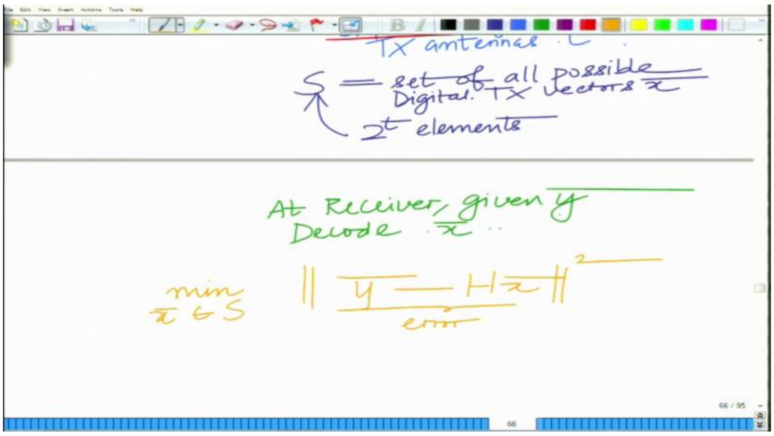
Now, if you look at each symbol x_i this has to be drawn from a constellation that is it cannot take any possible value. So each x_i is drawn from a suitable digital constellation for a digital wireless system, example, BPSK this is basically your binary phase shift keying, which implies there are two phases that means, each x_i has two possible values. So each x_i can be plus or minus 1, that means, each x_i has two possible values.

(Refer Slide Time: 14:43)



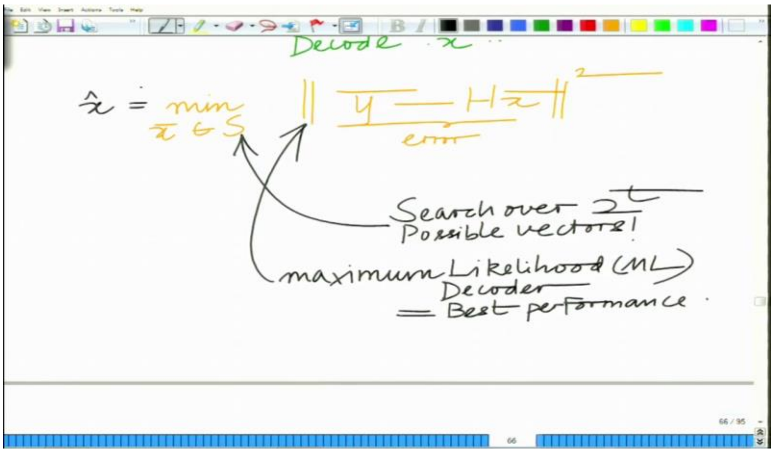
So basically the size of this set of the vector constellation is 2^t which is growing exponentially in the number of transmit antennas t.

(Refer Slide Time: 16:42)

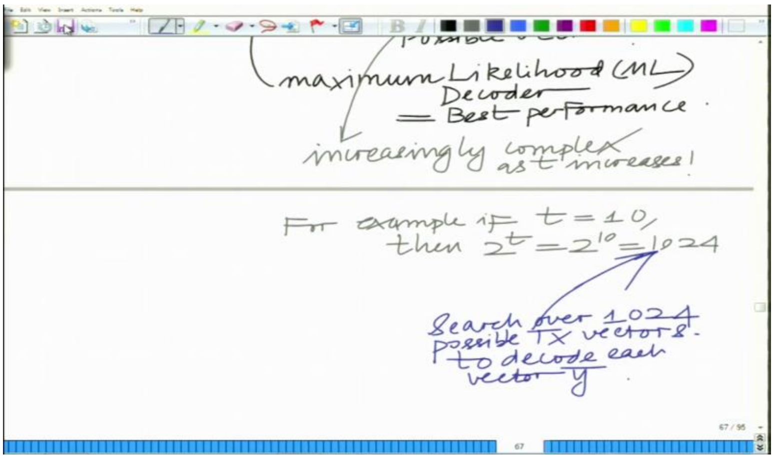


So now, let us say we have the set S which is the set of all possible digital transmit vectors $\bar{x}$ and this set has 2^t elements and therefore the problem is at the receiver once you receive $\bar{y}$ we have to decode $\bar{x}$. So therefore you use the best possible decoder which is known as the ML decoder. So we have $\min_{x \in S} \left\| \bar{y} - H\bar{x} \right\|^2$. The problem is you have to search over all t possible vectors to find x which basically minimizes this error and this is known as the maximum likelihood decoder.

(Refer Slide Time: 19:10)

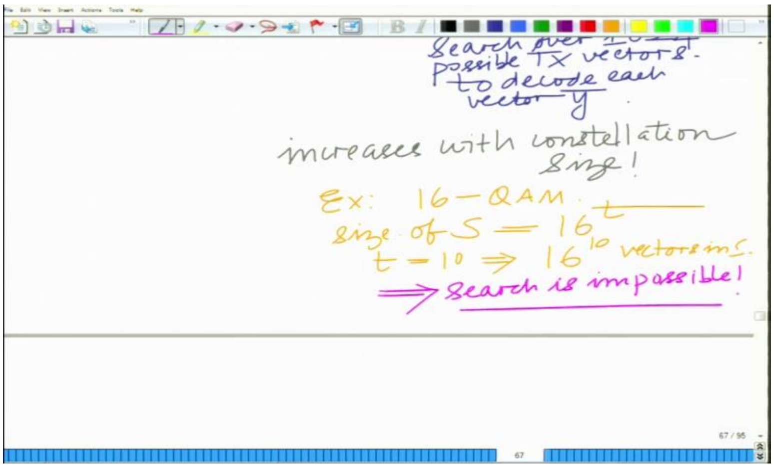


(Refer Slide Time: 20:07)



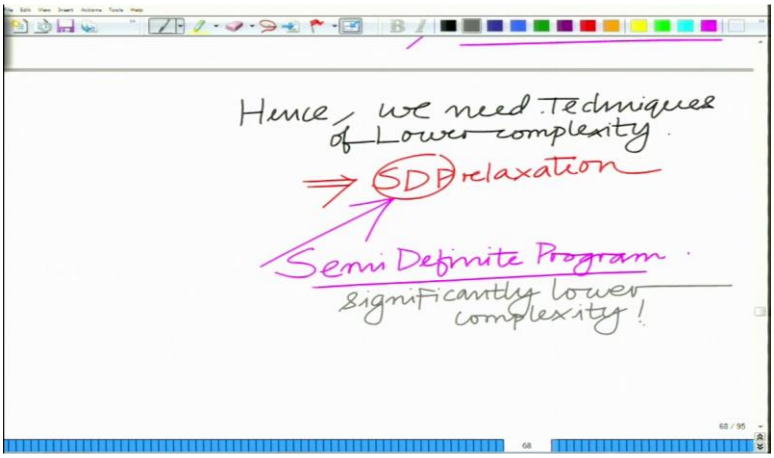
Now this search this increases with the constellation. For instance, if you have 16 QAM, then the number of vectors becomes 16^t.

(Refer Slide Time: 21:44)



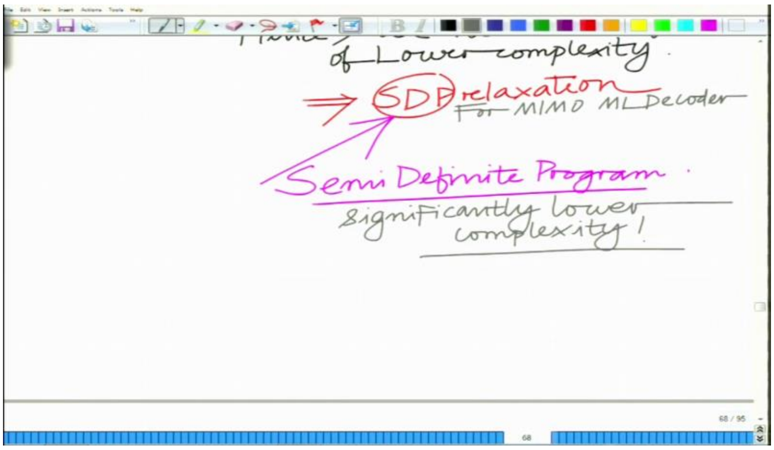
So this implies that this search is impossible or next to impossible and therefore, we have to come up with low complexity techniques.

(Refer Slide Time: 22:58)



Hence, one such technique is basically what is termed as SDP relaxation we relax this ML decoder problem as a Semi - Definite Program. We are going to formulate this MIMO ML decoder as a Semi - Definite Program which has a significantly lower complexity. So this is very useful for practical implementation.

(Refer Slide Time: 24:22)



So this has significantly lower complexity in comparison to the optimal ML decoder which has exponential complexity which is virtually impossible for a large number of transmit antennas and large constellation sizes. So the resulting ML decoder or the approximate ML decoder has a significantly lower complexity. So we will stop here and we will look at this in the subsequent module. Thank you very much.