

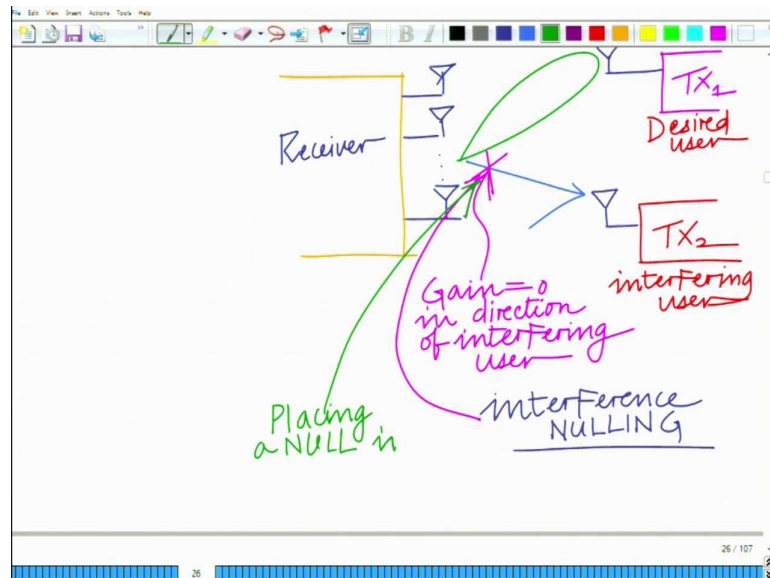
**Applied Optimization for Wireless, Machine Learning, Big Data**  
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**Lecture – 37**

**Practical Application: Zero-Forcing (ZF) Beamforming with Interfering User**

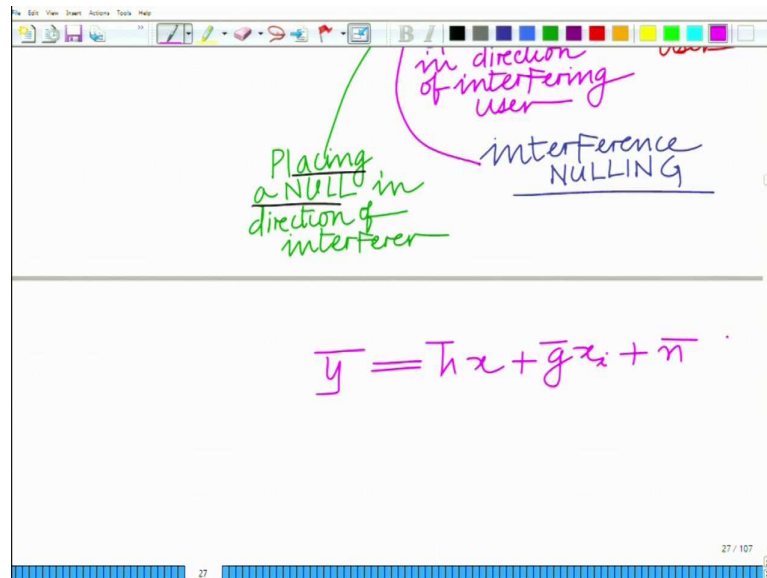
Hello, welcome to another module in this massive open online course. Let us look at another kind of beam forming which is known as Zero-Forcing (ZF) beam forming. In zero-forcing beamforming, the interference is null. In other words, interference is forced to zero in ZF-beamformer therefore this is also known as interference nulling.

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Consider a multiple antenna array system in which there are several receivers. First transmitter TX<sub>1</sub> is the desired user and the second transmitter TX<sub>2</sub> is the interfering user. So the signal gain must be maximized in the direction of the desired user while in the direction of the interfering user, the signal gain must be equal to zero and in this way, the interference is nulled.

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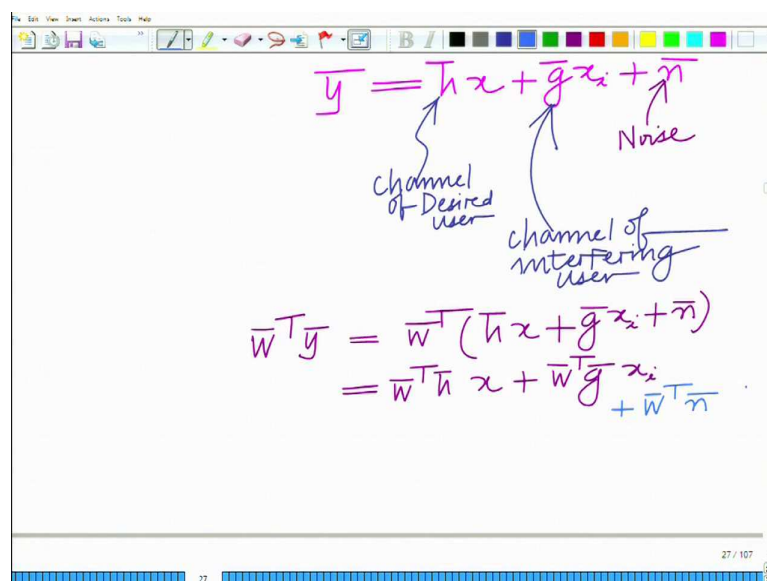


Let us discuss its procedure. Let us again go back to the system model,

$$\bar{y} = \bar{h}x + \bar{g}x_i + \bar{n}$$

Where  $\bar{y}$  is the received signal,  $\bar{h}$  is the channel vector of the desired user,  $x$  is the transmitted signal and  $\bar{n}$  is the additive white Gaussian noise. The term  $\bar{g}x_i$  defines the interference caused by interferer such that  $\bar{g}$  is the channel vector of interfering user.

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For electronic steering,

$$\begin{aligned}\bar{w}^T \bar{y} &= \bar{w}^T (\bar{h}x + \tilde{n}) \\ &= \bar{w}^T \bar{h}x + \bar{w}^T (\bar{g}x_i + \bar{n}) \\ &= \bar{w}^T \bar{h}x + \bar{w}^T \bar{g}x_i + \bar{w}^T \bar{n}\end{aligned}$$

Here,  $\bar{w}^T \bar{h}$  is the signal gain and  $\bar{w}^T \bar{g}$  is the interfering signal gain. So in ZF beamformer,  $\bar{w}^T \bar{g}$  should be zero along with unity signal gain.

$$\bar{w}^T \bar{h} = 1 \text{ and } \bar{w}^T \bar{g} = 0$$

So, this basically nulls the interference.

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**ZF BEAMFORMER:**

interference  
Zero Forcing interference.

optimization problem

objective

convex optimization problem

s.t.

affine

constraints

ZF constraint

$\min \sigma^2 \|\bar{w}\|^2$

$\bar{w}^T \bar{h} = 1$

$\bar{w}^T \bar{g} = 0$

Therefore, the optimization problem for zero-forcing beam former is

$$\begin{aligned}\min \quad & \sigma^2 \|\bar{w}\|^2 \\ \text{such that} \quad & \bar{w}^T \bar{h} = 1, \\ & \bar{w}^T \bar{g} = 0\end{aligned}$$

As usual  $\sigma^2$  is constant, therefore this optimization problem can be written as

$$\begin{aligned} \min \quad & \bar{w}^T \bar{w} \\ \text{such that} \quad & \bar{w}^T \bar{h} = 1, \\ & \bar{w}^T \bar{g} = 0 \end{aligned}$$

Here there are 2 constraints where second constraint the ZF constraint which is an affine set. Both the constraints are linear. Therefore this is again a convex optimization problem.

In fact, if there are more than one interfering user then there will be multiple constraints.

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Handwritten notes on a whiteboard showing the optimization problem and its vector form:

$$\begin{aligned} \min \quad & \bar{w}^T \bar{w} \\ \text{s.t.} \quad & \begin{bmatrix} \bar{h}^T \\ \bar{g}^T \end{bmatrix} \bar{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ & \underline{C}^T \bar{w} = \bar{e}_1 \\ & C = \begin{bmatrix} \bar{h} & | & \bar{g} \end{bmatrix} \\ & \bar{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \end{aligned}$$

Let us combine these two constraints in the vector form. So

$$\begin{bmatrix} \bar{h}^T \\ \bar{g}^T \end{bmatrix} \bar{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$c^T \bar{w} = \bar{e}_1$$

Such that

$$c^T = \begin{bmatrix} \bar{h} & | & \bar{g} \end{bmatrix} \quad \text{and} \quad \bar{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So the optimization problem is minimized as follows.

$$\begin{aligned} \min \quad & \bar{w}^T \bar{w} \\ \text{such that} \quad & c^T \bar{w} = \bar{e}_1 \end{aligned}$$

This is the optimization problem for zero-forcing Beam Forming. In fact, this is known as a quadratic program (QP) because here is a quadratic objective function.

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The image shows a handwritten derivation of the Lagrangian function for a quadratic program. The derivation is as follows:

$$\begin{aligned} f(\bar{w}, \bar{\lambda}) &= \bar{w}^T \bar{w} + \bar{\lambda}^T \left( \begin{bmatrix} \bar{h}^T \\ \bar{g}^T \end{bmatrix} \bar{w} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \\ \bar{\lambda} &= \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \quad \begin{array}{l} \text{2 Lagrange multipliers} \\ \text{= one for each constraint} \end{array} \\ &= \bar{w}^T \bar{w} + \bar{\lambda}^T (c^T \bar{w} - \bar{e}_1) \\ &= \bar{w}^T \bar{w} + \bar{\lambda}^T c^T \bar{w} - \bar{\lambda}^T \bar{e}_1 \end{aligned}$$

Let us form its Lagrangian.

$$F(\bar{w}, \bar{\lambda}) = \bar{w}^T \bar{w} + \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix} \left( \begin{bmatrix} \bar{h}^T \\ \bar{g}^T \end{bmatrix} \bar{w} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

There are two Lagrange multipliers  $\lambda_1$  and  $\lambda_2$ . So let us take a vector of these Lagrange multipliers.

$$\bar{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Therefore

$$\begin{aligned} F(\bar{w}, \bar{\lambda}) &= \bar{w}^T \bar{w} + \bar{\lambda}^T (c^T \bar{w} - \bar{e}_1) \\ &= \bar{w}^T \bar{w} + \bar{\lambda}^T c^T \bar{w} - \bar{\lambda}^T \bar{e}_1 \\ &= \bar{w}^T I \bar{w} + \bar{c}^T \bar{w} - \bar{\lambda}^T \bar{e}_1 \end{aligned}$$

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The image shows a digital whiteboard with handwritten mathematical derivations. At the top, the Lagrangian function is written as  $L = \bar{w}^T \bar{w} + \frac{\lambda^T C^T \bar{w} - \lambda^T \bar{e}_1}{\bar{z}}$ , with a note  $\bar{z} = C\bar{w}$  and the label "Lagrangian Function". Below this, the derivative of the first term is shown:  $\bar{w}^T I \cdot \bar{w} \xrightarrow{\frac{d}{d\bar{w}}} 2 \cdot I \cdot \bar{w} = 2\bar{w}$ . The bottom part of the whiteboard shows the gradient of the Lagrangian with respect to  $\bar{w}$  set to zero:  $\frac{dF}{d\bar{w}} = 2\bar{w} + C\bar{\lambda} - 0 = 0$ .

This is your Lagrangian function. Put the gradient of the Lagrangian function with respect to  $\bar{w}$  equal to zero.

$$\begin{aligned} \frac{dF}{d\bar{w}} &= 0 \\ 2\bar{w} + C\bar{\lambda} - 0 &= 0 \\ \bar{w} &= -\frac{1}{2}C\bar{\lambda} \end{aligned}$$

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The image shows a digital whiteboard with handwritten mathematical derivations. It starts with the equation  $2\bar{w} = -C\bar{\lambda}$ , which is then boxed and rearranged to  $\bar{w} = -\frac{1}{2}C\bar{\lambda}$ . A red note says "To find  $\bar{\lambda}$  use constraint". Below this, the constraint equation  $C^T \bar{w} = \bar{e}_1$  is substituted with the expression for  $\bar{w}$ :  $\Rightarrow C^T \left(-\frac{1}{2}C\bar{\lambda}\right) = \bar{e}_1$ , which simplifies to  $\Rightarrow -\frac{1}{2}(C^T C)\bar{\lambda} = \bar{e}_1$ .

To find the value of  $\bar{\lambda}$ , use constraint. Therefore

$$\begin{aligned}c^T \bar{w} &= \bar{e}_1 \\c^T \left( -\frac{1}{2} c \bar{\lambda} \right) &= \bar{e}_1 \\-\frac{\bar{\lambda}}{2} &= (c^T c)^{-1} \bar{e}_1\end{aligned}$$

Therefore the optimal ZF Beamformer vector  $\bar{w}^*$  is as follows.

$$\bar{w}^* = c (c^T c)^{-1} \bar{e}_1$$

So, this is the zero-forcing beam former which basically places a null in the direction of the interferer and hence it completely blocks the interference from the interfering user. In fact; it is also a very popular technique that is employed in practical wireless communication systems, especially in the presence of a large number of interfering users. This can also be used in a cognitive radio scenario for the secondary user during the on-going primary user transmission. So, one can block the interference caused by the primary transmitter by using zero-forcing wave forming.

Also because of its low complexity, ZF beamforming tends to be one of the popular beamforming techniques in the presence of interference.