1. Given a selective DF cooperative communication system with source power P_1, relay power P_2, noise power σ^2 and average link powers $\delta_{sd}^2, \delta_{sr}^2, \delta_{rd}^2$ for the source-destination, source-relay and relay-destination links, respectively. As shown in the lectures, the quantity $\Pr(e|\bar{\phi})$ is

$$\frac{3\sigma^4}{4P_1P_2\delta_{sd}^2\,\delta_{rd}^2}$$

Ans b

2. All of the given statements are true for compressive sensing

Ans d

3. The OMP solution is obtained via the following steps. Let the problem be represented as

$$\mathbf{y}_{\text{res}} = \mathbf{y} = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 0 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}^T\mathbf{y} = \mathbf{A}^T\mathbf{y}_{\text{res}} = [-1 \quad -1 \quad 1 \quad -4 \quad -3 \quad 2]^T$$

Maximum magnitude occurs for **A**(:,4) i.e. 4th column. Hence, $\mathbf{A}^{(1)} = \mathbf{A}(:,4)$. In first iteration, the sparse estimate is

$$\mathbf{x}^{(1)} = \left(\left(\mathbf{A}^{(1)}\right)^{\boldsymbol{T}}\mathbf{A}^{(1)}\right)^{-\mathbf{1}}\left(\mathbf{A}^{(1)}\right)^{\boldsymbol{T}}\mathbf{y} = -2,$$

$$\mathbf{y}_{\text{res}} = \mathbf{y} - \mathbf{A}^{(1)}\mathbf{x}^{(1)} = [2 \quad 1 \quad -1 \quad 0]^T$$

Next support is found as

$$\mathbf{A}^T\mathbf{y}_{\text{res}} = [1 \quad 1 \quad 3 \quad 0 \quad -1 \quad 2]^T$$

Max occurs for 3rd column. Form $\mathbf{A}^{(2)}$ by augmenting $\mathbf{A}^{(1)}$ with 3rd column as

$$\mathbf{A}^{(2)} = [\mathbf{A}(:,3) \quad \mathbf{A}(:,4)]$$

The LS estimate is

$$\mathbf{x}^{(2)} = \left(\left(\mathbf{A}^{(2)}\right)^{\boldsymbol{T}}\mathbf{A}^{(2)}\right)^{-\mathbf{1}}\left(\mathbf{A}^{(2)}\right)^{\boldsymbol{T}}\mathbf{y} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

The residue is

$$\mathbf{y} - \mathbf{A}^{(2)}\mathbf{x}^{(2)} = [0 \quad 0 \quad 0 \quad 0]^T$$

Hence, the estimate of the sparse vector is

$$\hat{\mathbf{x}} = [0 \quad 0 \quad 2 \quad -3 \quad 0 \quad 0]^T$$

The non-zero signal coefficients are x_3, x_4

Ans a

4. As seen from the solution above, values of the non-zero signal coefficients in the sparse solution are $2,\ -3$

Ans c

5. Consider a selective DF cooperative communication system with source power P_1, relay power P_2, noise power σ^2 and average link powers $\delta_{sd}^2, \delta_{sr}^2, \delta_{rd}^2$ for the source-destination, source-relay and relay-destination links, respectively. Let ϕ denote the error event at the relay, while e denotes error at the destination. $\Pr(e|\phi)\Pr(\phi)$ is $\frac{\sigma^4}{4P_1^2\delta_{sd}^2\,\delta_{sr}^2}$

   Ans c

6. The OMP solution is obtained via the following steps. Let the problem be represented as

$$\mathbf{y}_{\text{res}} = \mathbf{y} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 5 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}^T\mathbf{y} = \mathbf{A}^T\mathbf{y}_{\text{res}} = [2 \quad 8 \quad 5 \quad 3 \quad 5 \quad 7]^T$$

   Maximum magnitude occurs for $\mathbf{A}(:,2)$ i.e. 2nd column. Hence, $\mathbf{A}^{(1)} = \mathbf{A}(:,2)$. In first iteration, the sparse estimate is

$$\mathbf{x}^{(1)} = \left(\left(\mathbf{A}^{(1)}\right)^{\boldsymbol{T}}\mathbf{A}^{(1)}\right)^{-\mathbf{1}}\left(\mathbf{A}^{(1)}\right)^{\boldsymbol{T}}\mathbf{y} = 4,$$

$$\mathbf{y}_{\text{res}} = \mathbf{y} - \mathbf{A}^{(1)}\mathbf{x}^{(1)} = [2 \quad -1 \quad 0 \quad 1]^T$$

   Next support is found as

$$\mathbf{A}^T\mathbf{y}_{\text{res}} = [2 \quad 0 \quad 1 \quad -1 \quad 1 \quad 3]^T$$

   Max occurs for 6th column. Form $\mathbf{A}^{(2)}$ by augmenting $\mathbf{A}^{(1)}$ with 6th column as

$$\mathbf{A}^{(2)} = [\mathbf{A}(:,2) \quad \mathbf{A}(:,6)]$$

   The LS estimate is

$$\mathbf{x}^{(2)} = \left(\left(\mathbf{A}^{(2)}\right)^{\boldsymbol{T}}\mathbf{A}^{(2)}\right)^{-\mathbf{1}}\left(\mathbf{A}^{(2)}\right)^{\boldsymbol{T}}\mathbf{y} =$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ 7 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

   The residue is

$$\mathbf{y} - \mathbf{A}^{(2)}\mathbf{x}^{(2)} = [0 \quad 0 \quad 0 \quad 0]^T$$

   Hence, the estimate of the sparse vector is

$$\hat{\mathbf{x}} = [0 \quad 3 \quad 0 \quad 0 \quad 0 \quad 2]^T$$

   The non-zero signal coefficients are x_2, x_6

   Ans d

7. As seen from the solution above, values of the non-zero signal coefficients in the sparse solution are 3, 2

Ans c

8. The dual optimization problem can be found as follows. The Lagrangian is formulated as

$$L(\bar{\mathbf{x}}, \bar{\mathbf{v}}) = \bar{\mathbf{x}}^T\bar{\mathbf{x}} + \bar{\mathbf{v}}^T\left(\mathbf{A}\bar{\mathbf{x}} - \bar{\mathbf{b}}\right)$$

Setting gradient equal to 0

$$\nabla_{\bar{\mathbf{x}}} L(\bar{\mathbf{x}}, \bar{\mathbf{v}}) = \mathbf{2}\bar{\mathbf{x}} + \mathbf{A}^T\bar{\mathbf{v}} = \mathbf{0} \Longrightarrow \bar{\mathbf{x}} = -\frac{1}{2}\mathbf{A}^T\bar{\mathbf{v}}$$

Substituting in gradient one obtains dual problem as

$$\max g(\bar{\mathbf{v}}) = \max -\frac{1}{4}\bar{\mathbf{v}}^T\mathbf{A}\mathbf{A}^T\bar{\mathbf{v}} - \bar{\mathbf{b}}^T\bar{\mathbf{v}}$$

Ans b

9. The linear support vector machine (SVM) that maximizes the separation between these sets is given as

$$\begin{aligned} &\min. \|\bar{\boldsymbol{a}}\|_2 \\ \text{s.t. } &\bar{\mathbf{a}}^T\bar{\mathbf{x}}_i + b \geq 1, i = 1,2, \dots, N \\ &\bar{\mathbf{a}}^T\bar{\mathbf{y}}_i + b \leq -1, i = 1,2, \dots, N \end{aligned}$$

Ans a

10. MIMO channel with singular values $\sigma_i = 1, \frac{1}{2}$ for i = 1,2. The noise power $\sigma^2 = 2$. The optimal power allocated to the singular modes can be evaluated as follows.

$$P_i = \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_i^2}\right)^+$$

$$\sum_{i=1}^{2} P_i = 4 \Longrightarrow \frac{2}{\lambda} = 4 + 2 + 8 = 14 \Longrightarrow \frac{1}{\lambda} = 7$$

Observe $7 - 8 = -1 < 0$. Hence, all the power is allocated to the dominant mode i.e. 4 to mode 1 and 0 to mode 2.

Ans a