Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 44
Practical Application: Image deblurring

Keywords: *Image deblurring*

Hello welcome to another module in this massive open online course. So we are looking at various applications of least squares and in this module let us look at another interesting application in the context of image processing specifically in the context of Image deblurring.

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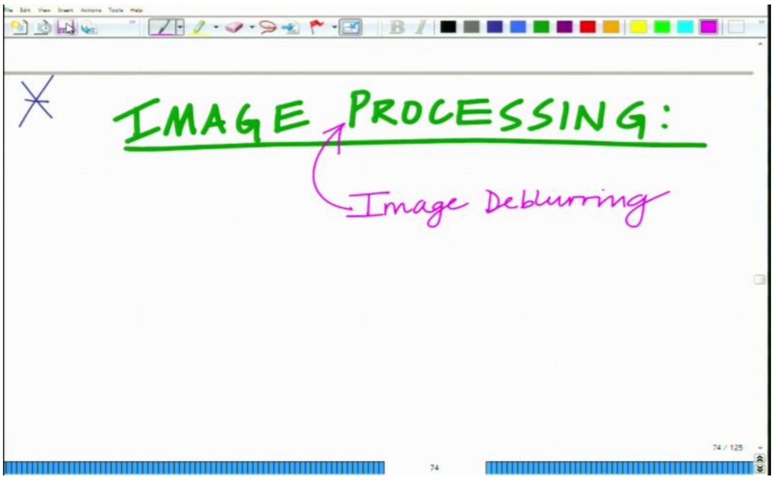

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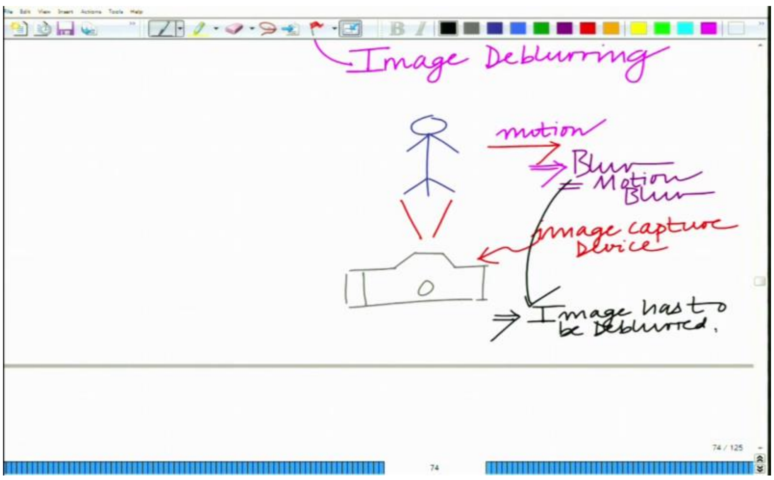

Now let us look at what happens when you try to capture the image of a person as shown in slide. Now, if this person or this object is in motion then this leads to a blur. So the blur effect is basically associated with motion and it is basically degrading the image and it can also arise from several other factors such as environmental factors, atmospheric factors, motion of the wind motion etc. So in general motion leads to blur and this is termed as motion blur and therefore, to get clean image, implies image has to be deblurred.

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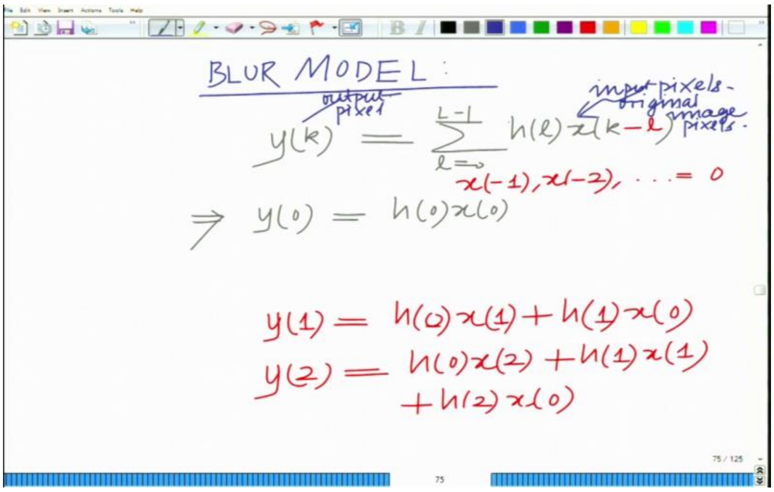

Now one way to model the motion blur is the following, so the blur model can be described as follows. Let say you have an output pixel y(k) that can be described as $y(k) = \sum_{l=0}^{L-1} h(l)x(k-l)$, so this can be written as shown in slide. So this is a signal which is 0 for n < 0. So what you can see is that each output pixel is a linear combination of several input pixels and that is what gives the blurring effect that is when you combine these pixels we get the blur effect.

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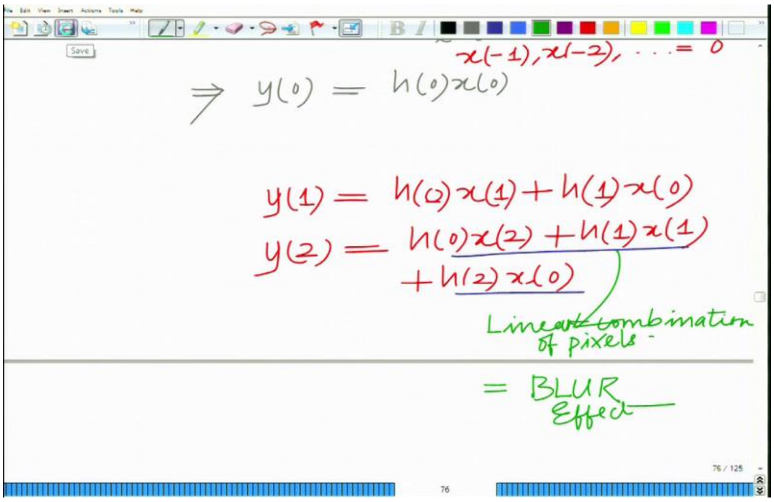

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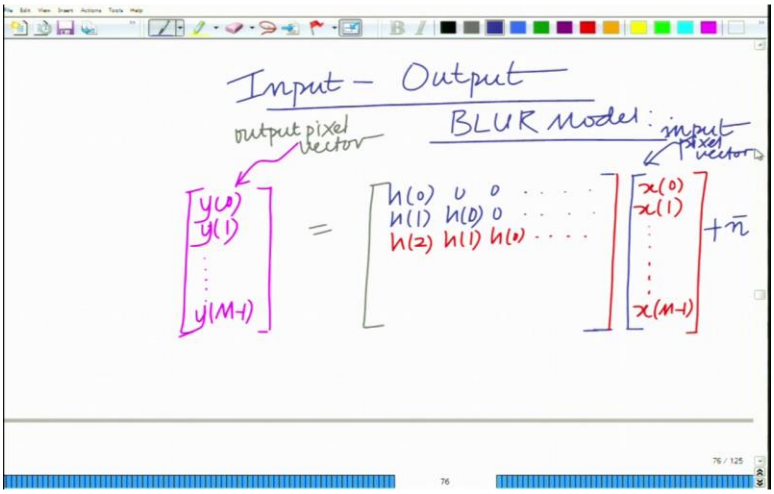

And therefore the input output blur model can be represented as follows, let us say the total number of pixels is M and we are considering a single of column of an image as shown in slide. Now, in addition you can also have noise as shown.

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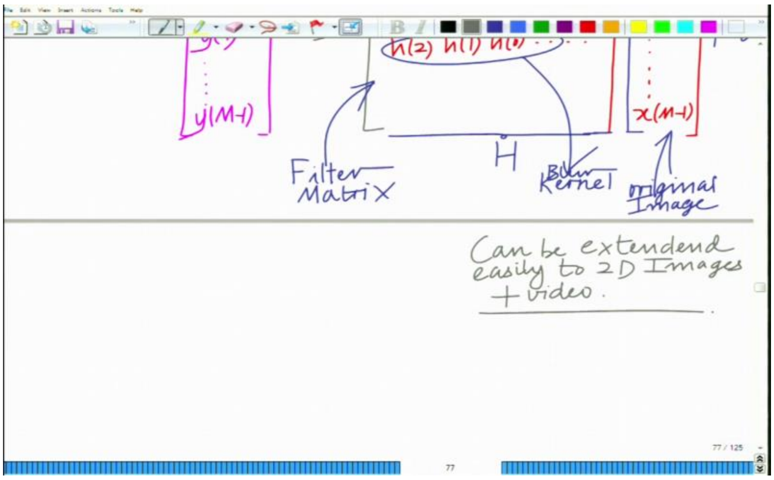

So each pixel is a linear combination of the original pixel and some neighbouring pixel and that is what gives the blur effect. And this matrix is also known as a filter matrix, because you are representing the blur by this linear transformation characterized by this matrix. This matrix is denoted as matrix H which is the filter matrix or blur matrix. And the entries repeating along the row is also called the kernel or the blur kernel. So I can represent the blurring effect in the image as this linear system. So this can be used in both the ways, either to recover the original image from the given output vector $\bar{y}$ or given input vector $\bar{x}$ to introduce the blur effect, one can use this linear input output system model. Now here what we are considering is, given a blur image, how do deblur it.

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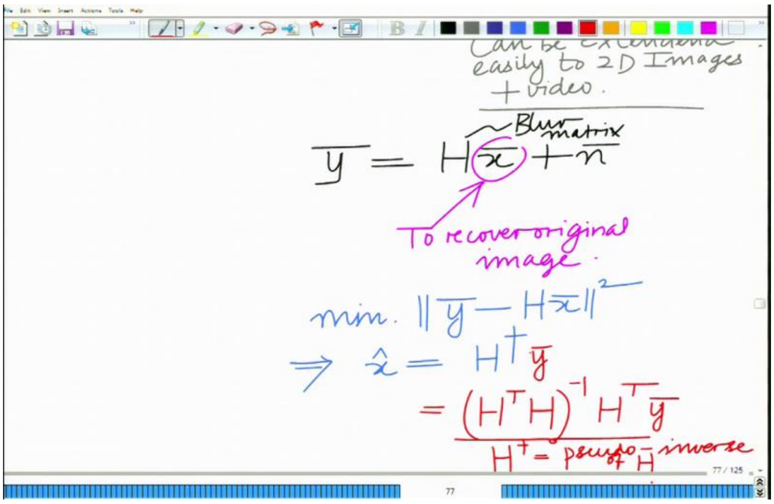

Now, we have this model as $\bar{y} = H\bar{x} + \bar{n}$ and to reconstruct the original image or to recover we now apply the least squares. So we have $\min \left\| \bar{y} - H\bar{x} \right\|^2$ implies the estimate or the reconstructed image or the deblurred image is the pseudo inverse of H. We have $x = H^{\dagger}\bar{y}$ which is $H^{\dagger} = \left(H^T H\right)^{-1} H^T$ and this acts as a left inverse and is the pseudo inverse of H.

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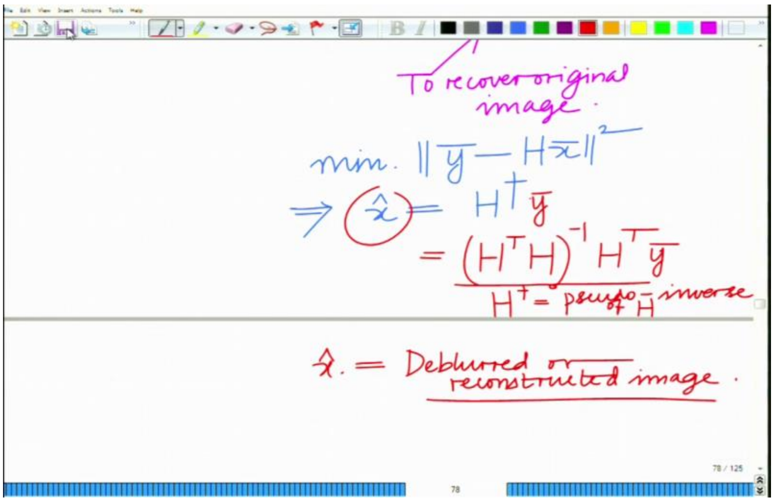

This x is the deblurred or reconstructed image and this is another interesting application of the least squares paradigm. Therefore, the least squares paradigm in general has several applications, in different areas of signal processing and communication. So we will stop here and continue in the subsequent modules. Thank you very much.