Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 76
Application: SDP for MIMO Maximum Likelihood (ML) Detection

***Keywords**: Semi Definite Program, MIMO Maximum Likelihood Detection*

Hello, welcome to another module in this Massive Open Online Course. So we are looking at the SDP that is Semi Definite Programming and its application in the context of MIMO detection that is how to reduce the complexity of the MIMO detector.

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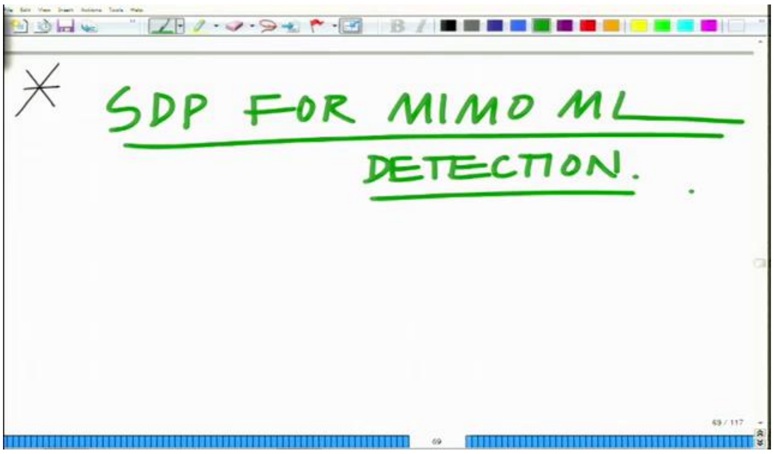

So we are looking at SDP for MIMO ML detection.

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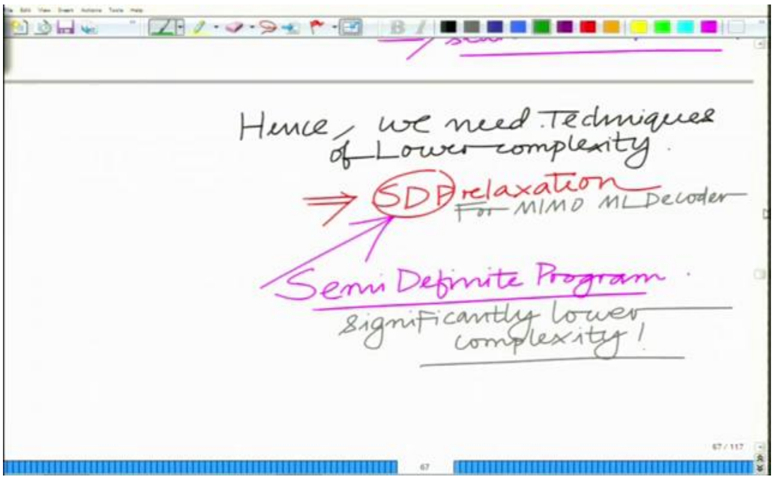

And SDP employs a positive semi definite constraint, that is the linear combination of matrices has to be positive semi definite and this is termed as a linear matrix inequality, LMI.

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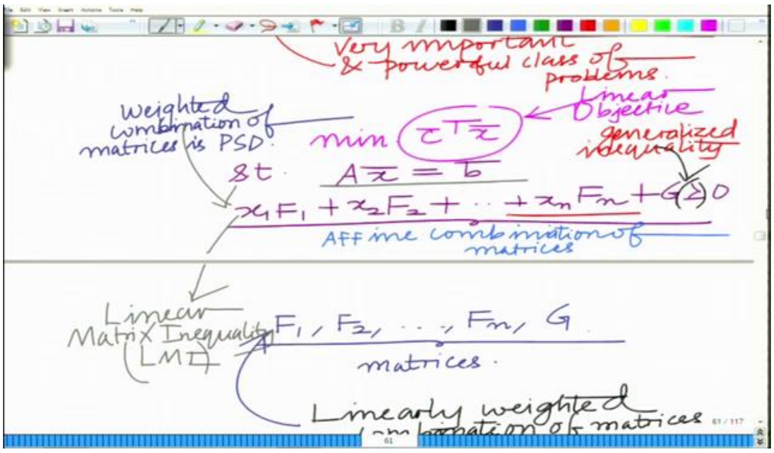

So SDP enforces a linear matrix inequality that is what is novel about SDP.

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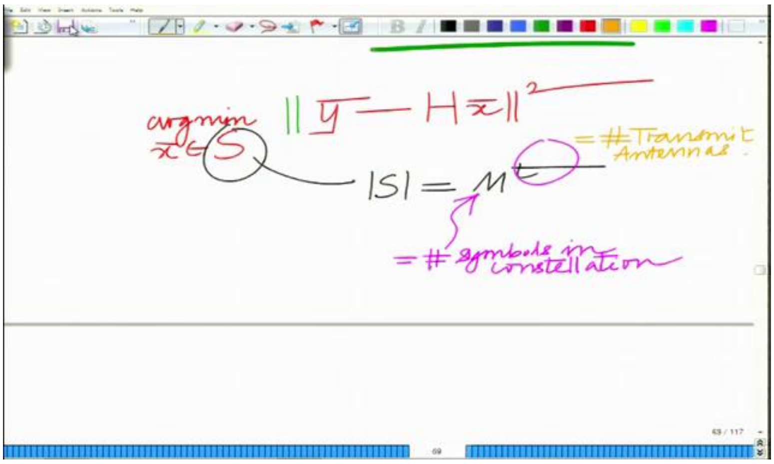

We have the MIMO detection problem as $\min\limits_{x \in S} \left\| \bar{y} - H\bar{x} \right\|^2$.

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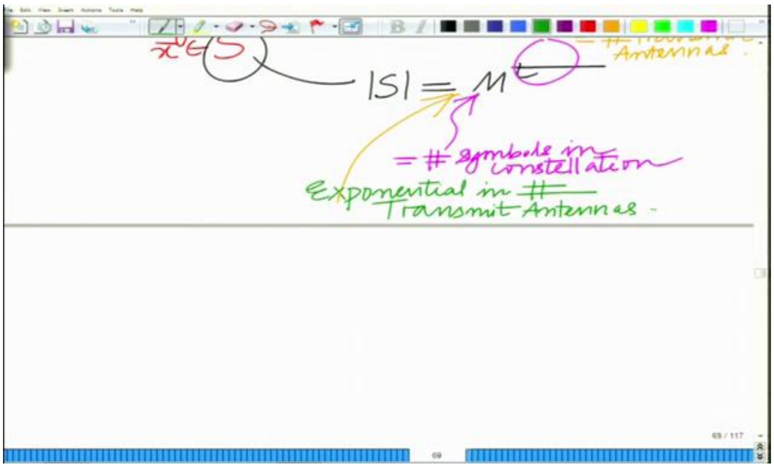

And so basically this is exponential in the number of transmit antennas, which is of very high complexity.

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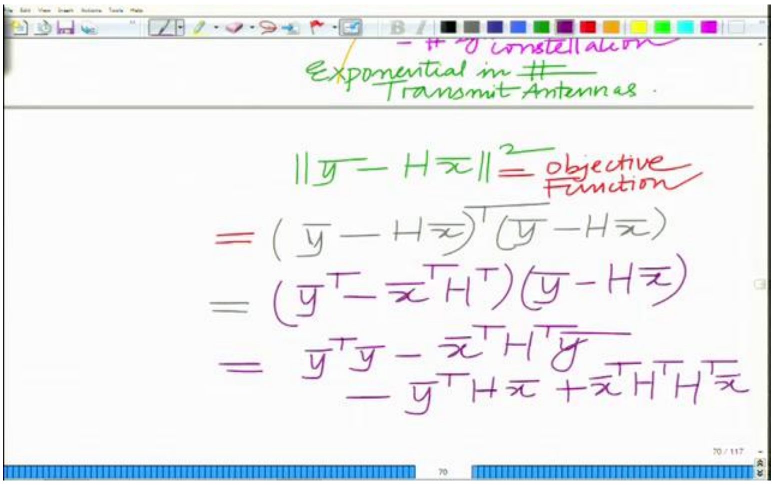

Now the cost function is simplified as shown in slide. So we have the simplified cost function as $\bar{y}^T \bar{y} - \bar{x}^T H^T \bar{y} - \bar{y}^T H \bar{x} + \bar{x}^T H^T H \bar{x}$.

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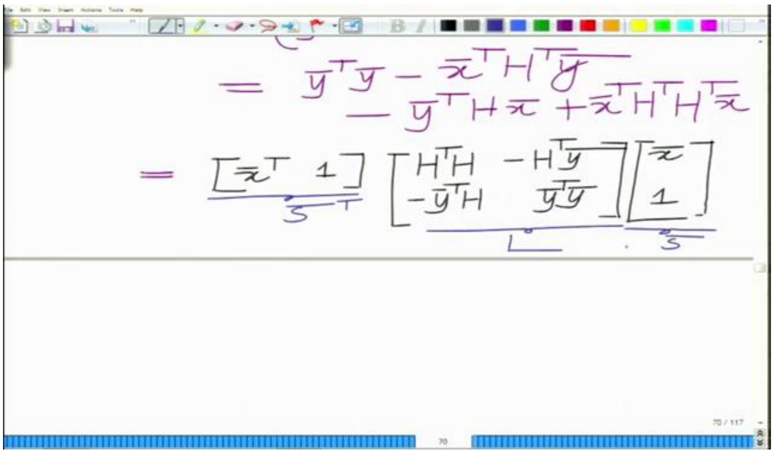

This can be written as shown in slide. So I am making a column vector by stacking it along with this number 1. So we have this t + 1 dimensional vector, $\bar{s}$.

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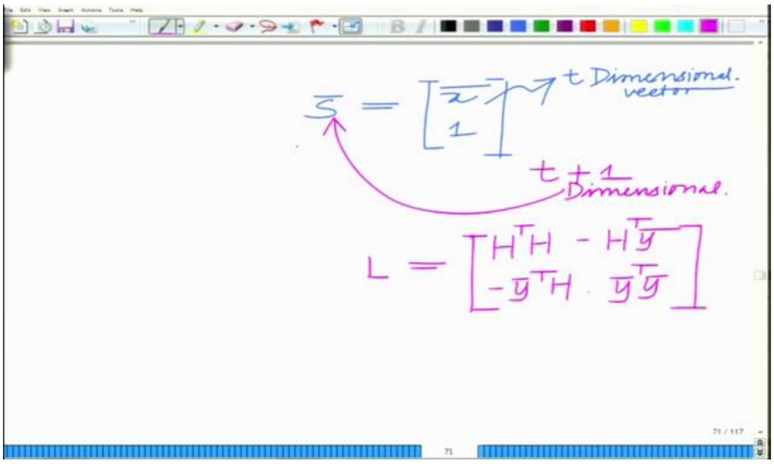

Now the matrix L is given as $\begin{bmatrix} H^T H & -H^T \bar{y} \\ -\bar{y}^T H & \bar{y}^T \bar{y} \end{bmatrix}$.

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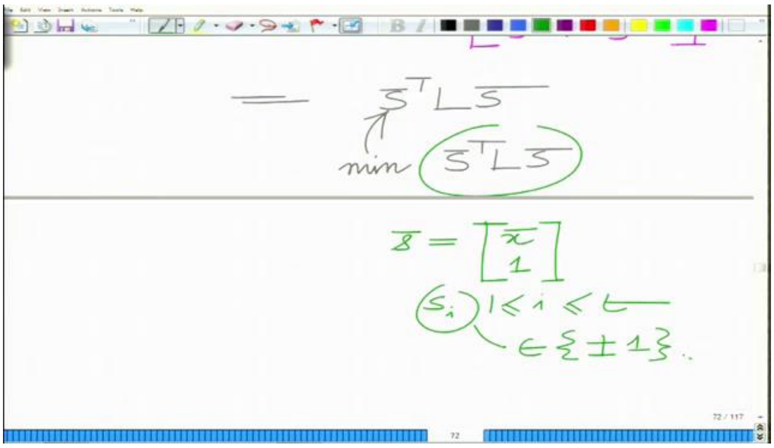

So we have written this as $\bar{s}^T L \bar{s}$. So this L can be thought of as a weighting matrix. So let us say this is BPSK constellation.

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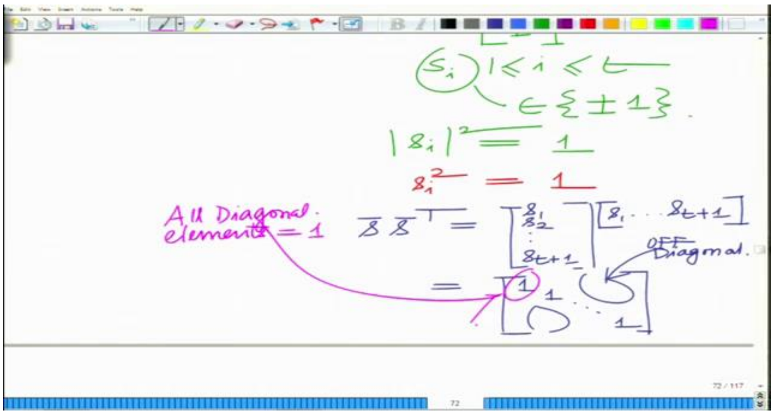

Now this proceeds as shown in slide.

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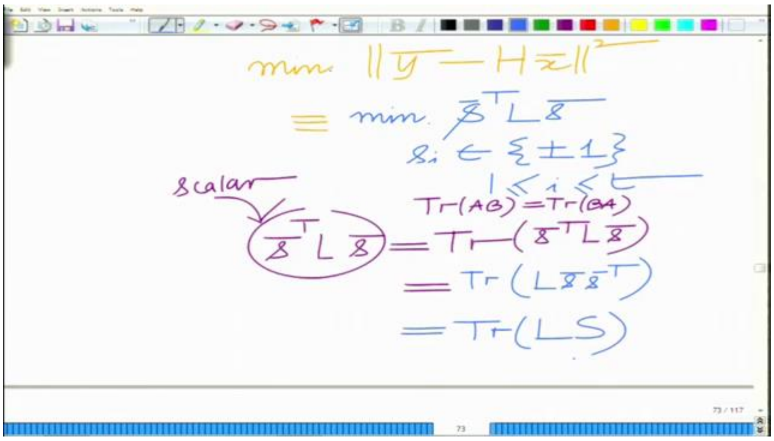

Now the above problem can be equivalently written as $\min\limits_{\substack{s_i \in \{\pm 1\} \\ 1 \leq i \leq t}} \bar{s}^T L \bar{s}$. Now this $\bar{s}^T L \bar{s}$ is a scalar quantity. So I can write this as $\bar{s}^T L \bar{s} = Tr(LS)$ since trace is the sum of the diagonal elements for a square matrix. So a single number is a special case of square matrix. So the trace will yield the number itself.

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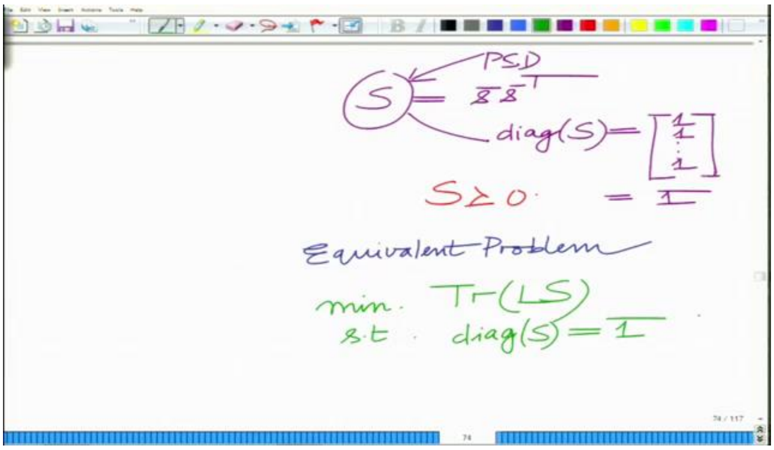

Here $S = \bar{s}\bar{s}^T$. So the equivalent problem will become
$$\begin{aligned} &\min Tr(LS) \\ &s.t\ diag(S) = \bar{1} \\ &\quad S \geq 0 \\ &\quad S = \bar{s}\bar{s}^T \end{aligned}.$$

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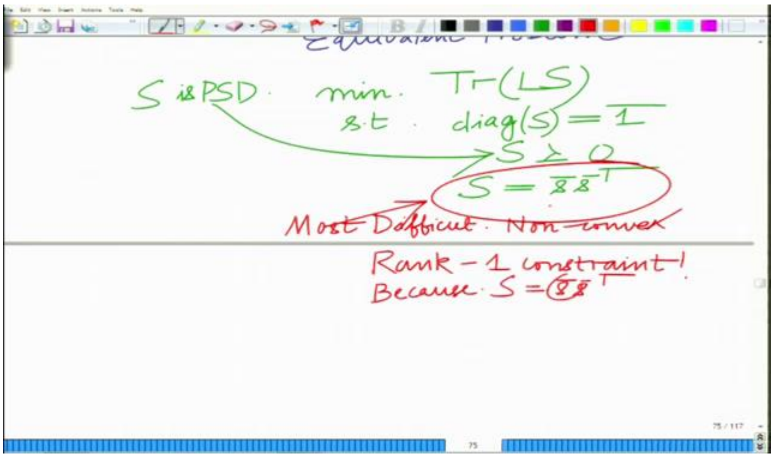

S is a positive semi definite matrix and of all the constraints this is the most difficult non-convex constraint. This is known as a rank-1 constraint because $S = \bar{s}\bar{s}^T$. So since this is very difficult to impose we simplify this and in this case we simply ignore this. This is known as an SDP relaxation, so we relax it. So this rank-1 constraint makes it non-convex, so it makes it non SDP. So we relax it as an SDP that is we ignore this rank-1 constraint.

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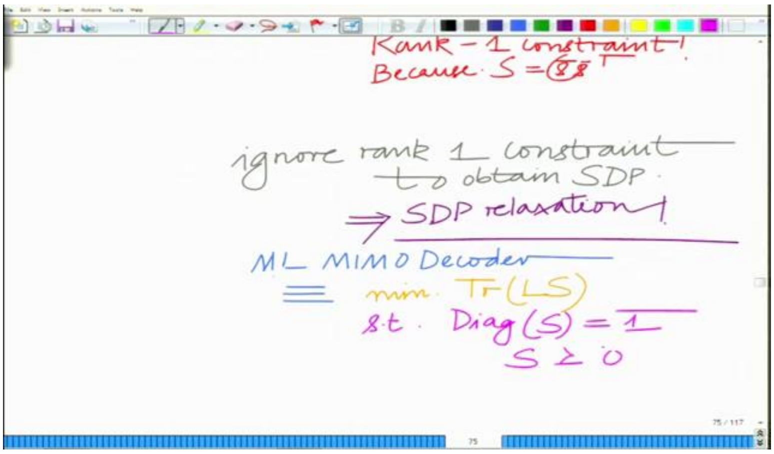

So this is termed as SDP relaxation. So our ML decoder can be equivalently written as,

$$\min Tr(LS)$$
$$s.t\ diag(S) = \bar{1}$$ and this yields an approximate solution.
$$S \geq 0$$

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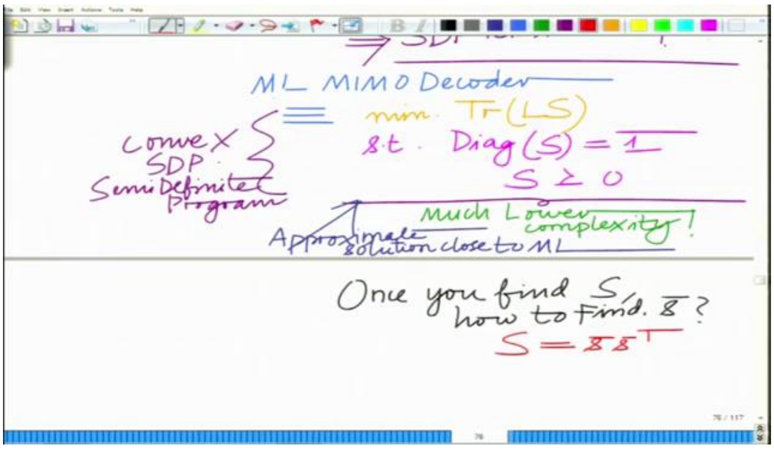

So this is significantly of lower complexity and therefore, it is very amenable to implement this in practice. The only thing is it yields an approximate solution close to the ML solution. Now, once you find S how to find $\bar{s}$. So the point is because we have ignored the rank 1 constraint, S is not guaranteed to be $S = \bar{s}\bar{s}^T$.

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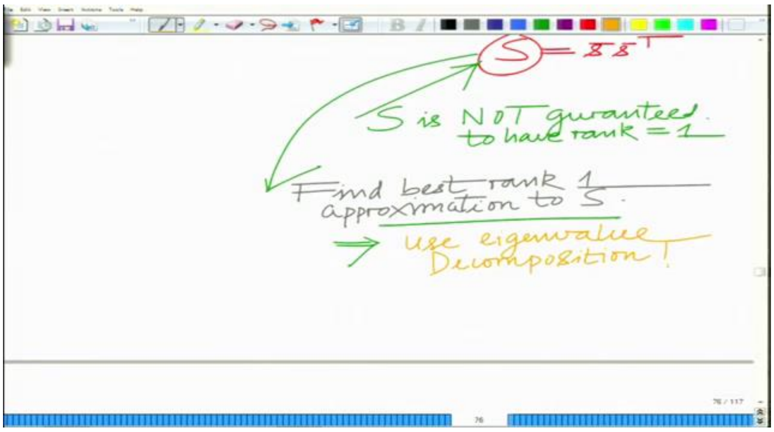

So in this context we need to find $\bar{s}$ and the key here is to find best rank 1 approximation to S, for that we use the Eigenvalue decomposition of S.

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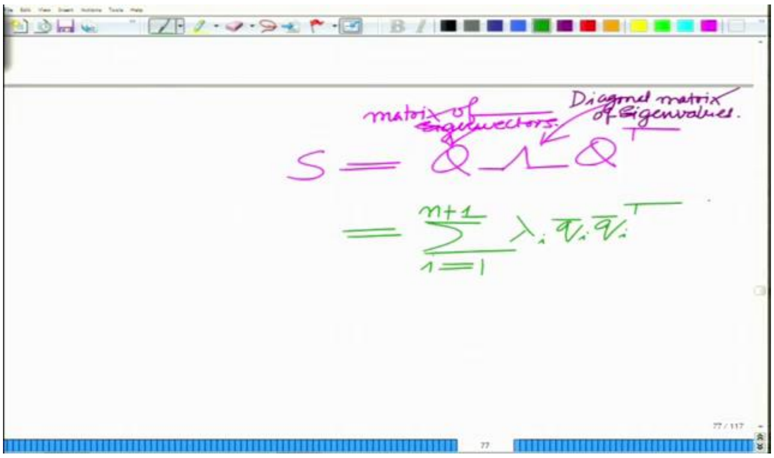

The Eigenvalue decomposition is as follows, we can write S as $S = Q \Lambda Q^T$ where Q is the matrix of eigenvectors and Λ is the diagonal matrix of Eigenvalues and this is then proceeded as shown in slide.

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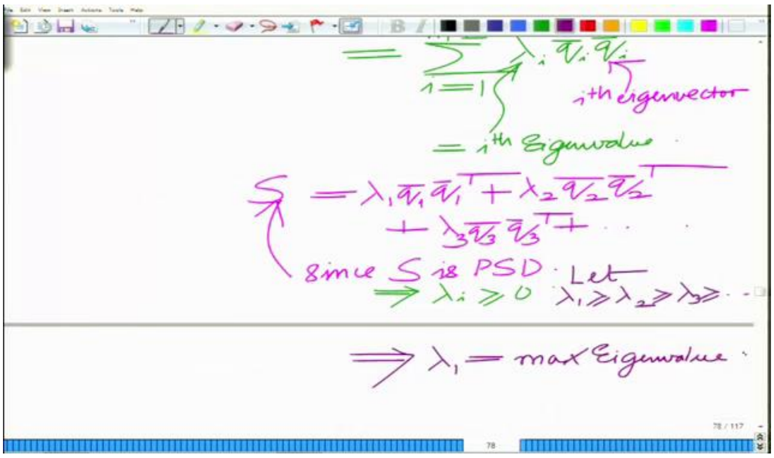

Now, since S is positive semi definite, note that $\lambda_i \geq 0$. So I can always arrange them in decreasing order.

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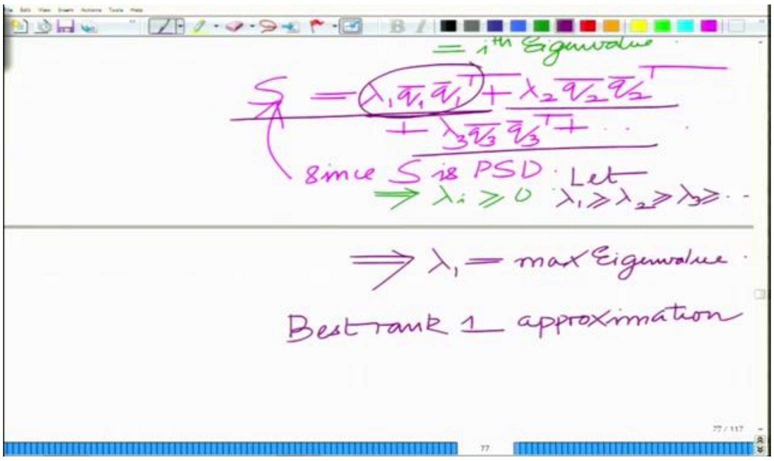

Then, the best rank – 1 approximation is simply choose S equal to the largest Eigenvalue.

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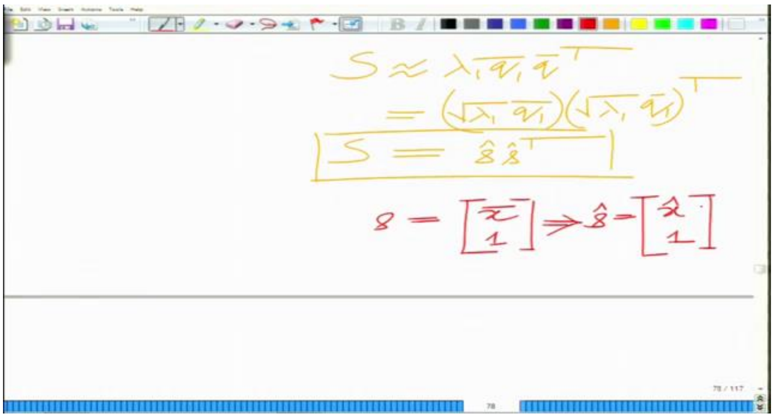

So this is as shown in slide and the final step is $\hat{s} = \begin{bmatrix} x \\ 1 \end{bmatrix}$, so by choosing the first t symbols of $\hat{s}$ you get transmitted symbols.

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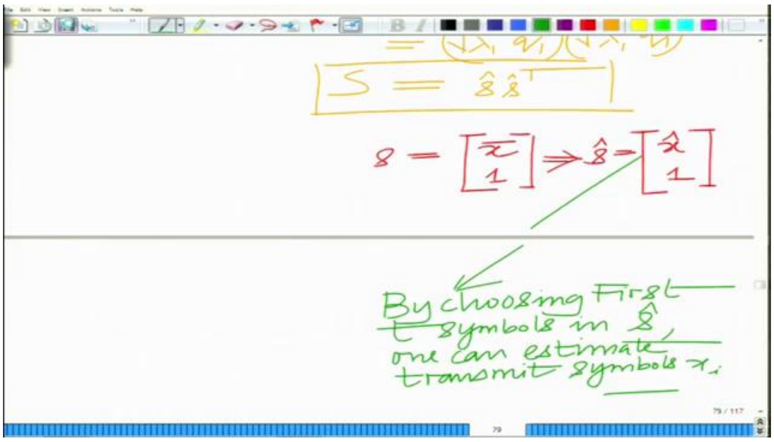

So you take the original ML decoder, recast it in a different form and then you relax the rank 1 constraint that makes it a semi definite program, this process is known as SDP relaxation. From the SDP relaxation you get S which is a positive semi definite matrix, from that you perform the Eigenvalue decomposition thereby getting the best rank 1 approximation, so that will be nothing but the principle Eigenvector of S and from that you take the top t symbols. So let us stop here and continue in the subsequent modules. Thank you very much.