Applied Optimization for Wireless, Machine Learning, Big Data

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Lecture – 43
Practical Application: Multi antenna channel estimation

Keywords: *Multi antenna channel estimation*

Hello, welcome to another module in this massive open online course. So we have looked at several aspects including the intuition behind this least squares, let us look at some practical applications of this least squares.

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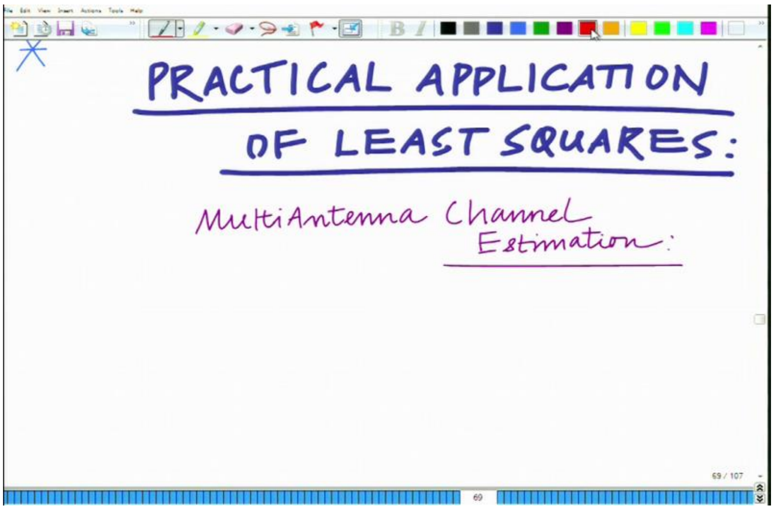

Let us consider the first problem which is related to wireless communication that is the problem of multi antenna channel estimation.

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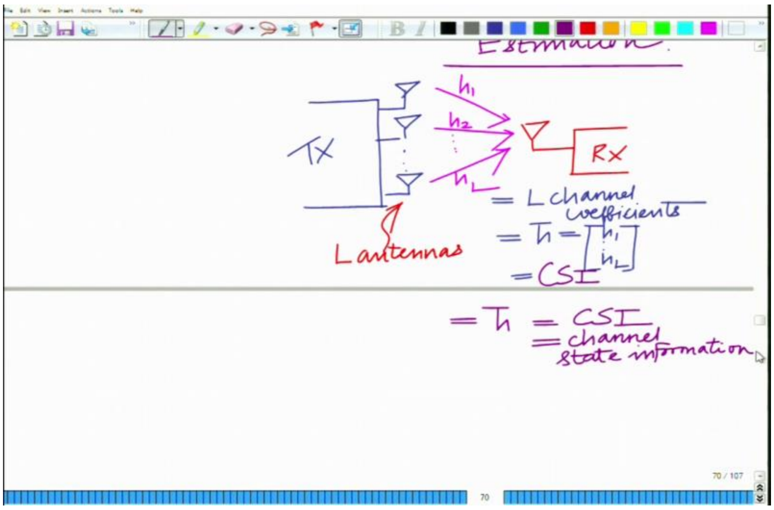

Now let us consider a system with multiple transmit antennas, let us say you have L antennas at the transmitter and for simplicity let us say you have a single receive antenna. Now, there are L channel coefficients h_1, h_2 upto h_L. Now the channel state information or CSI is unknown and has to be estimated. So this channel vector $\bar{h}$ which basically constitutes the channel state information has to be estimated.

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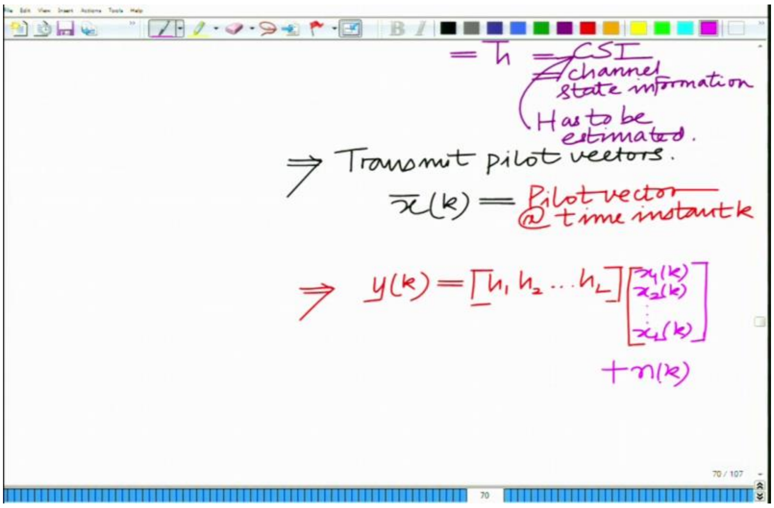

Now, to estimate the channel state information we transmit pilot symbols. Now, these are symbols that are known at the receiver. So let us say $\bar{x}(k)$ is the kth pilot vector. So we have the receive symbol at time instant k equals the channel coefficients times the pilot symbols along with the noise samples.

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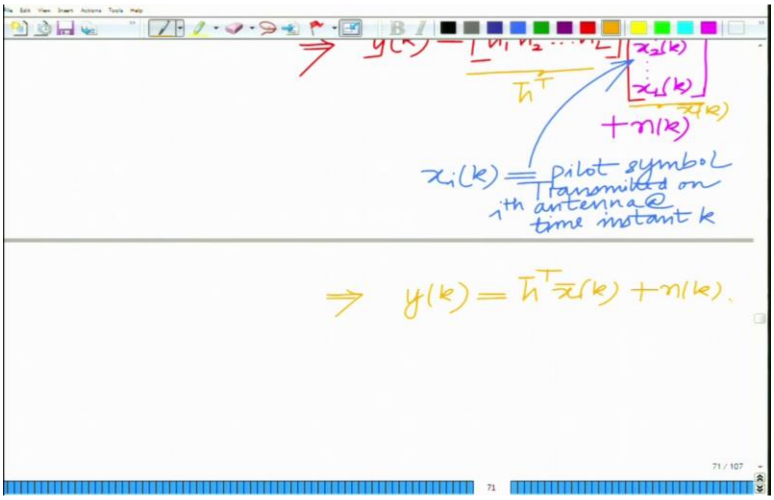

So therefore this implies $y(k) = \bar{h}^{T}\bar{x}(k) + \mathrm{n}(\mathrm{k}) = \bar{x}^{T}(k)\bar{\mathrm{h}} + n(k)$.

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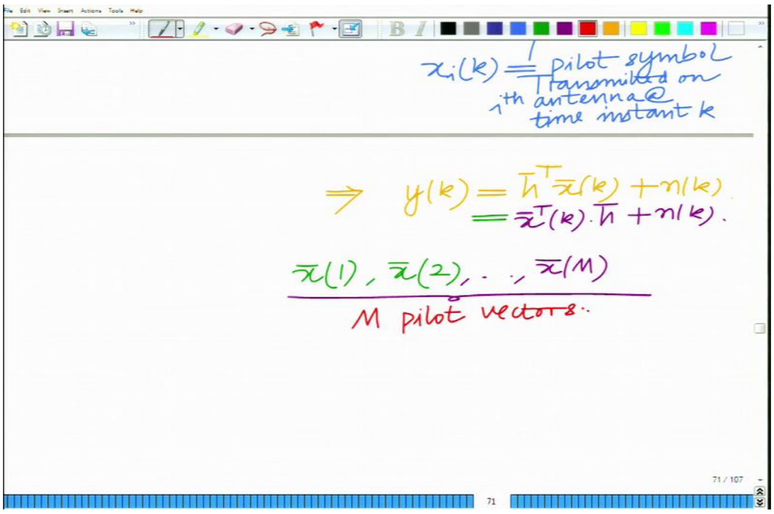

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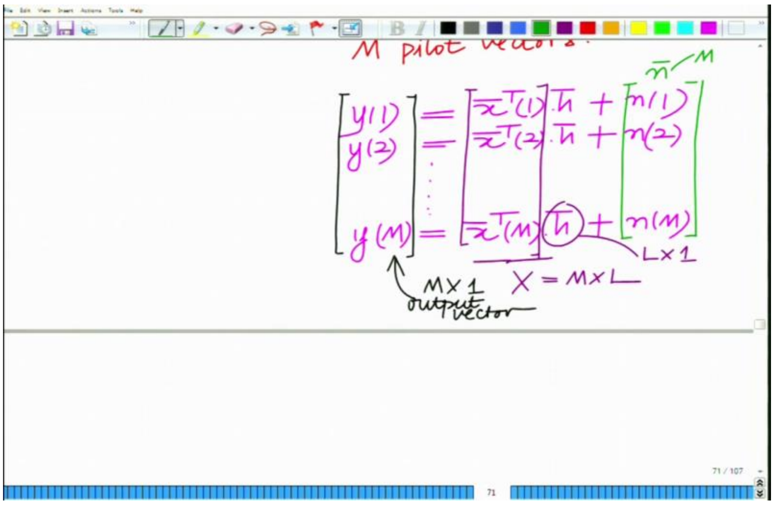

Consider the transmission of M pilot vectors, the equivalent system can be written as shown in slide. We can call this as the pilot matrix which has M rows each of size L, so this pilot matrix X will be of size $M \times L$, $\bar{h}$ is a channel vector which is of size $L \times 1$ and this noise elements will be $\bar{n}$ of size $M \times 1$.

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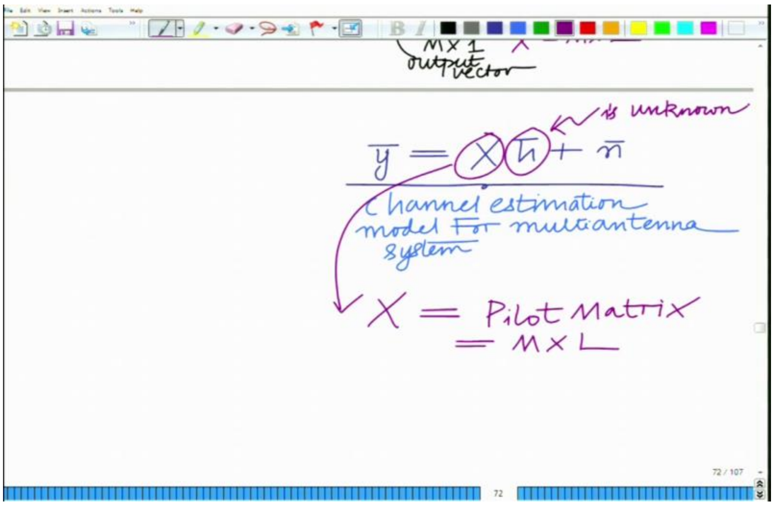

So this is the model for channel estimation which is $\bar{y} = X\bar{h} + \bar{n}$, in fact, this is the channel estimation model for the multi antenna system. So this X is the pilot matrix of size $M \times L$, this vector $\bar{h}$ represents the CSI and is unknown and this is the noisy observation model. So you have to look at the best vector $\bar{h}$ which explains the observed vector $\bar{y}$ corresponding to the transmitted pilot symbols in the matrix X.

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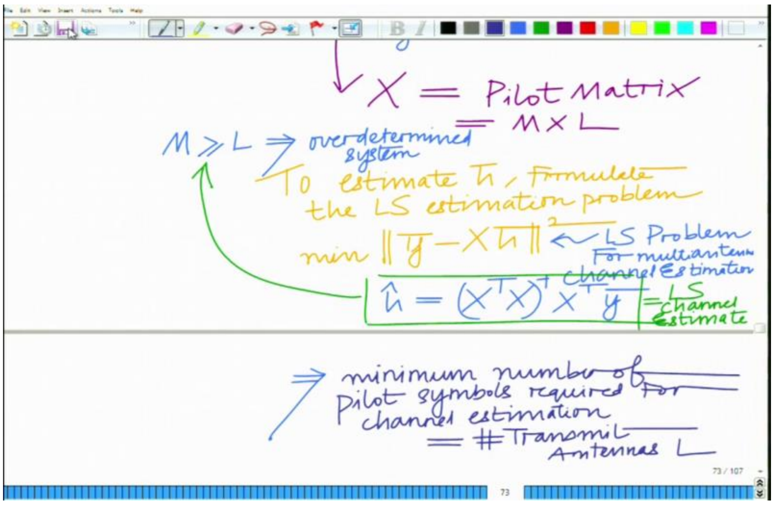

So to estimate $\bar{h}$ formulate the least square estimation problem, that is $\min \left\| \bar{y} - X\bar{h} \right\|^2$. So this is the LS problem for the multi antenna channel estimation. The least square solution is $h = \left(X^T X\right)^{-1} X^T \bar{y}$ and this is also known as the least squares channel estimate. The other thing we are assuming is that this is an over determined system where the number of pilot symbols, M is greater than or equal to the number of transmit antennas, L. So you can also say that for channel estimation using the least square technique the number of pilot symbols required is at least equal to that of the number of transmit antenna.

So this is one of the most popular applications of the least squares solution especially in the context of signal processing for a practical wireless communication system. So we will stop here and look at other application is the subsequent modules. Thank you very much.