Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 47
Convex Optimization Problem representation: Canonical form, Epigraph form

Keywords: *Canonical form, Epigraph form*

Welcome to another module in this massive open online course. So we are going to set out the formulation of a Convex Optimization Problem.

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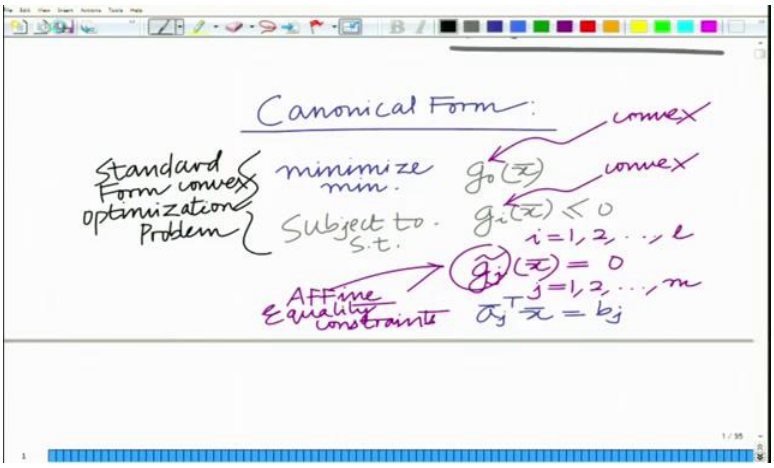

So this can be thought of as a canonical form or the standard form of a convex optimization problem and the convex optimization problem can be stated as follows, that is you minimise an objective function, can be objective function of a vector or can be objective function of a scalar, so we have it as follows

$$\begin{aligned} &\min\; g_0(\bar{x}) \\ &\text{s.t}\; g_i(\bar{x}) \leq 0 \\ &\quad i = 1, 2, \ldots, l \\ &\quad g_j(\bar{x}) = 0 \\ &\quad j = 1, 2, \ldots, m \end{aligned}$$

. And now this objective function has to be convex and these inequality constraints are also convex and these equality constraints have to be affine which basically implies that they are hyperplane. So this is a standard form of a convex optimization problem.

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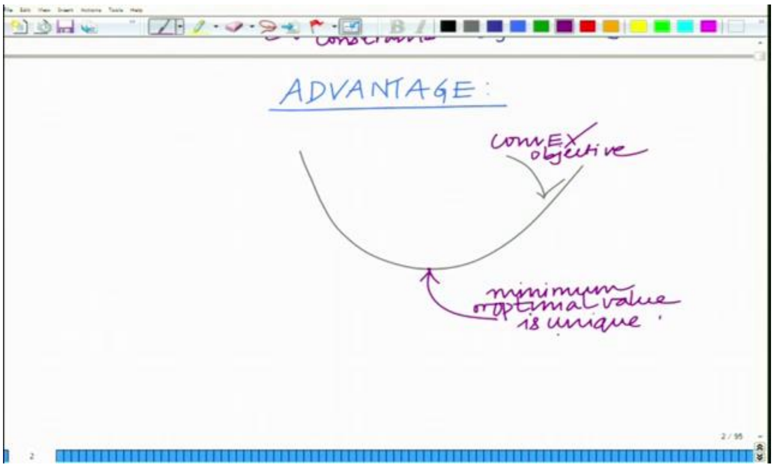

The advantage of a convex optimization problem is that when you look at a convex objective function and you minimise it, the optimum value is unique, the minimizer need not be unique.

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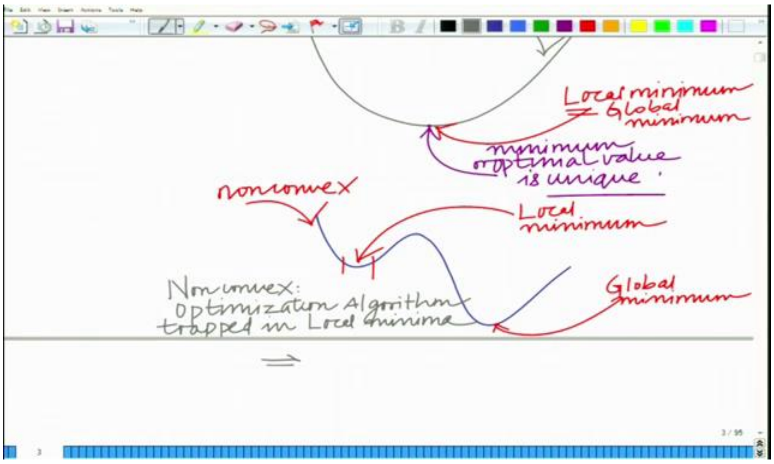

So you have the concept of a global minimum and the local minimum when the objective is non-convex. However, here any local minimum is the global minimum. So that is the advantage of convex optimization. Here in non-convex there can be many local minima and only one global minimum. So the problem is that the optimization algorithms that you employ can get trapped in local minima and they can yield spurious solutions which are not actually the minimum of the objective function.

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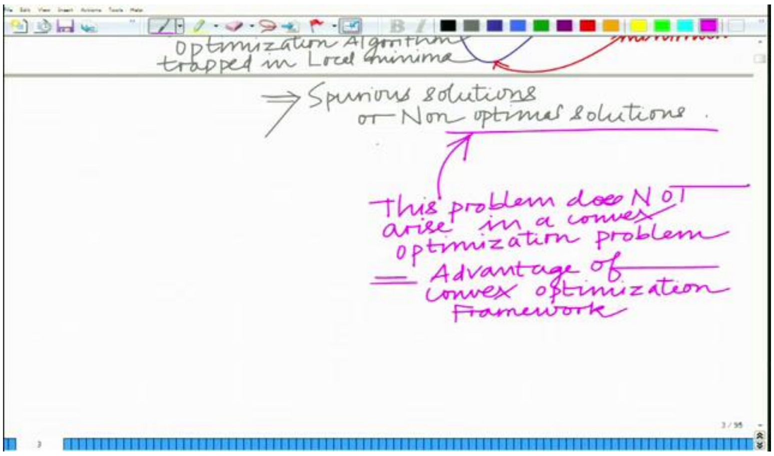

So this problem of spurious minima or getting trapped in local minima is entirely avoided by a convex optimization problem.

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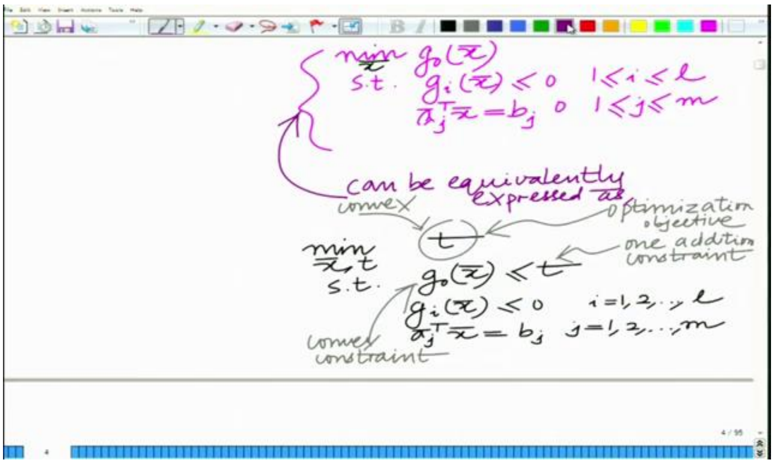

Now a convenient reformulation of a convex optimization is known as the epigraph form. So a convex optimization problem can be recast in epigraph form given as follows.

So we have $\begin{aligned} &\min\ g_0(\bar{x}) \\ \text{s.t}\ & g_i(\bar{x}) \leq 0 \\ & i = 1, 2, \ldots, l \\ & \bar{a}^T_j(\bar{x}) = b_j \\ & j = 1, 2, \ldots, m \end{aligned}$ and this can be written in epigraph form as $\begin{aligned} &\min\ t \\ & g_0(\bar{x}) \leq t \\ \text{s.t}\ & g_i(\bar{x}) \leq 0 \\ & i = 1, 2, \ldots, l \\ & \bar{a}^T_j(\bar{x}) = b_j \\ & j = 1, 2, \ldots, m \end{aligned}$.

So this is a convex constraint and therefore, this is still a convex optimization problem which is a very simple and elegant reformulation that simplifies many complex convex optimization problems.

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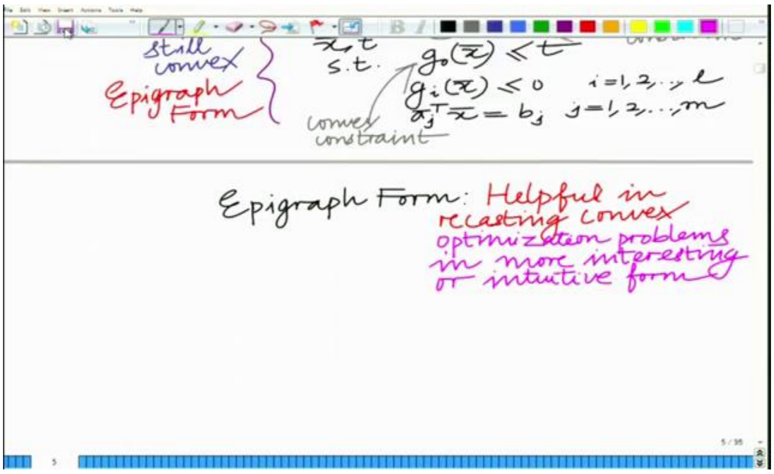

So it yields the same solution, however in the second problem you are optimising both with respect to $\bar{x}$ and t and this is termed as the epigraph form.

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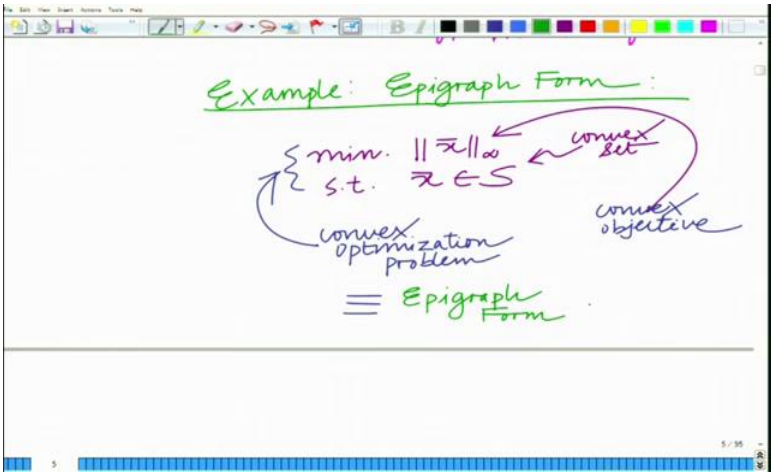

Now let us look at a simple example to understand this better, so let us consider this problem $\begin{array}{l}\min \left\|\bar{x}\right\|_{\infty} \\ s.t\ \ \bar{x} \in S\end{array}$ so this stands for the infinity norm and it is a combination of linear and affine constraints. So the constraints are convex and this is a convex norm, so the objective is a convex objective ok and therefore this is in fact, a convex optimization problem. Now we want to formulate the equivalent epigraph form and that can be derived as shown in slide.

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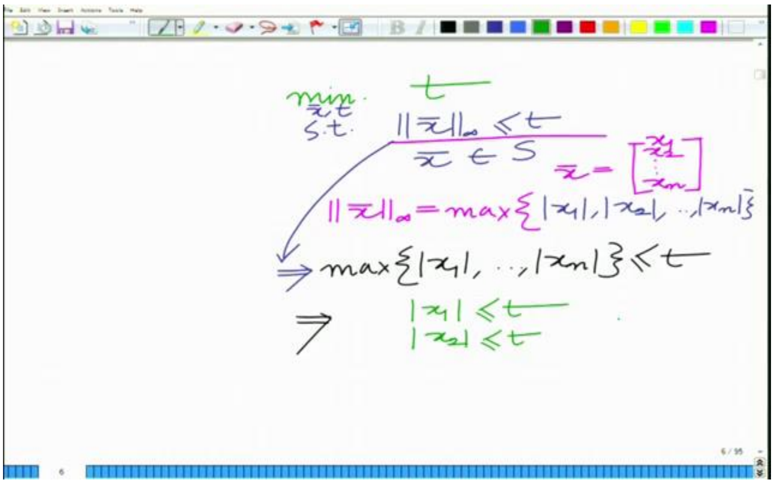

So this can be written as

$$\begin{aligned} &\min t \\ &s.t \; \|\bar{x}\|_\infty \leq t \\ &\quad x \in S \end{aligned}$$

and this is the epigraph form. This can be also modified as shown in slide.

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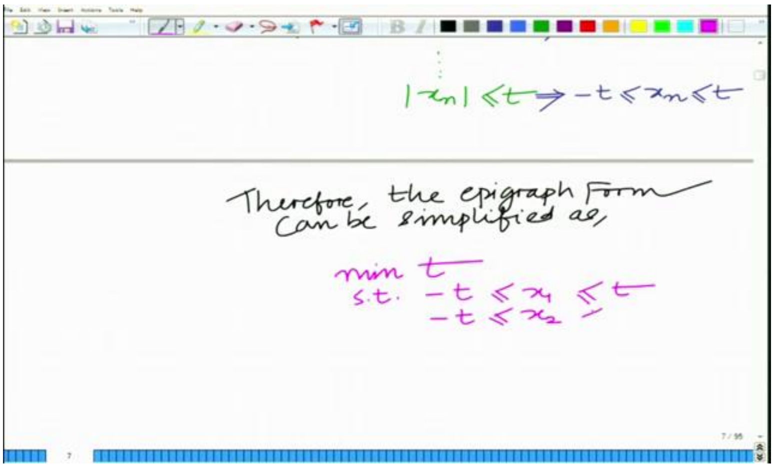

Therefore the epigraph form can be simplified as shown in slide.

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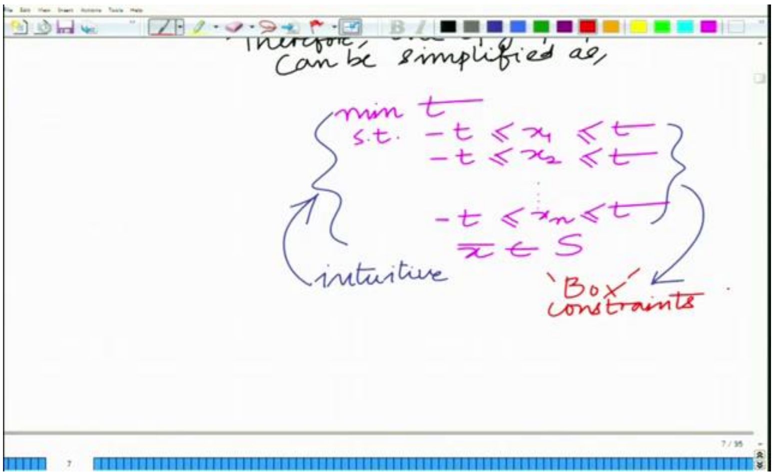

And these are some sort of box constraints and you can think of $\bar{x}$ to lie in a box of dimensions 2t.

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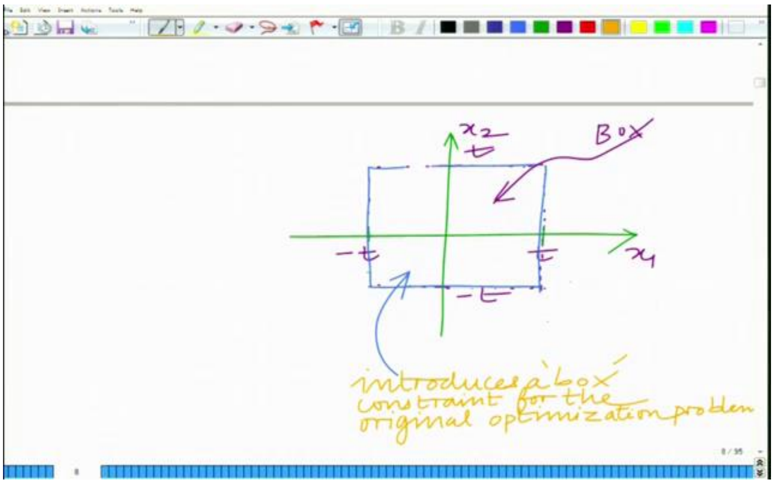

So it sort of introduces a box constraint for the original optimization problem. So this modified optimization problem is easy to interpret and analyse. So we will stop here and continue in the subsequent modules. Thank you very much.