Applied Optimization for Wireless, Machine Learing, Big data

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Lecture - 63
Concept of Duality

Keywords: *Duality*

Hello, welcome to another module in this massive open online course. So we are looking at different topics and concepts in convex optimization and particularly from an applied perspective. In this module, let us start with a new topic and that is Duality.

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So what this does is it formalizes the framework of Lagrange multipliers. So recall a standard form optimization problem given as follows.

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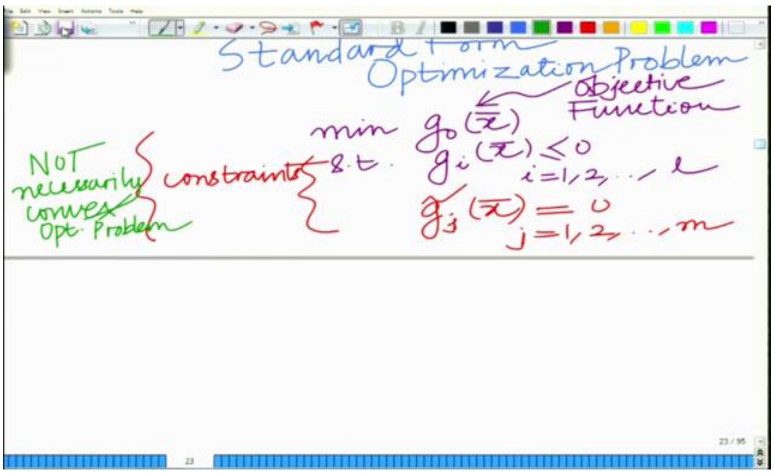

So we have $\begin{array}{ll} \min & g_0(\bar{x}) \\ & g_i(\bar{x}) \le 0 \\ \text{s.t} & i = 1, 2, .., l \\ & g_j(\bar{x}) = 0 \\ & j = 1, 2, .., m \end{array}$. And we have seen that the objective function is convex, the inequality constraints are convex and the equality constraints are affine so it becomes a convex optimization problem.

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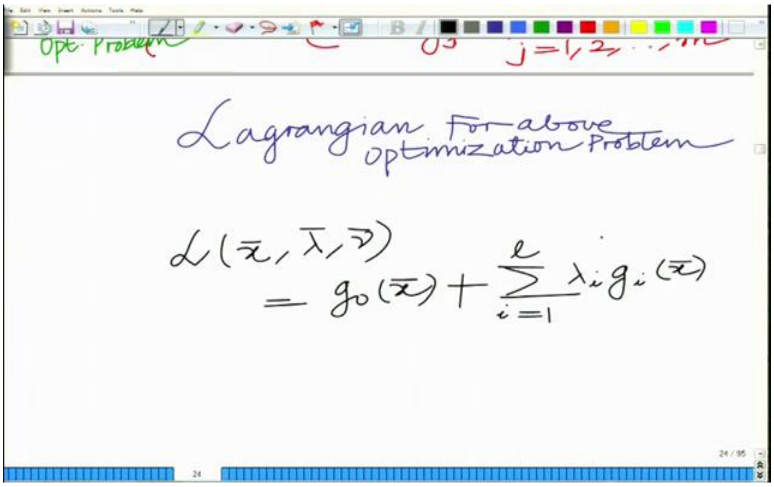

Now for this optimization problem, the Lagrangian function can be formulated as $L\left(\bar{x}, \bar{\lambda}, \bar{\nu}\right) = g_0(\bar{x}) + \sum_{i=1}^{l} \lambda_i g_i(\bar{x}) + \sum_{j=1}^{m} \nu_j g_j(\bar{x})$.

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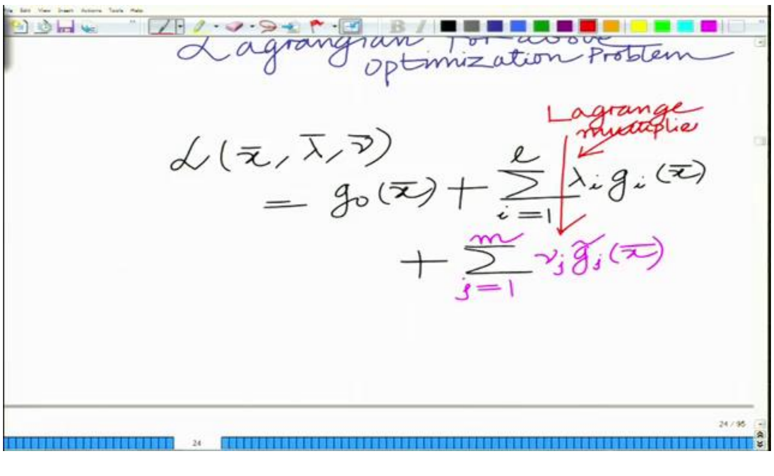

Now these quantities are the Lagrange multipliers.

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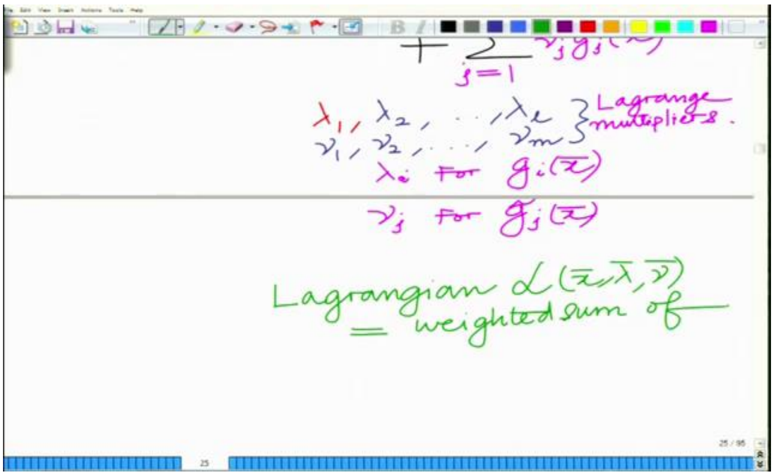

So this is a weighted sum of the objective function and the constraints and the weights are basically the Lagrange multipliers.

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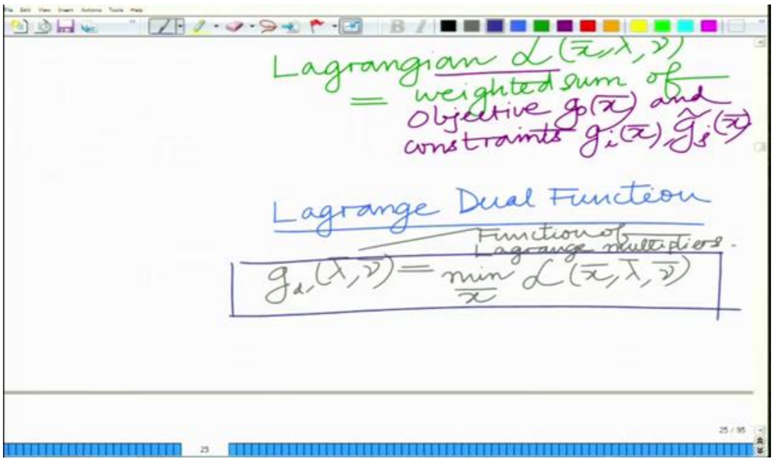

Now, the Lagrange dual function is $g_d(\bar{\lambda},\bar{\nu}) = \min_{x} L\left(\bar{x},\bar{\lambda},\bar{\nu}\right)$ so this is a function of the Lagrange multipliers.

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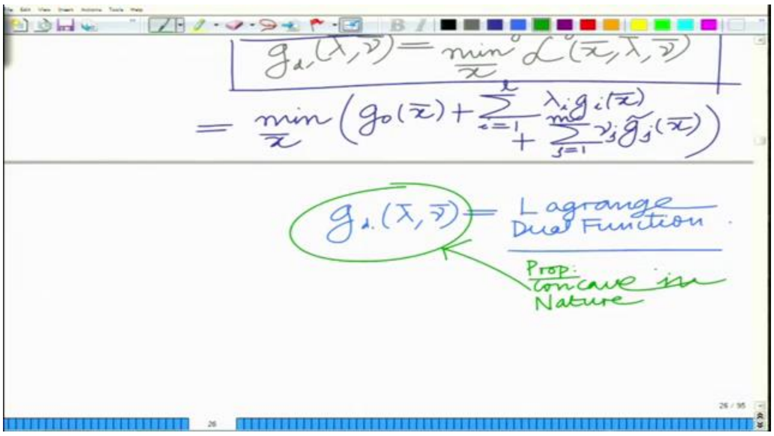

So this can be again written as $\min_{\bar{x}} \left(g_0(\bar{x}) + \sum_{i=1}^{l} \lambda_i g_i(\bar{x}) + \sum_{j=1}^{m} \nu_j g_j(\bar{x}) \right)$. Now it is important to remember that we have started with the standard form optimization problem which is not necessarily convex. This Lagrangian dual function has a very interesting property that is this can be shown to be concave in nature, irrespective of the original optimization problem which need not be convex.

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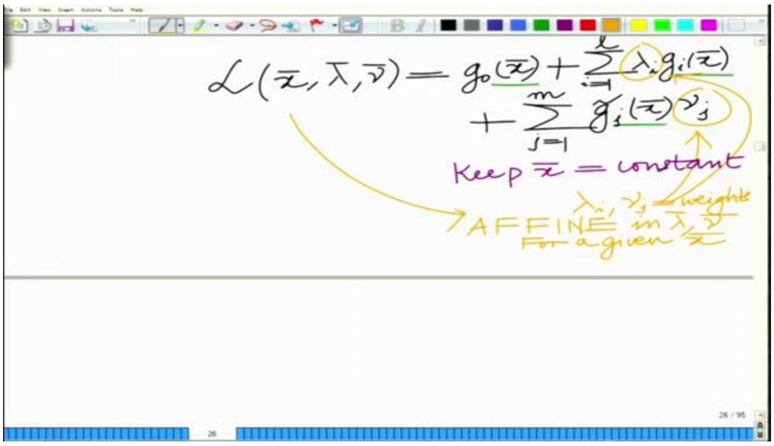

And if you closely observe this function you can observe that even though this is a complicated function of $\bar{x}$, this is affine in the Lagrange multipliers which are nothing but the weights.

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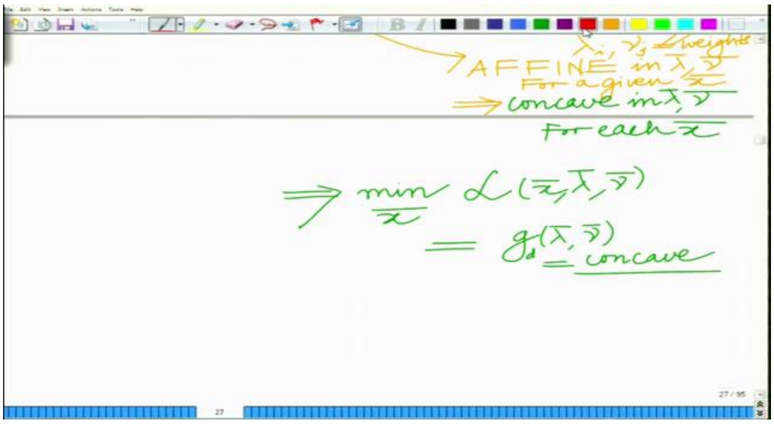

So this is affine in the sense that this is a hyper plane and therefore, this is a concave function. And what is the dual doing, this is taking the minimum over $\bar{x}$. So this is concave.

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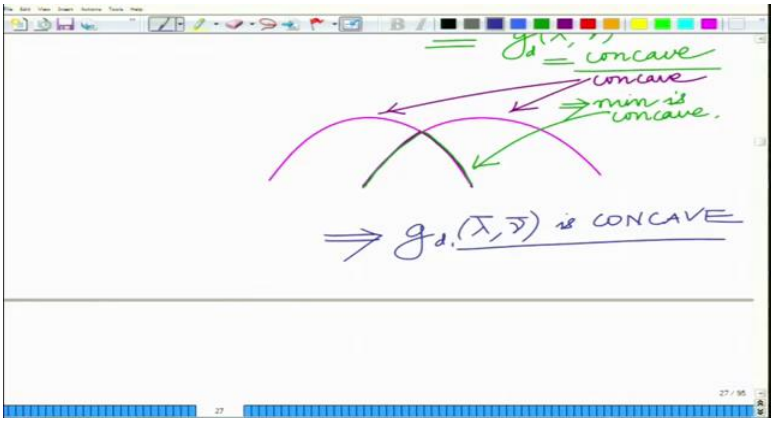

So this Lagrangian dual function is a concave function. So even when the original problem is not necessarily convex one can convert a standard, possibly non convex optimization problem into an equivalent concave optimization problem. So that is the power of the duality framework. So in fact, can use this to simplify several possibly non convex optimization problems, as we are going to see subsequently. Thank you very much.