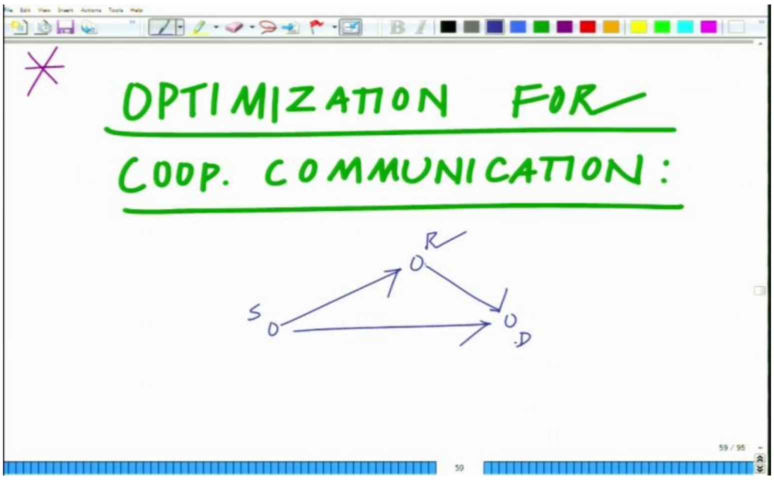
Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 54
Practical Application: Optimal power allocation factor determination for Co-operative Communication

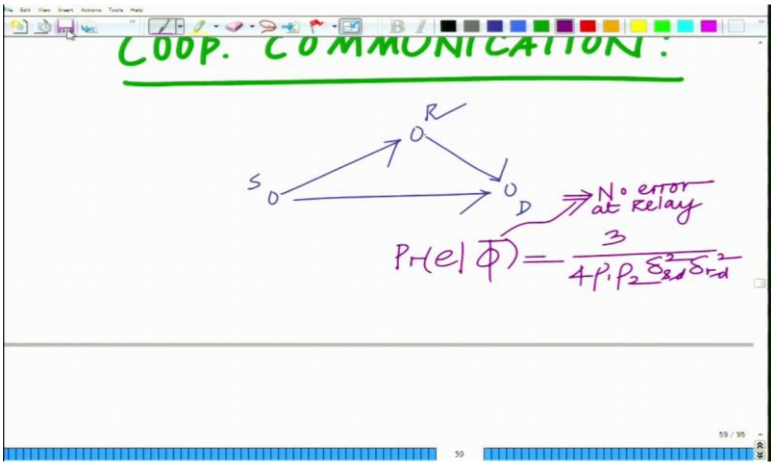
***Keywords**: Co-operative Communication, Optimal power allocation factor*

Hello, welcome to another module in this massive open online course. So we are looking at optimization for co-operative communication.

(Refer Slide Time: 00:25)

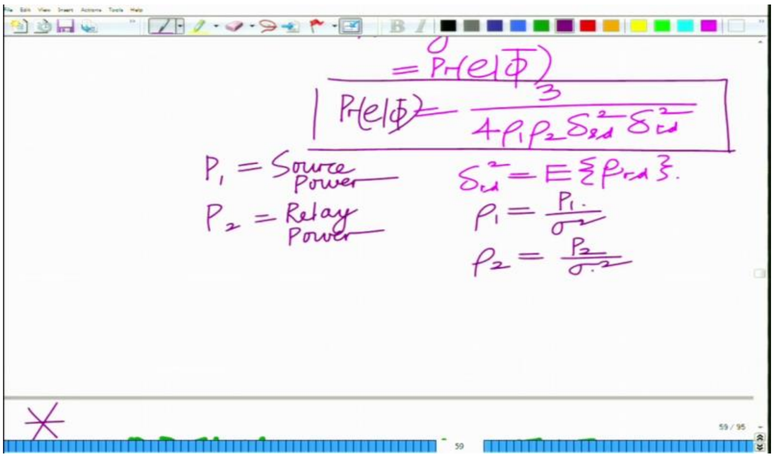


(Refer Slide Time: 01:13)



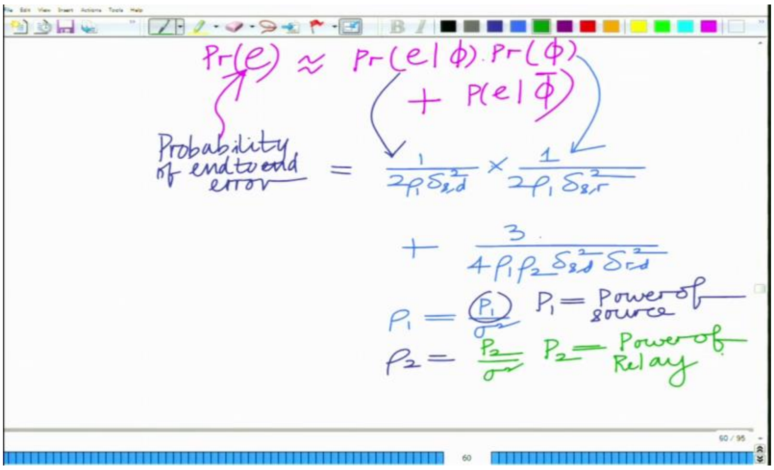
So we have derived the expression for the probability of error at destination given there is no error at the relay.

(Refer Slide Time: 01:21)



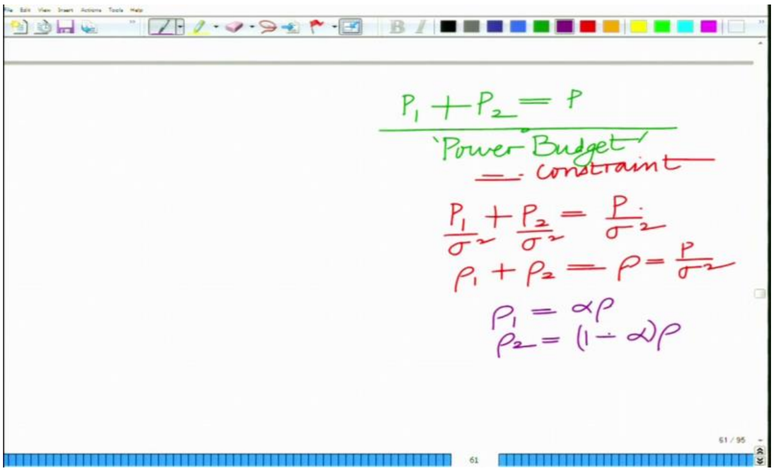
This is given as $Pr(e/\bar{\phi}) = \dfrac{3}{4\rho_1\rho_2\delta_{sd}^2\delta_{rd}^2}$.

(Refer Slide Time: 02:00)



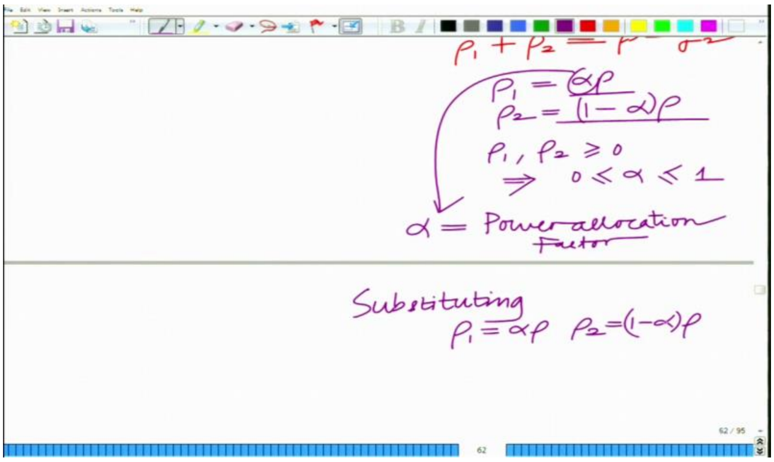
We already know the expression for the end to end error and this is given as $Pr(e) \approx Pr(e/\phi)\,Pr(\phi) + Pr(e/\bar{\phi})$. We said this is a good approximation which is tight at high SNR. Now we are going to substitute these expressions to find the end to end error. This is as shown in slide. And now we want to make an optimization problem, where we want to minimize the bit error rate, the end to end rate and this means increasing the power infinitely and when the power becomes infinite, the bit error rate becomes 0. So we will impose a power budget on this corporative communication system which is that the power of the source and the power of the relay is a constant.

(Refer Slide Time: 05:18)



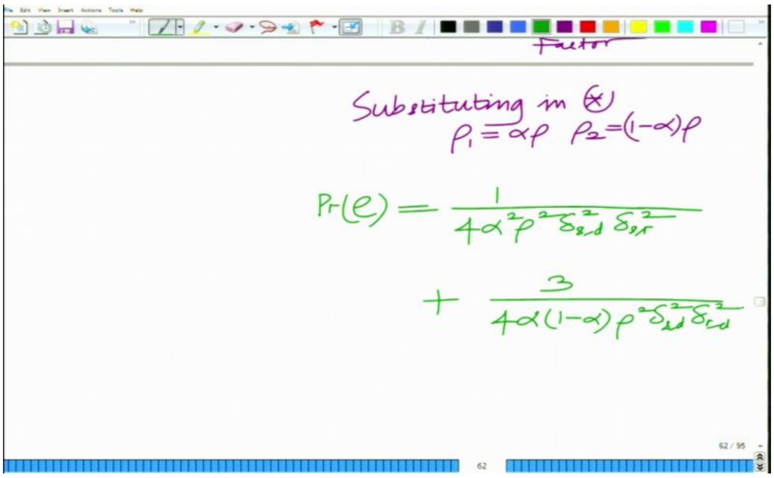
So this is given as shown in these slides.

(Refer Slide Time: 06:25)



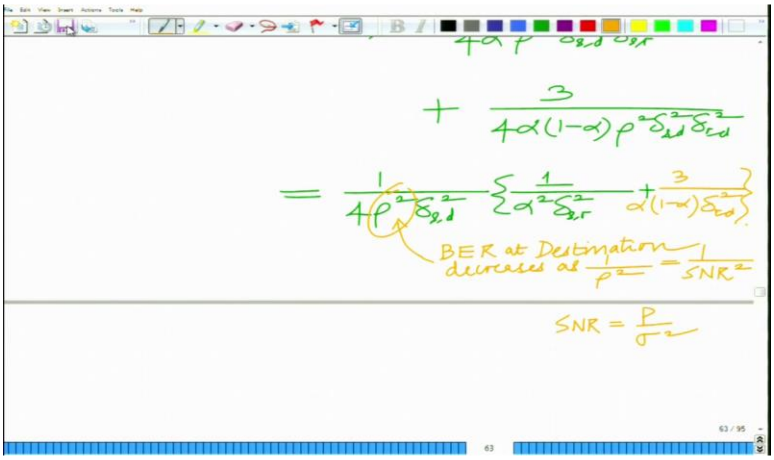
Now we can think of this α as the power allocation factor which lies between 0 and 1. Now, substituting these values we get the probability of end to end error.

(Refer Slide Time: 07:27)



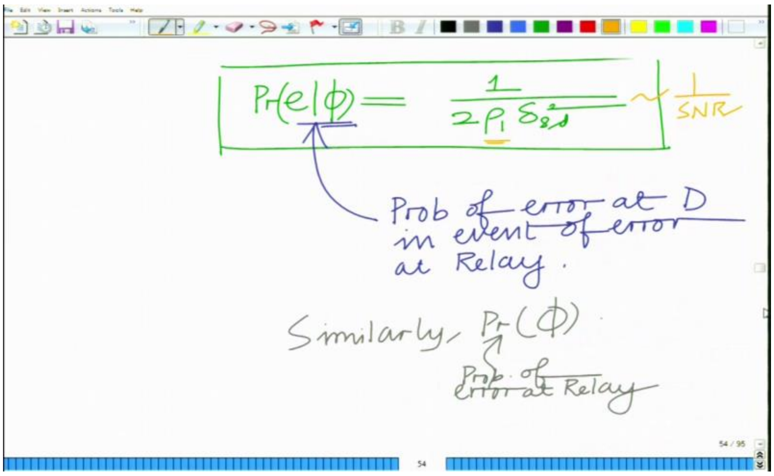
So the probability of end to end error is given as $Pr(e) = \frac{1}{4\alpha^2\rho^2\delta_{sd}^2\delta_{sr}^2} + \frac{3}{4\alpha(1-\alpha)\rho^2\delta_{sd}^2\delta_{rd}^2}$.

(Refer Slide Time: 08:15)

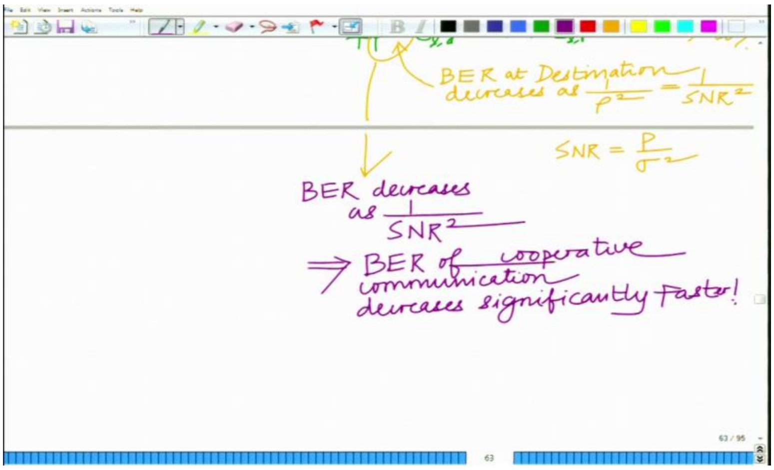


And now if you take this $\frac{1}{4\rho^2\delta_{sd}^2}$ and if we look at this bit error rate expression now, we will notice that the effective end to end bit error rate decreases as $\frac{1}{\rho^2} = \frac{1}{SNR^2}$. So if you look at simply the source destination link, the probability of error decreases as $\frac{1}{\rho_1}$. Therefore, this is known as diversity order 1, which is the exponent of the SNR in the bit error rate expression.

(Refer Slide Time: 10:12)

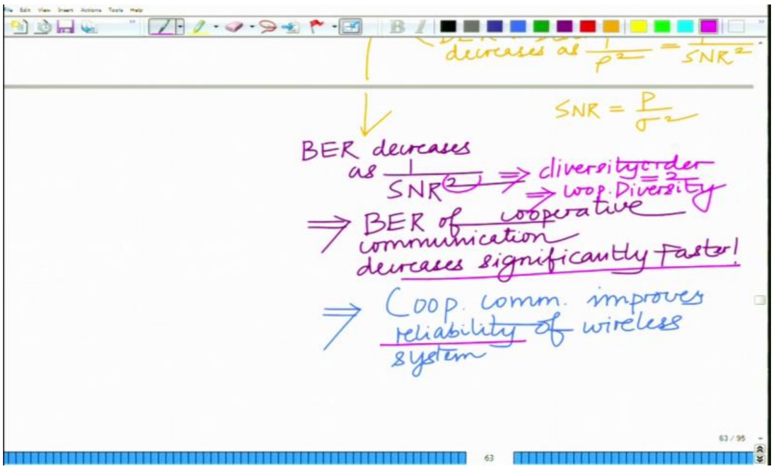


(Refer Slide Time: 10:24)



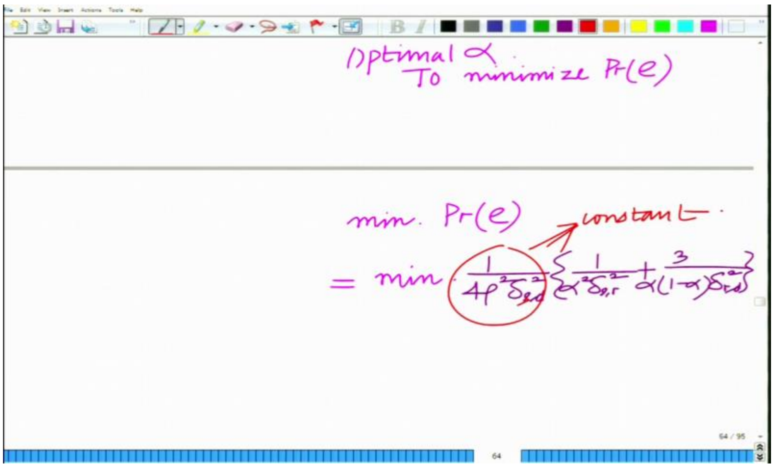
However, once you are adding a relay in this co-operative communication system, the bit error rate in co-operative communication system decreases as $\frac{1}{SNR^2}$ and thus the BER decreases much faster. Thus corporative communication leads to a significant decrease in the bit error rate of a wireless communication system thereby improving the reliability.

(Refer Slide Time: 11:36)



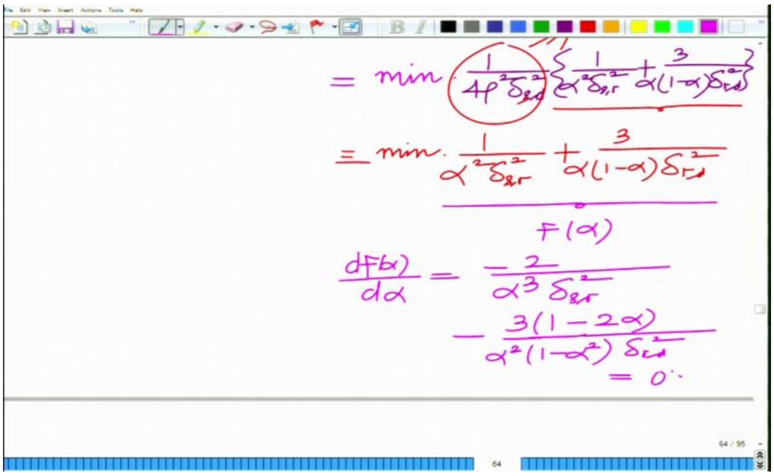
For a co-operative communication system the diversity order is 2 and this is also termed as co-operative diversity. So co-operative diversity helps to improve the reliability for wireless communication system, by making the bit error rate at the destination decrease significantly faster than it would have happened in the presence of only a source destination link.

(Refer Slide Time: 13:00)



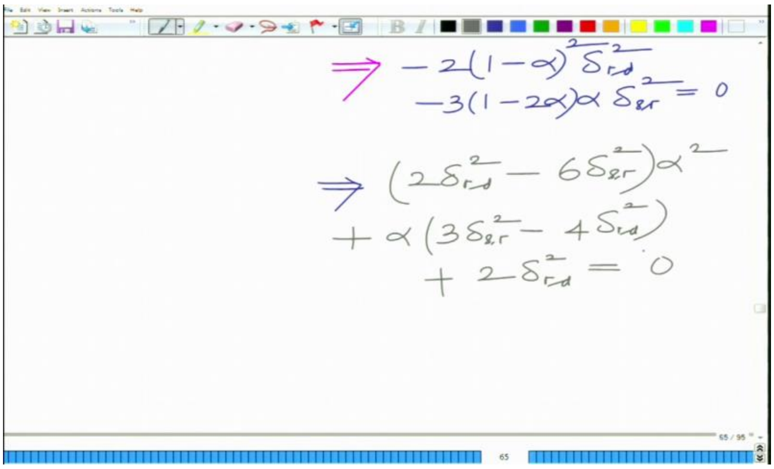
And now we want to find the optimal power allocation factor α to minimize the bit error rate or probability of error. So the optimization problem is $\min \Pr(e) = \min \frac{1}{4\rho^2\delta_{sd}^2}\left\{\frac{1}{\alpha^2\delta_{sr}^2} + \frac{3}{\alpha(1-\alpha)\delta_{rd}^2}\right\}$ and the constraint is now incorporated in α .

(Refer Slide Time: 14:40)

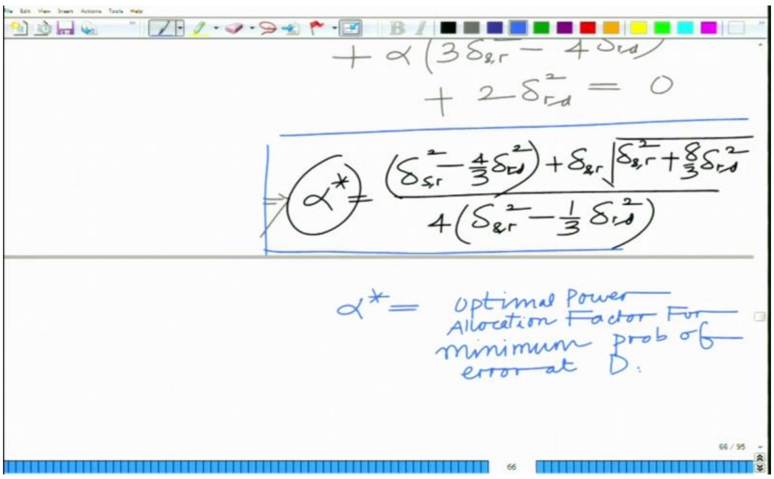


So we need to only minimize the part other than the fixed parameters as shown in slide.

(Refer Slide Time: 15:51)

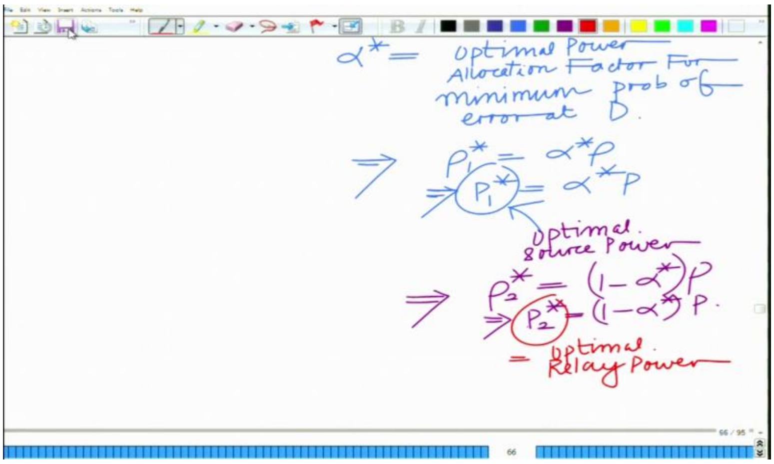


(Refer Slide Time: 17:00)

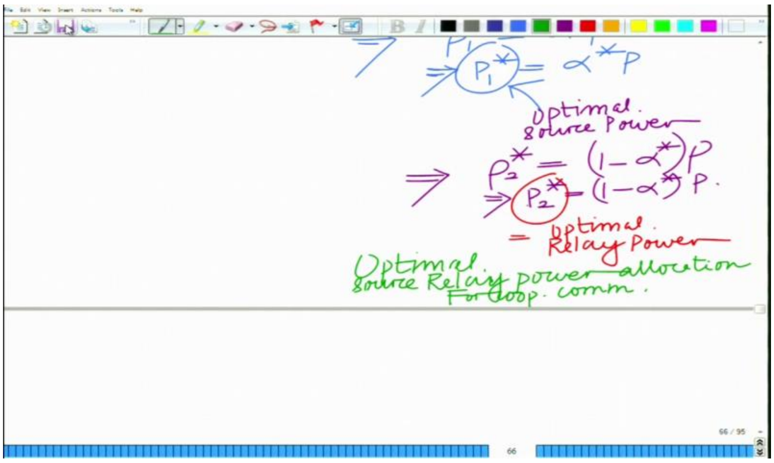


Now on solving this we get the optimal power allocation factor for minimum probability of error at destination and that is given by $\alpha^* = \dfrac{\left(\delta_{sr}^2 - \frac{4}{3}\delta_{rd}^2\right) + \delta_{sr}\sqrt{\delta_{sr}^2 + \frac{8}{3}\delta_{rd}^2}}{4\left(\delta_{sr}^2 - \frac{1}{3}\delta_{rd}^2\right)}$.

(Refer Slide Time: 18:28)



(Refer Slide Time: 19:44)



So this is the optimal source relay power allocation for co-operative communication, optimal in the sense that it minimizes the end-to-end bit error rate. So basically that completes our discussion on this co-operative communication system. And we have shown that because of corporative diversity, the bit error rate decreases and the bit error rate of co-operative communication is significantly lower and therefore, the reliability is significantly higher in comparison to that of having only a source destination link. Thank you very much.