

**Applied Optimization for Wireless, Machine Learning, Big Data**  
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**Lecture - 78**  
**Matrix Completion Problem in Big Data: Netflix-I**

**Keywords:** *Matrix Completion Problem*

Hello, welcome to another module in this massive open online course. So we are looking at an application of Convex Optimization Big Data in particular to the Netflix problem. Let us consider a very simple version of that problem.

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**NETFLIX PROBLEM:**

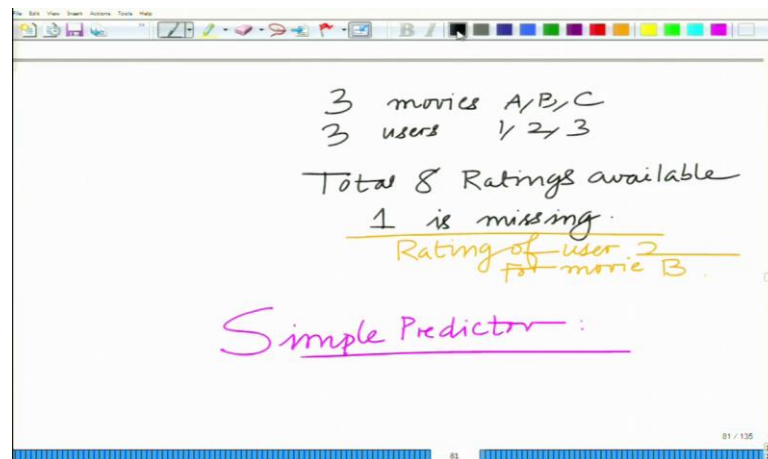
Simple

	A	B	C
1	3	5	3
2	4	?	3
3	2	5	4

missing Matrix completion problem

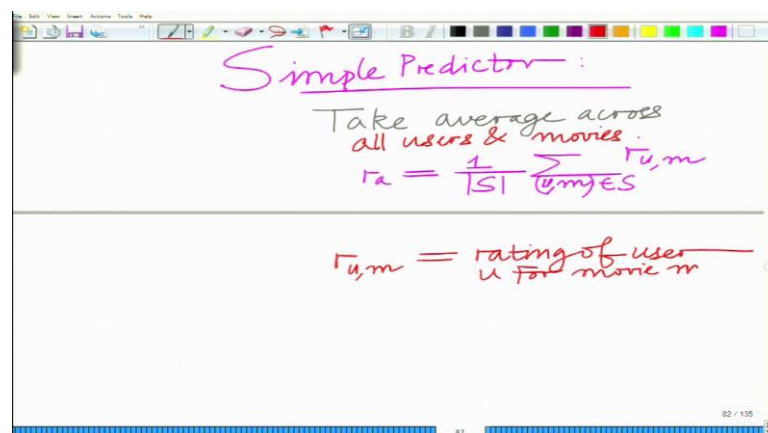
The Netflix problem as I said involves about half a million users and about 20000 movies. We are going to consider an extremely simple version of that which can be generalized. Let us say that you have 3 movies A B C and 3 users 1 2 3, similar to the table that we had seen yesterday. Let us consider again a simple example, where user 1 has rated movies A B C, the ratings are 3, 5, 3. User 3 has also rated movies A B C, ratings are 2, 5, 4. But user 2 has only rated movies A and C and his ratings are 4 and 3. Now, what you can clearly see is that one rating is missing, if you look at this as a matrix, this is missing. And therefore, we have to predict this rating to complete this matrix of users and ratings that is why it is also known as a matrix completion problem.

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There are 3 movies A B C and 3 users. In total 8 ratings are available, one is missing, that is the rating of user 2 for movie B and this which we have to predict. Now let us consider a simple predictor and the best predictor is nothing but the mean that is the average rating of each movie per user.

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So we have rating  $r_a = \frac{1}{|S|} \sum_{(u,m) \in S} r_{u,m}$  where S is the set of all ratings that are available. So

$r_{u,m}$  is the rating of user u for movie m.

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all users & movies

average rating  $\rightarrow r_a = \frac{1}{|S|} \sum_{(u,m) \in S} r_{u,m}$

$r_{u,m}$  = rating of user  $u$  for movie  $m$ .

$S = \{(u,m) \mid \text{user } u \text{ has rated movie } m\}$

$|S|$  = Total # ratings  
= # elements in  $S$ .

$r_a$  = LAZY PREDICTOR

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Now this is also termed as a Lazy Predictor as this is only the average rating.

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$r_a$  = LAZY PREDICTOR

$$r_a = \frac{1}{8} (3 + 5 + 3 + 4 + 3 + 2 + 5 + 4)$$
$$= \frac{29}{8} = 3.625$$

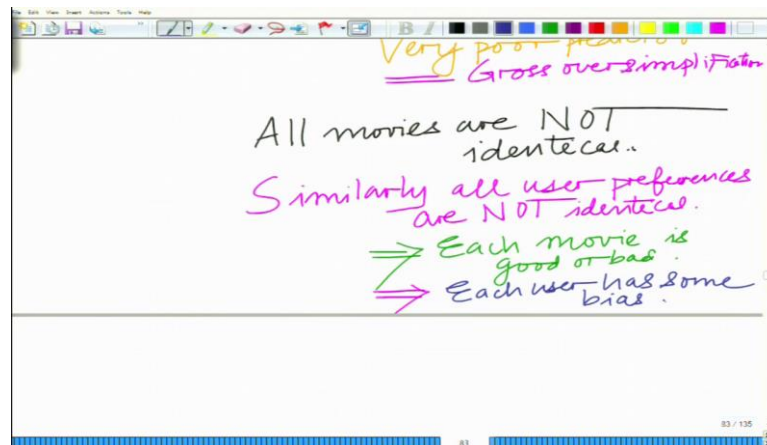
$r_a = 3.625$

very simple  
Very poor predictor  
Gross oversimplification

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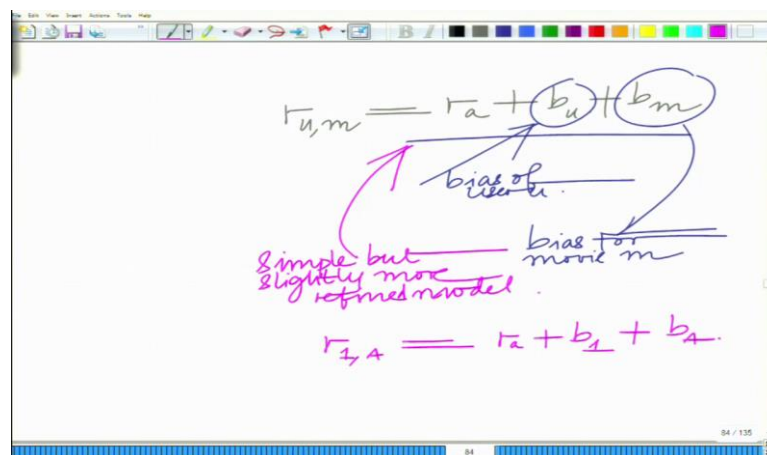
And for this example, the average is  $r_a = 3.625$  as shown in slide. Its performance is going to be very poor because this is a gross oversimplification.

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The reason for that is each movie is unique. So each user has a certain bias, each movie is good, but by and large, a large number of people think some movies are better than some other movies. So some movies are consistently rated the best. Some users are same way more lenient towards their ratings while some users might be very harsh. So to each user each movie is either good or bad.

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So we can model the rating of each user as  $r_{u,m} = r_a + b_u + b_m$  which is the average, the bias of each user and the bias of each movie respectively. So this is a slightly more refined model and probably a more natural model in capturing the behaviour.

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simple but slightly more refined model. bias for movie m

$$3 = r_{1,A} = r_a + b_1 + b_A$$

$r_a = 3.625$

$$3 - 3.625 = b_1 + b_A$$

$$\Rightarrow -0.625 = b_1 + b_A$$

Accordingly, we can have various equations for the above problem as shown in slide.

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$$3 - 3.625 = b_1 + b_A$$

$$\Rightarrow -0.625 = b_1 + b_A$$

$$r_{1,B} = 5 = r_a + b_1 + b_B$$

$$\Rightarrow 5 - 3.625 = b_1 + b_B$$

$$\Rightarrow 1.375 = b_1 + b_B$$

So there are 8 such equations as shown in slide.

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$$\begin{aligned} -0.625 &= b_1 + b_A \\ 1.375 &= b_1 + b_B \\ -0.625 &= b_1 + b_C \\ 0.375 &= b_2 + b_A \\ -0.625 &= b_2 + b_C \\ -1.625 &= b_3 + b_A \\ 1.375 &= b_3 + b_B \\ 0.375 &= b_3 + b_C \end{aligned}$$

system of Linear Equations

In fact, this is a system of linear equations which can be written in the form of a matrix as shown in slide.

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Matrix Form:

$$\begin{bmatrix} -0.625 \\ 1.375 \\ -0.625 \\ 0.375 \\ -0.625 \\ -1.625 \\ 1.375 \\ 0.375 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

$\vec{F} = A \vec{b}$   $\leftarrow 6 \times 1$   
 $8 \times 1 \quad 8 \times 6$

So here we have more equations than unknowns.

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$\vec{F} = A \vec{b}$   $\leftarrow 6 \times 1$   
 $8 \times 1 \quad 8 \times 6$   
 # Equations = 8  
 # unknowns = 6

$\Rightarrow$  more Equations Than unknowns!  
 $\Rightarrow$  overdetermined system

$\| \vec{F} - A \vec{b} \|^2$

We know that this is an over determined system and hence we solve this over determined system by using least squares  $\| \vec{r} - A \vec{b} \|^2$  but we cannot solve this exactly and we have to solve it approximately. So we find the best vector  $\vec{b}$  which minimizes the norm of the approximation error  $\| \vec{r} - A \vec{b} \|^2$ .

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The image shows a digital whiteboard with handwritten mathematical notes. At the top, the least squares problem is defined as  $\min. ||r - AB||^2$ , with "Least Squares Problem" written below it. An arrow points to the equation  $\hat{b} = (A^T A)^{-1} A^T b$ , where  $\hat{b}$  is circled in red. Below this, a linear model is written in red:  $\hat{r}_{2,B} = r_a + b_2 + b_B$ . Above this equation, the parameters  $b_1, b_2, b_3, b_A, b_B, b_C$  are listed. A green arrow points from the text "slightly better" to the model equation. At the bottom, the text "But NOT the best!" is written in green. The whiteboard interface includes a toolbar at the top and a status bar at the bottom showing "85 / 135".

$$\min. ||r - AB||^2$$

Least Squares Problem

$$\hat{b} = (A^T A)^{-1} A^T b$$

$b_1, b_2, b_3, b_A, b_B, b_C$

$$\hat{r}_{2,B} = r_a + b_2 + b_B$$

slightly better

But NOT the best!

So this is your least squares problem implies  $\bar{b} = (A^T A)^{-1} A^T$ . So from this you get  $b_1, b_2, b_3, b_A, b_B, b_C$ . So we have got first stage, we started with the average, we refined it, but we are not done yet. We are going to refine this model further to predict it as closely as possible and that we are going to do in the subsequent module. Thank you very much.