

1. Given a robust LP with the constraint $\Pr(\bar{\mathbf{c}}_i^T \bar{\mathbf{x}} \leq d_i) \geq \eta$, where \mathbf{c}_i is a Gaussian random vector with mean $\bar{\boldsymbol{\mu}}_i$ and covariance matrix $\mathbf{R}_i = \tilde{\mathbf{R}}_i \tilde{\mathbf{R}}_i^T$. This constraint can be equivalently represented as

$$\|\tilde{\mathbf{R}}_i^T \bar{\mathbf{x}}\| \leq \frac{(d_i - \bar{\mathbf{x}}_i^T \bar{\boldsymbol{\mu}}_i)}{Q^{-1}(1 - \eta)}$$

Ans c

2. The epigraph form of the optimization problem is

$$\begin{aligned} \min t \\ \text{s.t. } g_0(\bar{\mathbf{x}}) &\leq t \\ g_i(\bar{\mathbf{x}}) &\leq 0, i = 1, 2, \dots, l \\ \mathbf{a}_j^T \bar{\mathbf{x}} &= b_j, j = 1, 2, \dots, m \end{aligned}$$

Ans b

3. The given optimization problem $\min \|\mathbf{A}\bar{\mathbf{x}} - \bar{\mathbf{b}}\|_\infty$ can be formulated as the LP below

$$\begin{aligned} \min t \\ -t\bar{\mathbf{1}} \preceq \mathbf{A}\bar{\mathbf{x}} - \bar{\mathbf{b}} \preceq t\bar{\mathbf{1}} \end{aligned}$$

Ans b

4. Given a MIMO communication system with channel matrix \mathbf{H} . The optimal transmit and receive beamformers, respectively, for maximum SNR at the receiver, are principal eigenvectors of $\mathbf{H}^H \mathbf{H}$ and $\mathbf{H} \mathbf{H}^H$

Ans b

5. Given the vectors $\bar{\mathbf{u}} = [1 \ 1 \ 1]^T$, $\bar{\mathbf{v}} = [1 \ 2 \ 3]^T$. The set of orthonormal vectors spanning the same subspace is $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. This can be seen as follows

$$\bar{\mathbf{v}} - \frac{\bar{\mathbf{u}} \bar{\mathbf{u}}^T}{\|\bar{\mathbf{u}}\| \|\bar{\mathbf{u}}\|} \bar{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{3}} \times 6 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Corresponding unit-norm vector is $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Ans c

6. Given matrix $\mathbf{A} = 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} [1 \ 1] + 3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} [1 \ -1]$. It can be seen that \mathbf{A} is PSD, since it is of the form $\sum_{i=1}^n \mathbf{a}_i \mathbf{a}_i^T$. Further, since $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are orthogonal, these are the eigenvectors. Corresponding eigenvalues are 4, 6. Hence, solution to the optimization problem is the maximum eigenvalue 6

Ans d

7. The minimum value of $\bar{\mathbf{x}}^T \mathbf{A} \bar{\mathbf{x}}$ for $\|\bar{\mathbf{x}}\| = 1$ is given by the minimum eigenvalue = 4.

Ans c

8. Given the noise vector $\bar{\mathbf{n}} = [n_1 \ n_2 \ \dots \ n_L]^T$, with its elements i.i.d. zero-mean Gaussian that have variance σ^2 . Let $\bar{\mathbf{w}}$ denote a weight vector. The quantity $\bar{\mathbf{w}}^T \bar{\mathbf{n}}$ has variance $\sigma^2 \|\bar{\mathbf{w}}\|^2$

Ans b

9. Given the MIMO channel matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The largest eigenvalue is $\mathbf{H}^T \mathbf{H}, \mathbf{H} \mathbf{H}^T$ are 4. The corresponding principal eigenvectors

are $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$, respectively, which are the optimal transmit and receive beamforming

vectors

Ans d

10. For the compressive sensing problem shown, the possible values of k for the l_k norm are $k = 0, 1$

Ans b