Applied Optimization for Wireless, Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 39
Practical Application: Robust Beamformer Design for Wireless Systems

Hello. Welcome to another module in this massive open online course. So, we are looking at Robust Beam forming as an application of convex optimization. The optimization problem for robust beam forming in multiple antenna system is

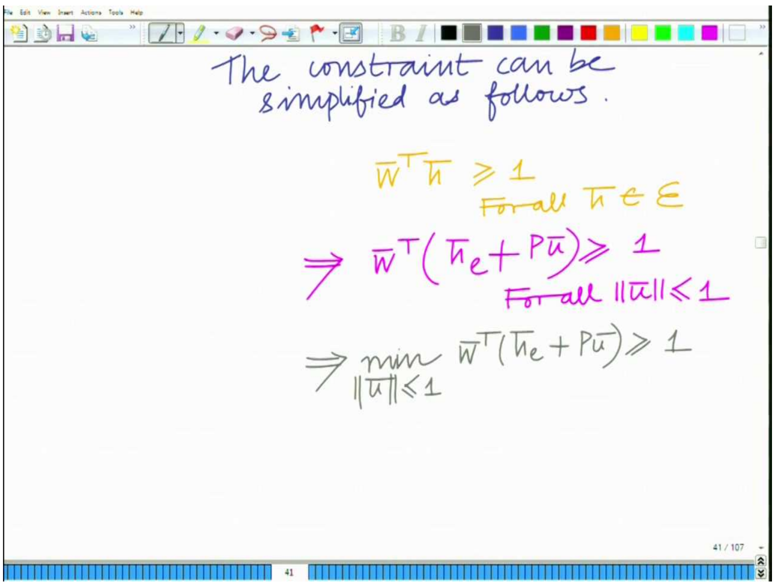
$$\min \ \bar{w}^T R \bar{w}$$
$$\text{such that } \ \bar{w}^T \bar{h} \geq 1 \qquad \text{for all } \bar{h} \in \varepsilon$$

Where $\bar{h}$ is the channel vector of the desired user, $\bar{\varepsilon}$ is the error in the CSI estimate, R is the noise plus interference covariance matrix and $\bar{w}$ is the beamforming vector. This uncertainty $\bar{\varepsilon}$ in the channel is modelled as follows.

$$\bar{\varepsilon} = \left\{ \bar{h}_e + P\bar{u} \mid \|\bar{u}\| \leq 1 \right\}$$

This ellipsoid is termed as the uncertainty ellipsoid which has the centre at $\bar{h}_e$ that is the nominal channel or the estimated channel.

(Refer Slide Time: 02:30)



The constraint in the above optimization problem can be simplified as follows.

$$\bar{w}^T\left(\bar{h}_e + P\bar{u}\right) \geq 1 \qquad \text{for all } \|\bar{u}\| \leq 1$$

$$\min_{\|\bar{u}\|\leq 1} \bar{w}^T\left(\bar{h}_e + P\bar{u}\right) \geq 1$$

$$\min_{\|\bar{u}\|\leq 1}\left(\bar{w}^T\bar{h}_e + \bar{w}^T P\bar{u}\right) \geq 1$$

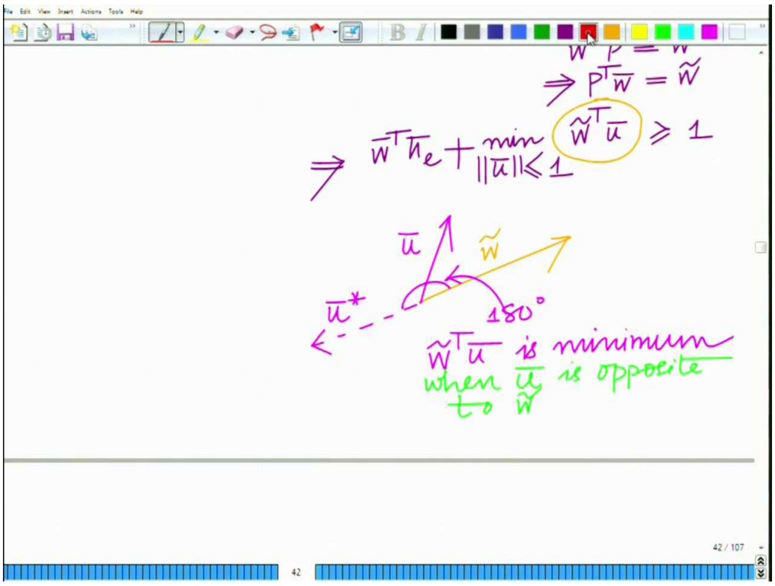
$$\bar{w}^T\bar{h}_e + \min_{\|\bar{u}\|\leq 1}\left(\bar{w}^T P\bar{u}\right) \geq 1$$

$$\bar{w}^T\bar{h}_e + \min_{\|\bar{u}\|\leq 1}\left(\tilde{w}^T\bar{u}\right) \geq 1$$

Where

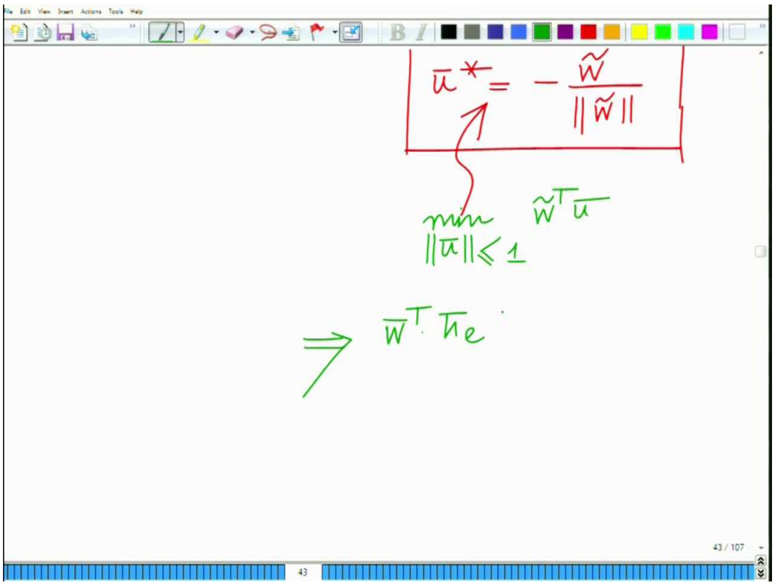
$$\tilde{w} = P^T\bar{w}$$

(Refer Slide Time: 06:32)



Remember the constraint ensures a minimum gain of unity for all CSI vector belonging to the uncertainty ellipsoid. So from the above expression it is clear that So, the dot product $\tilde{w}^T\bar{u}$ is minimum, when $\bar{u}$ forms a 180^0 angle with $\tilde{w}$. Therefore $\tilde{w}^T\bar{u}$ is minimum when $\bar{u}$ is in opposite direction to $\tilde{w}$.

(Refer Slide Time: 08:35)



So using above observation that $\bar{u}$ is in opposite direction to $\tilde{w}$; let us normalize this $\bar{u}$ as follows.

$$\bar{u}^* = -\frac{\tilde{w}}{\|\tilde{w}\|}$$

Therefore

$$\bar{w}^T \bar{h}_e + \tilde{w}^T \left(-\frac{\tilde{w}}{\|\tilde{w}\|} \right) \geq 1$$

$$\bar{w}^T \bar{h}_e - \|\tilde{w}\| \geq 1$$

$$\bar{w}^T \bar{h}_e - \|P\bar{w}\| \geq 1$$

So, this is the simplified version of the constraint.

$$\bar{w}^T \bar{h}_e - 1 \geq \|P\bar{w}\|$$

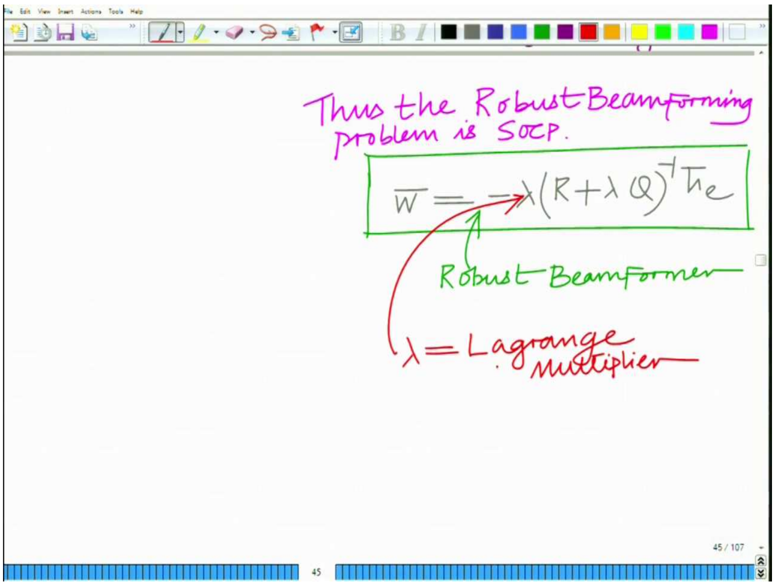
Here $\bar{w}^T \bar{h}_e - 1$ is the affine set and $\|P\bar{w}\|$ is the norm and here norm is less than or equal to some affine set. Therefore this constraint represents a conic region.

So the equivalent convex optimization problem for robust beamforming is as follows.

$$\min \; \bar{w}^T R \bar{w}$$
$$\text{such that } \left\| \bar{w}^T P \right\| \leq \bar{w}^T \bar{h}_e - 1$$

This is a conic constraint with second order objective therefore this is a second order quadratic optimization or second order cone program (SOCP). Thus the robust beam forming problem reduces to a very interesting optimization known as an SOCP problem.

(Refer Slide Time: 15:09)



And hence it can be shown that robust beam former is the inverse of the Lagrange multiplier λ.

$$\bar{w} = -\lambda \left(R + \lambda Q \right)^{-1} \bar{h}_e$$

And the matrix Q is

$$Q = PP^T - \bar{h}_e \bar{h}_e^T$$

This means that matrix Q depends on the nominal estimate of the channel $\bar{h}_e$.