Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 37
Practical Application: Zero-Forcing (ZF) Beamforming with Interfering User

Hello, welcome to another module in this massive open online course. Let us look at another kind of beam forming which is known as Zero-Forcing (ZF) beam forming. In zero-forcing beamforming, the interference is null. In other words, interference is forced to zero in ZF-beamformer therefore this is also known as interference nulling.

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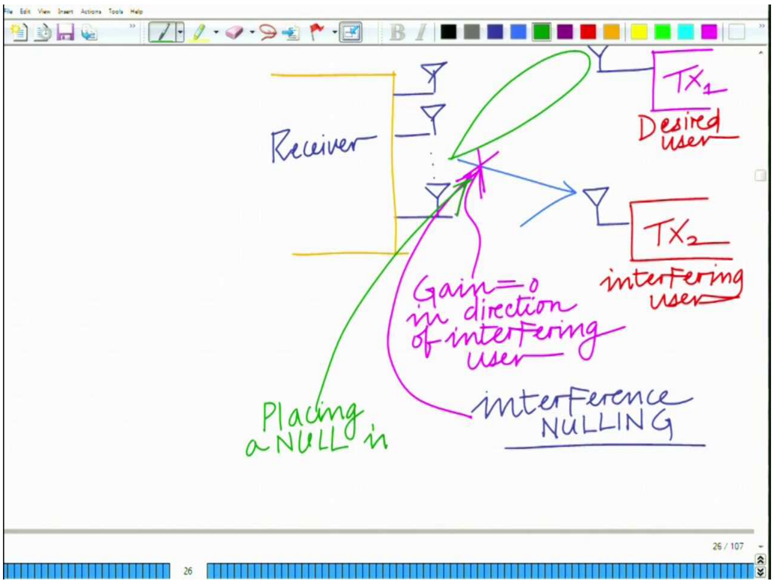

Consider a multiple antenna array system in which there are several receivers. First transmitter Tx_1 is the desired user and the second transmitter Tx_2 is the interfering user. So the signal gain must be maximized in the direction of the desired user while in the direction of the interfering user, the signal gain must be equal to zero and in this way, the interference is nulled.

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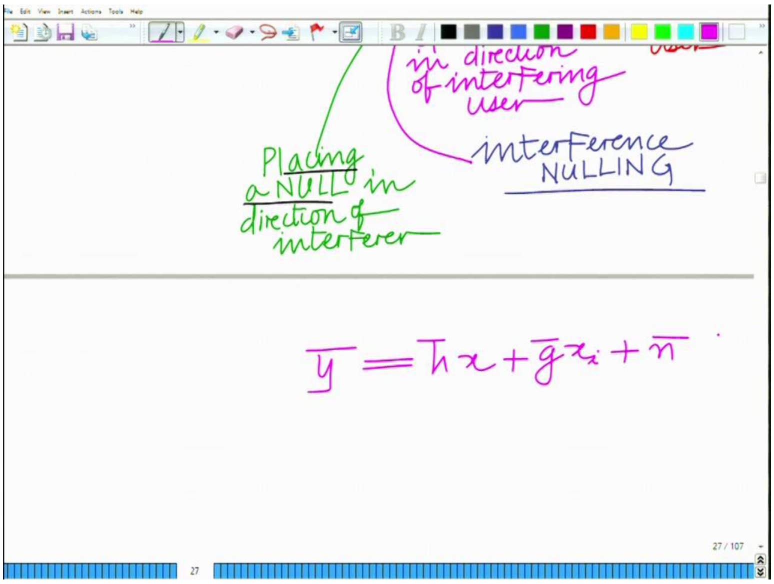

Let us discuss its procedure. Let us again go back to the system model,

$$\bar{y} = \bar{h}x + \bar{g}x_i + \bar{n}$$

Where $\bar{y}$ is the received signal, $\bar{h}$ is the channel vector of the desired user, x is the transmitted signal and $\bar{n}$ is the additive white Gaussian noise. The term $\bar{g}x_i$ defines the interference caused by interferer such that $\bar{g}$ is the channel vector of interfering user.

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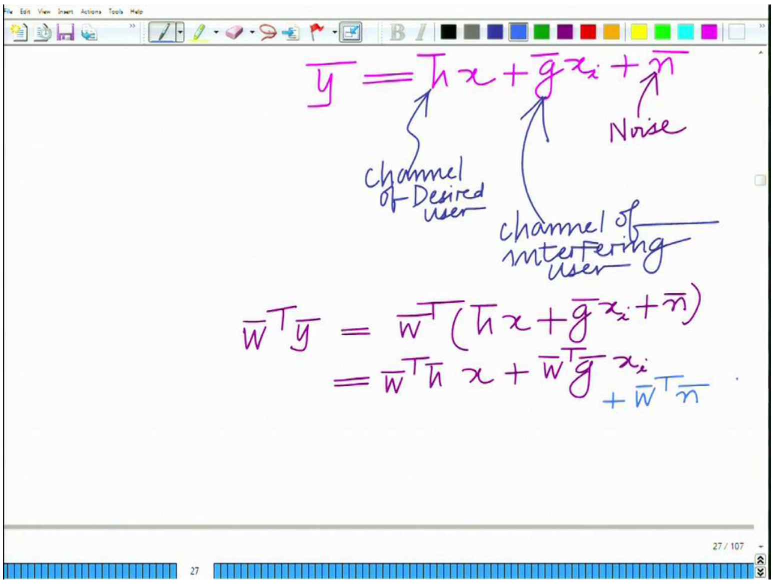

For electronic steering,

$$
\begin{aligned}
\bar{w}^T \bar{y} &= \bar{w}^T \left(\bar{h}x + \tilde{n}\right) \\
&= \bar{w}^T \bar{h}x + \bar{w}^T \left(\bar{g}x_i + \bar{n}\right) \\
&= \bar{w}^T \bar{h}x + \bar{w}^T \bar{g}x_i + \bar{w}^T \bar{n}
\end{aligned}
$$

Here, $\bar{w}^T \bar{h}$ is the signal gain and $\bar{w}^T \bar{g}$ is the interfering signal gain. So in ZF beamformer, $\bar{w}^T \bar{g}$ should be zero along with unity signal gain.

$\bar{w}^T \bar{h} = 1$ and $\bar{w}^T \bar{g} = 0$

So, this basically nulls the interference.

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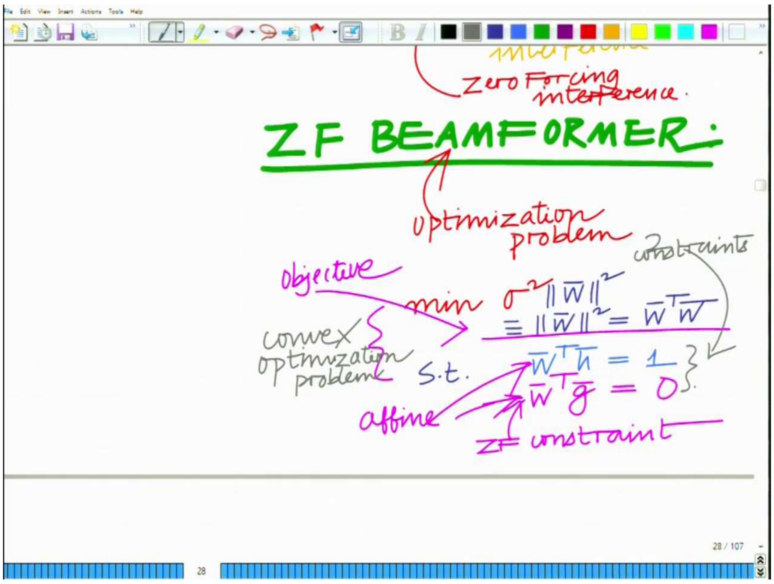

Therefore, the optimization problem for zero-forcing beam former is

$$
\begin{aligned}
\min \ & \sigma^2 \|\bar{w}\|^2 \\
\text{such that } & \bar{w}^T \bar{h} = 1, \\
& \bar{w}^T \bar{g} = 0
\end{aligned}
$$

As usual σ^2 is constant, therefore this optimization problem can be written as

$$\min \ \bar{w}^T \bar{w}$$
$$\text{such that } \ \bar{w}^T \bar{h} = 1,$$
$$\bar{w}^T \bar{g} = 0$$

Here there are 2 constraints where second constraint the ZF constraint which is an affine set. Both the constraints are linear. Therefore this is again a convex optimization problem.

In fact, if there are more than one interfering user then there will be multiple constraints.

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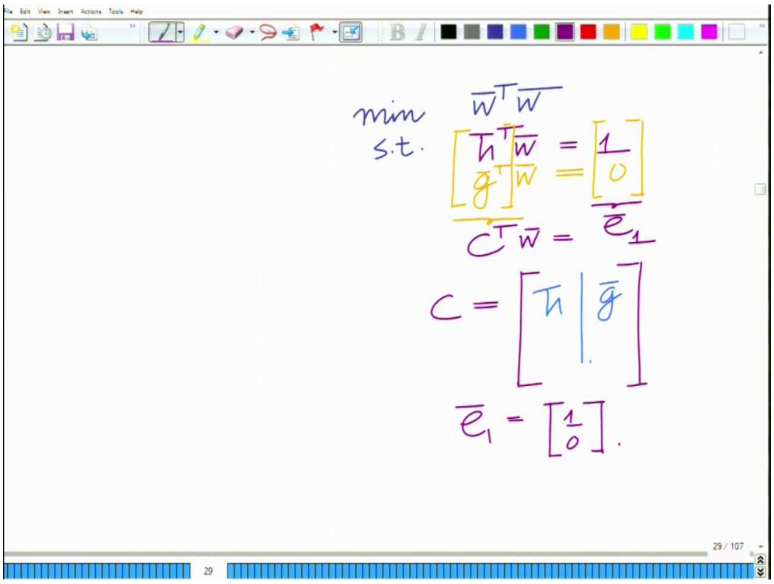

Let us combine these two constraints in the vector form. So

$$\begin{bmatrix} \bar{h}^T \\ \bar{g}^T \end{bmatrix} \bar{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$c^T \bar{w} = \bar{e}_1$$

Such that

$$c^T = \left[\bar{h} \mid \bar{g} \right] \quad \text{and} \quad \bar{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So the optimization problem is minimized as follows.

$$\min \ \bar{w}^T \bar{w}$$
$$\text{such that } \ c^T \bar{w} = \bar{e}_1$$

This is the optimization problem for zero-forcing Beam Forming. In fact, this is known as a quadratic program (QP) because here is a quadratic objective function.

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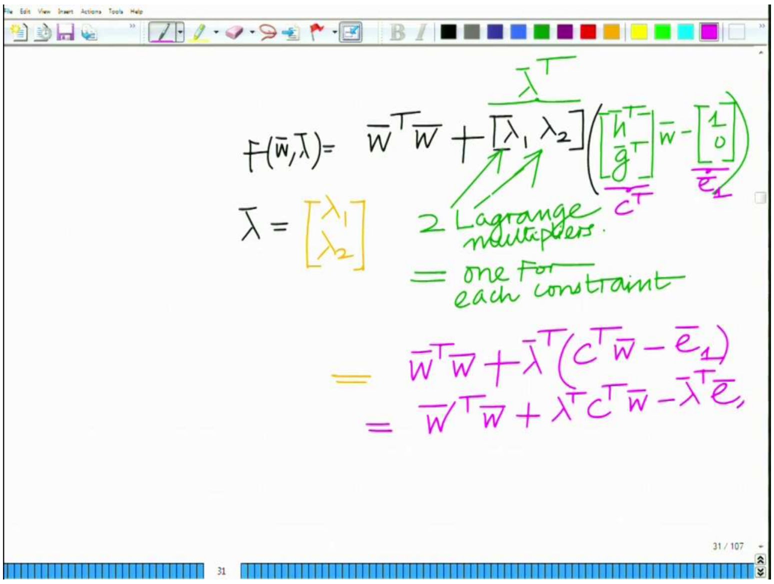

Let us form its Lagrangian.

$$F\left(\bar{w}, \bar{\lambda}\right) = \bar{w}^T \bar{w} + [\lambda_1 \quad \lambda_2]\left(\begin{bmatrix} \bar{h}^T \\ \bar{g}^T \end{bmatrix} \bar{w} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

There are two Lagrange multipliers λ_1 and λ_2. So let us take a vector of these Lagrange multipliers.

$$\bar{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Therefore

$$\begin{aligned} F\left(\bar{w}, \bar{\lambda}\right) &= \bar{w}^T \bar{w} + \bar{\lambda}^T \left(c^T \bar{w} - \bar{e}_1\right) \\ &= \bar{w}^T \bar{w} + \bar{\lambda}^T c^T \bar{w} - \bar{\lambda}^T \bar{e}_1 \\ &= \bar{w}^T I \bar{w} + \bar{c}^T \bar{w} - \bar{\lambda}^T \bar{e}_1 \end{aligned}$$

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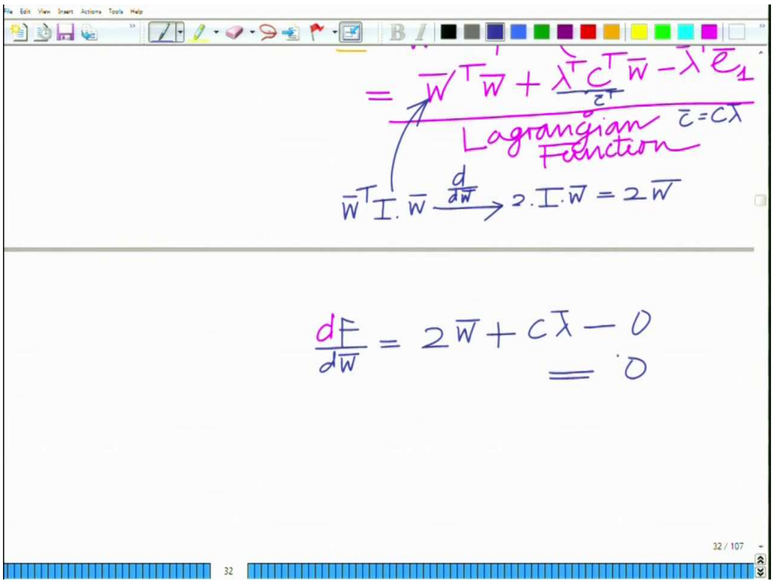

This is your Lagrangian function. Put the gradient of the Lagrangian function with respect to $\overline{w}$ equal to zero.

$$\frac{dF}{d\overline{w}} = 0$$
$$2\overline{w} + c\overline{\lambda} - 0 = 0$$
$$\overline{w} = -\frac{1}{2}c\overline{\lambda}$$

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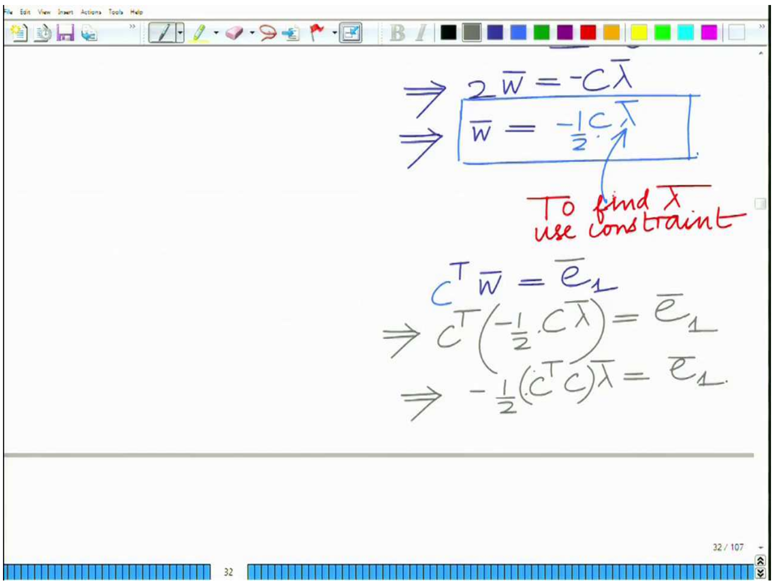

To find the value of $\bar{\lambda}$, use constraint. Therefore

$$
\begin{aligned}
c^T \bar{w} &= \bar{e}_1 \\
c^T\left(-\frac{1}{2}c\bar{\lambda}\right) &= \bar{e}_1 \\
-\frac{\bar{\lambda}}{2} &= \left(c^T c\right)^{-1}\bar{e}_1
\end{aligned}
$$

Therefore the optimal ZF Beamformer vector $\bar{w}^*$ is as follows.

$$\bar{w}^* = c\left(c^T c\right)^{-1}\bar{e}_1$$

So, this is the zero-forcing beam former which basically places a null in the direction of the interferer and hence it completely blocks the interference from the interfering user. In fact; it is also a very popular technique that is employed in practical wireless communication systems, especially in the presence of a large number of interfering users. This can also be used in a cognitive radio scenario for the secondary user during the on-going primary user transmission. So, one can block the interference caused by the primary transmitter by using zero-forcing wave forming.

Also because of its low complexity, ZF beamforming tends to be one of the popular beamforming techniques in the presence of interference.