1. A function $f(\bar{\mathbf{x}})$ is convex if and only if it satisfies the property $f(\theta_1\bar{\mathbf{x}}_1 + \theta_2\bar{\mathbf{x}}_2) \leq \theta_1 f(\bar{\mathbf{x}}_1) + \theta_2 f(\bar{\mathbf{x}}_2)$ for all values of θ such that $0 \leq \theta \leq 1$
   Ans d
2. The function $f(x) = x^3$ is Convex for $x \geq 0$ and concave for $x < 0$
   Ans b
3. The second derivative of $Q(x)$, which denotes the tail probability of the standard normal random variable, is $\frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
   Ans b
4. For a fixed transmit power and noise power at the receiver, average BER across a fading wireless channel is greater than the average BER of a wireline system
   Ans d
5. The conjugate function is
   $$\max_x yx - f(x) = \max_x yx + \ln x$$
   Differentiating $y + \frac{1}{x} = 0 \Rightarrow x = -\frac{1}{y}$ if $y \leq 0$. For $y \geq 0$, it is an increasing function and maximum is ∞. Hence, $f^*(y) = -1 - \ln(-y)$ for $y < 0$ and ∞ otherwise
   Ans c
6. The Hessian of $||\bar{\mathbf{x}}||$ can be evaluated as follows
   $$\frac{d}{d\bar{\mathbf{x}}} ||\bar{\mathbf{x}}|| = \frac{\bar{\mathbf{x}}}{||\bar{\mathbf{x}}||} \Longrightarrow \nabla^2 ||\bar{\mathbf{x}}|| = \frac{\mathbf{I}}{||\bar{\mathbf{x}}||} - \frac{\bar{\mathbf{x}}\bar{\mathbf{x}}^T}{||\bar{\mathbf{x}}||^3}$$
   Ans c
7. For a convex function f and random variable X, Jensen's inequality states that $f\big(E(X)\big) \leq E\big(f(X)\big)$
   Ans d
8. For a concave function f and random variable X, Jensen's inequality states that $f\big(E(X)\big) \geq E\big(f(X)\big)$
   Ans a
9. The second derivative of $Q\big(\sqrt{x}\big)$, where $Q(x)$ denotes the tail probability of the standard normal random variable, is
   $$\frac{1}{4\sqrt{2\pi}} \frac{1}{x^{3/2}} e^{-\frac{x}{2}} + \frac{1}{4\sqrt{2\pi}} \frac{1}{x^{1/2}} e^{-\frac{x}{2}}$$
   Ans b
10. As shown in the lectures, the conjugate function of $f(\bar{\mathbf{x}}) = \frac{1}{2}\bar{\mathbf{x}}^T\mathbf{Q}\bar{\mathbf{x}}$ is
    $$\frac{1}{2}\bar{\mathbf{y}}^T\mathbf{Q}^{-1}\bar{\mathbf{y}}$$
    Ans a