**Applied Optimization for Wireless, Machine Learning, Big Data**
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**Lecture - 53**
**Practical Applications Probability of Error Computation for Co-operative Communication**

***Keywords**: Co-operative Communication, Probability of Error Computation*

Hello, welcome to another module in this massive open online course. So we are looking at co-operative communication and specifically we would like to look at optimization for co-operative communication.

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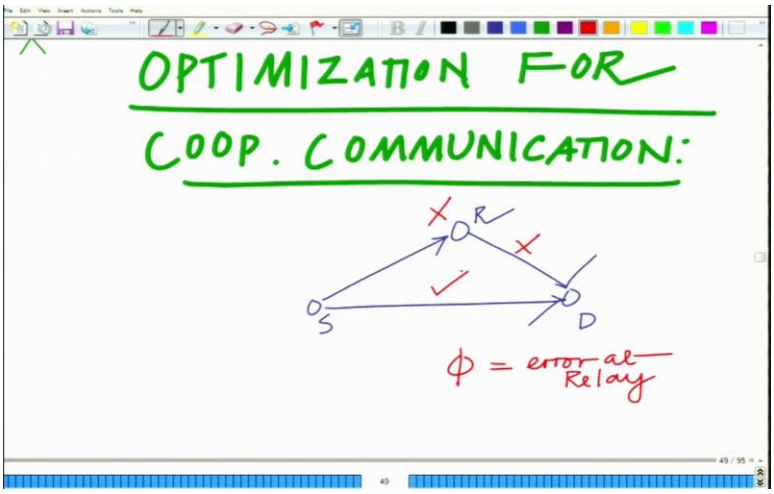


So we want to look at minimization of the error rate. And consider the cooperative communication system with the source, relay and destination nodes as shown in slide. When you have an error at the relay which means the relay is not able to decode the symbol correctly, in selective decode and forward, the relay simply does not retransmit. So the relay to destination link does not exist. So only source destination link exists and the decoding at the destination takes place on the signal received from the source.

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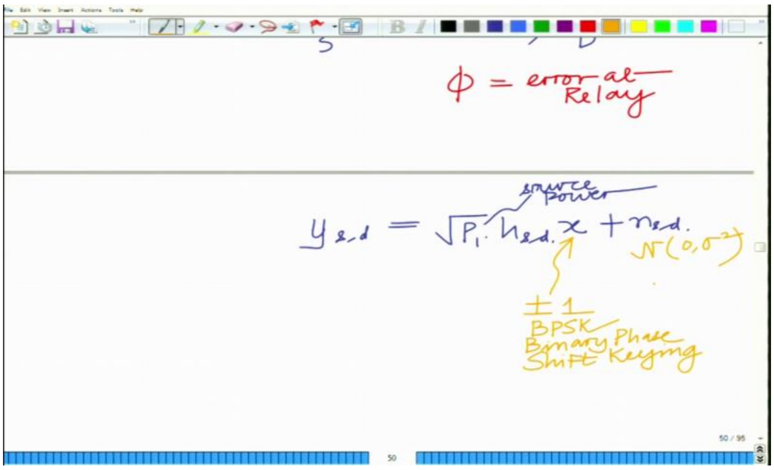


Now this link can be modelled as $y_{sd} = \sqrt{P_1} h_{sd} x + n_{sd}$ where $y_{sd}$ is the received symbol corresponding to transmission, because this is the source destination link, $P_{1is}$ the source power, $h_{sd}$ is the fading channel coefficient between the source and destination times and x is the transmitted. Let this x be a BPSK symbol that is this is $\pm 1$. This $n_{sd}$ is additive white Gaussian noise with mean 0 and variance or power $\sigma^2$.

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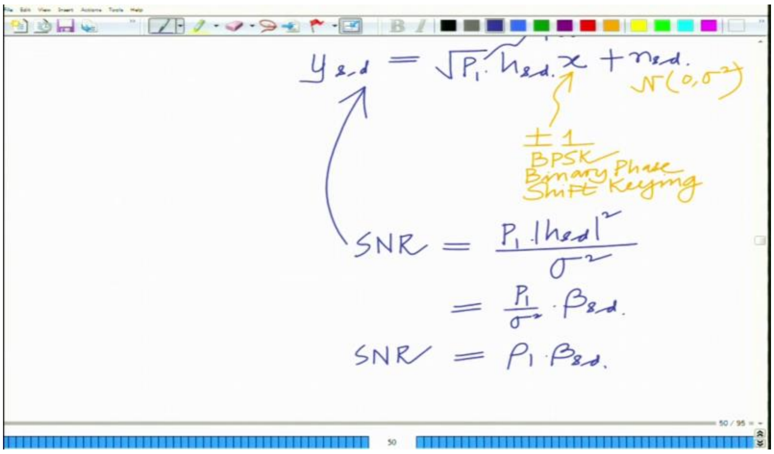


And now if you look at the output SNR this is $\frac{P_1 |h_{sd}|^2}{\sigma^2} = \frac{P_1}{\sigma^2} \beta_{sd}$. Now $\frac{P_1}{\sigma^2} = \rho_1$ so the SNR at the output when the relay is decoding an error that is there is only source destination link is $\rho_1 \beta_{sd}$.

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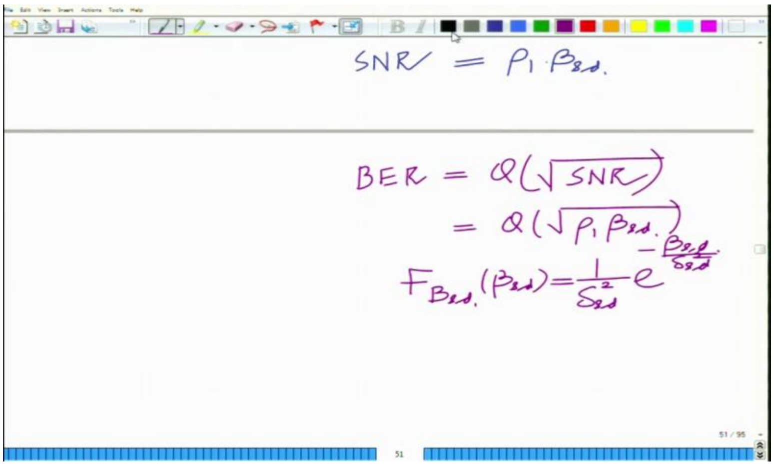


Now, since we are considering BPSK modulation, the bit error rate has a very simple expression given as $BER = Q\left(\sqrt{SNR}\right) = Q\left(\sqrt{\rho_1 \beta_{sd}}\right)$. We have $\beta_{sd}$ which is a random quantity and is exponentially distributed with average power $\delta_{sd}^2$.

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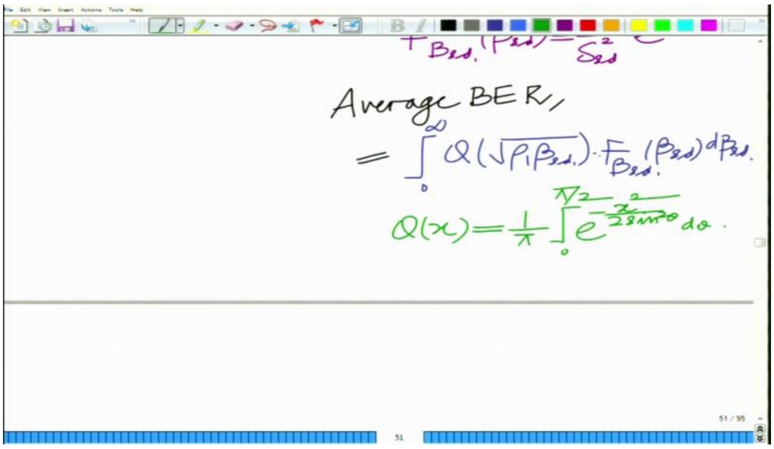


To find the average bit error rate corresponding to these observations or these decoded symbols at the destination over a long period of time we multiply the bit error by the probability density function and integrate it. So we have $\int_0^{\infty} Q\left(\sqrt{\rho_1 \beta_{sd}}\right) f_{\beta_{sd}}(\beta_{sd})\, \mathrm{d}\,\beta_{sd}$ and here, we are going to use the formula for the Q function, that is $Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\theta}} d\theta$ which is also known as the Craig's formula.

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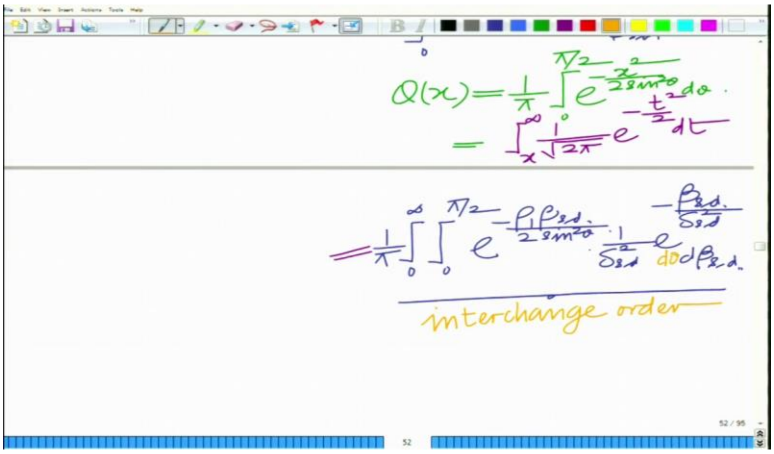


This is then solved as shown in slide.

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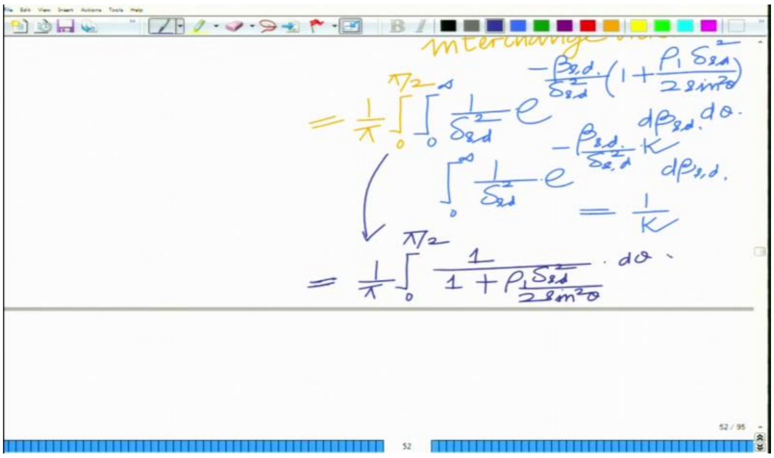


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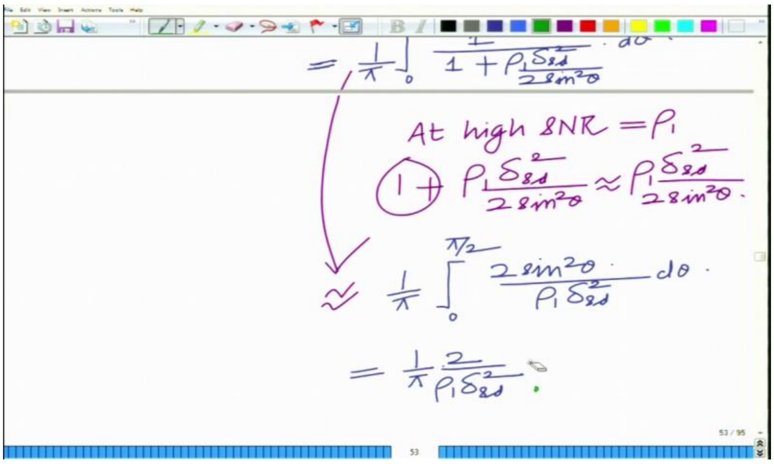

Now we the probability as $\Pr(e/\phi) = \frac{1}{2\rho_1\delta_{sd}^2}$.

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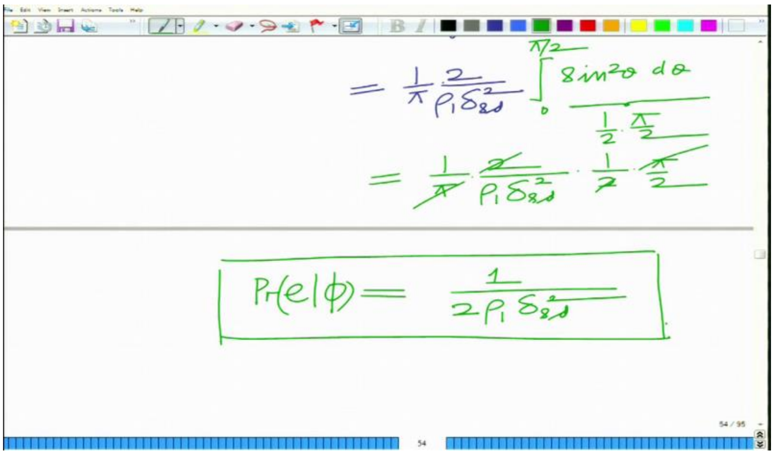


This is the probability of error, in the event of which there is no relay to destination transmission, so there is only source destination transmission and this is the probability of error at the destination for decoding the BPSK symbol transmitted by the source given the error at the relay.

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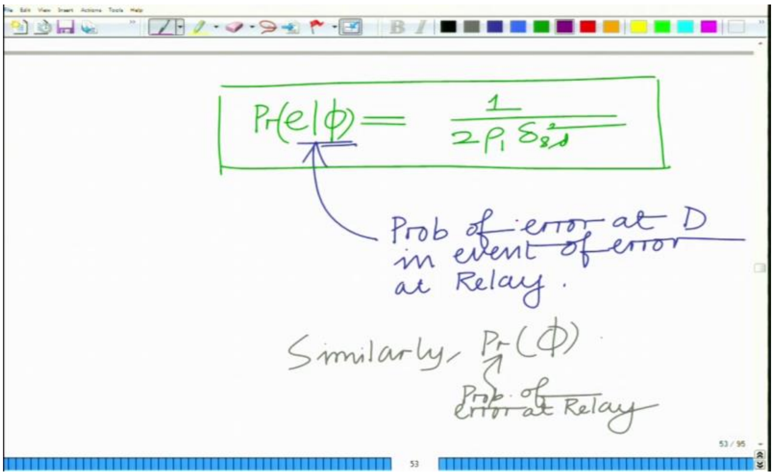


So we also need to find the probability of an error at relay $\Pr(\phi)$.

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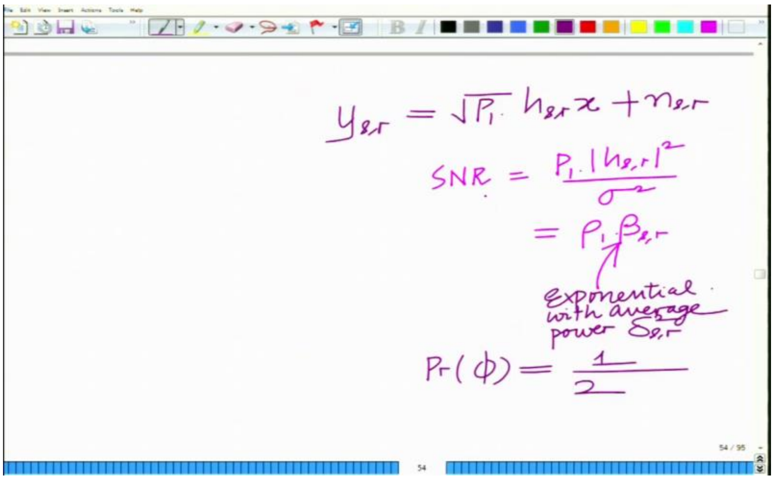


The source to relay link is also a fading link. So this can be modelled as $y_{sr} = \sqrt{P_1} h_{sr} x + n_{sr}$. Now SNR is exactly the same as that of source destination link and is given as $\rho_1 \beta_{sr}$, this $\beta_{sr}$ is also an exponential random variable with average power $\delta_{sr}^2$. So the probability of error is $\Pr(\phi) = \frac{1}{2\rho_1 \delta_{sr}^2}$.

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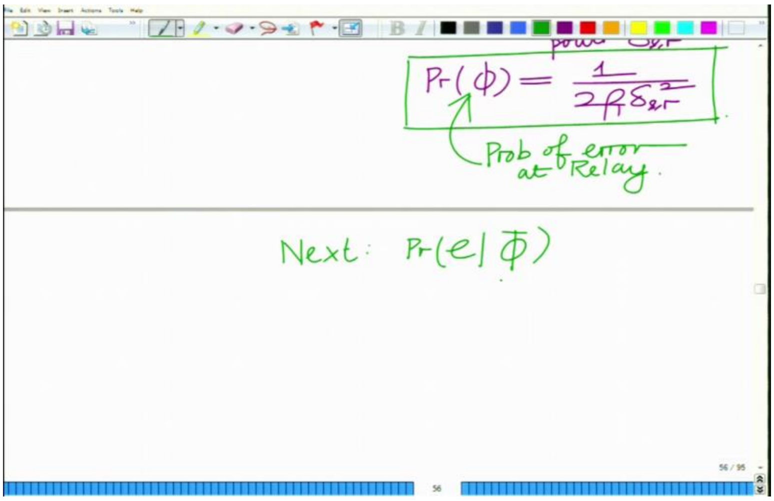


So this is the probability of error at the relay and now we want to find $\Pr(e/\bar{\phi})$.

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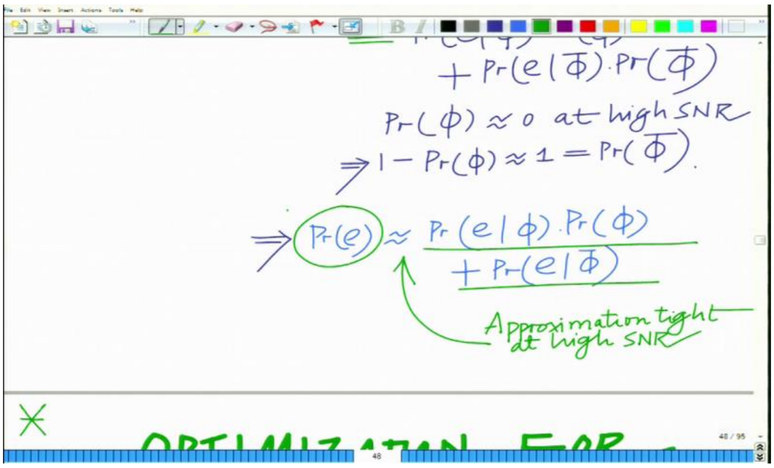


So we need to find the probability of error at destination given that the relay decodes accurately.

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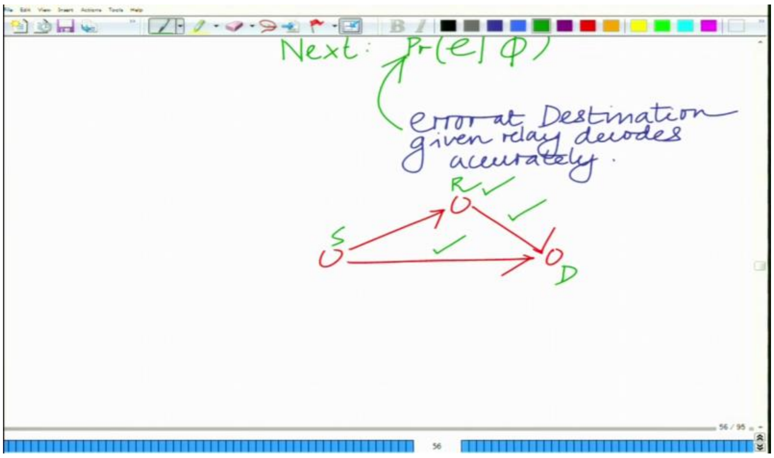


So if relay is decoding accurately, source transmits, relay also transmits. So at destination you have two signals, signal received by the source and signal received from the relay. Now, the destination has to employ some kind of combining. You can treat this as a beam forming problem with the multiple nodes.

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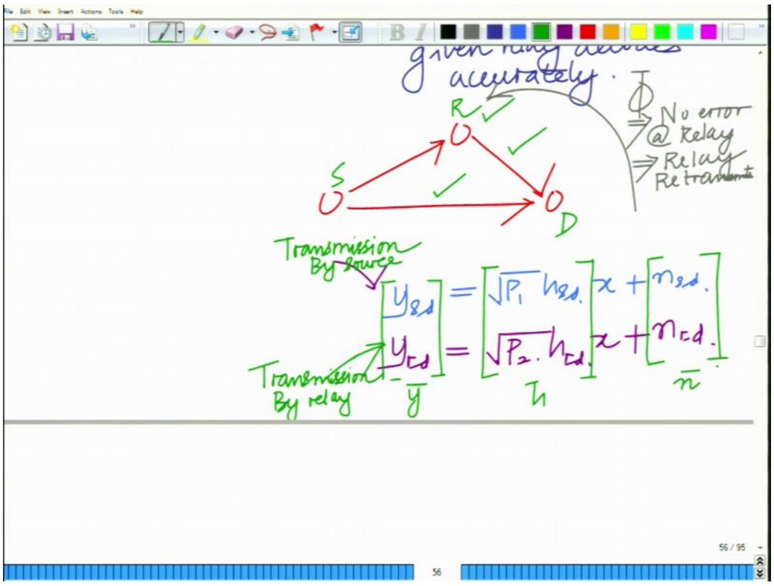


$\bar{\phi}$ implies there is no error at relay, implies relay retransmits, so now you have two symbols $y_{sd} = \sqrt{P_1} h_{sd} x + n_{sd}$ and $y_{rd} = \sqrt{P_2} h_{rd} x + n_{rd}$. And now we have to combine them in some kind of an optimal fashion and we know that in the beamforming problem, the optimal combiner is the maximal ratio combiner. Therefore, we treat these two received symbols as your receive vector $\bar{y}$, this as your channel vector $\bar{h}$ and this as your noise vector $\bar{n}$.

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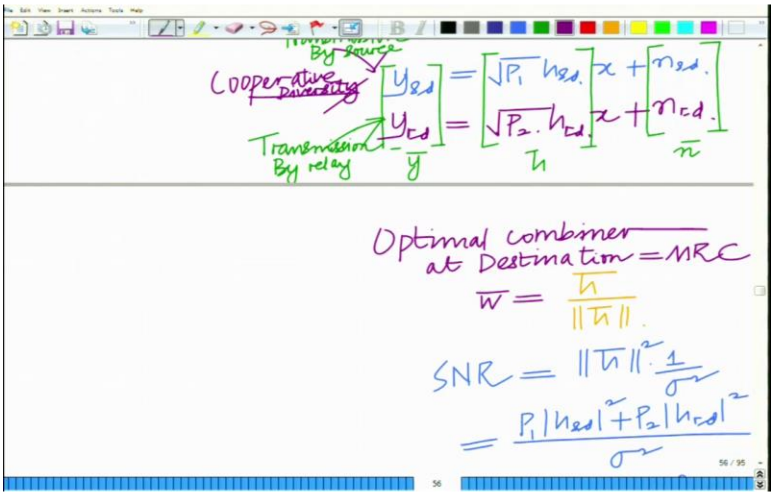

The optimal combiner at destination is the MRC that is the Maximal Ratio Combiner and we have the beam former $\bar{w} = \frac{\bar{h}}{\left\|\bar{h}\right\|}$. The SNR is given as shown in slide and is $\rho_1\beta_{sd} + \rho_2\beta_{rd}$. You can see this is the coherent combining that is combines the SNRs corresponding to both the source destination transmission and the relay destination and thereby you are enhancing the reliability. This is where you get the gain from co-operative communication, because you have the signals that are transmitted by two different sources, one is the original source and other is the relay, which is acting as the replica of the source, so now you see the co-operative diversity aspect emerging, because there is transmission by the source, there is transmission by the relay. So they are co-operating and you have two signal copies and that gives diversity in a wireless communication system, which leads to a significant decrease in the bit error rate.

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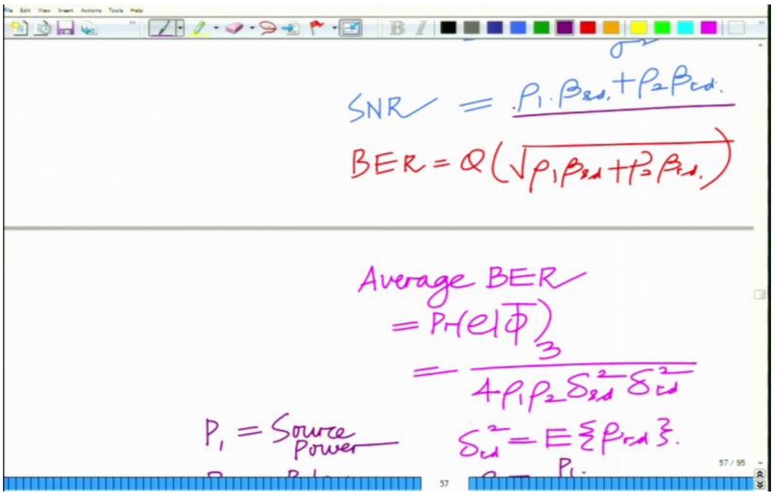


The bit error rate for BPSK will be obtained as in the previous case. So $BER = Q\left(\sqrt{\rho_1\beta_{sd} + \rho_2\beta_{rd}}\right)$ and you can average this over the probability density function. Thus the average bit error rate for this is $\Pr(e/\bar{\phi})$ and this is given as $\frac{3}{4\rho_1\rho_2\delta_{sd}^2\delta_{rd}^2}$.

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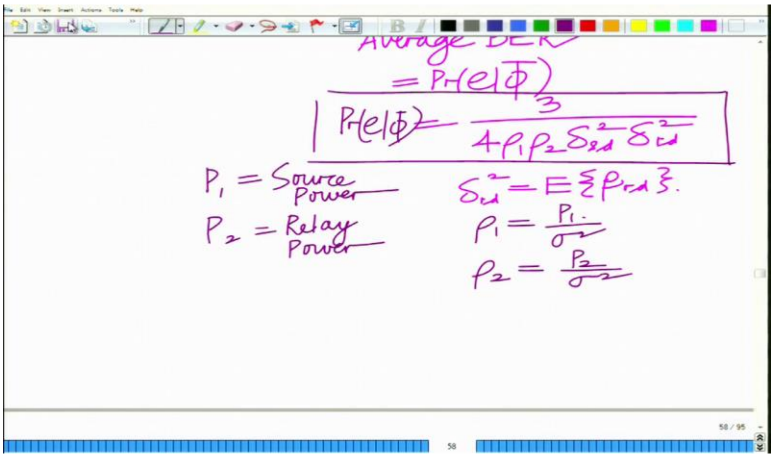


The source power can be very different from the relay power and we can derive this expression later. So now putting all these components together, one can derive the probability of error that is the final expression for the end to end error. And using that one can come up with a framework for optimal power distribution between the source and relay. The optimization problem pertains to how to distribute the power optimally between the source and relay in a wireless communication system, which we will deal with in the next module. Thank you very much.