Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 38
Practical Application: Robust Beamforming With Channel Uncertainty for Wireless

Hello, welcome to another module in this massive open online course. Let us discuss the practical applications of robust beamforming with channel uncertainty in wireless communication.

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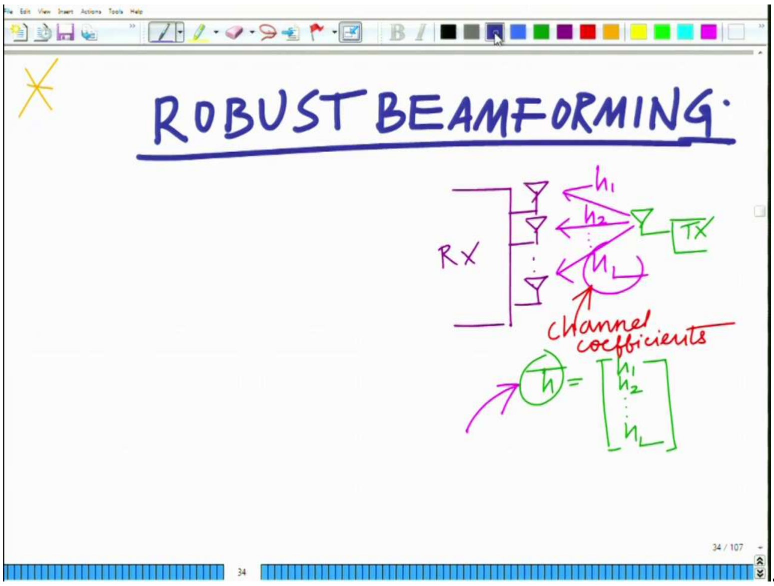

Again consider the multiple antenna arrays at the receiver. There is one desired transmitter as primary user with several secondary users as interferers. The channel coefficients of this system are

$$\bar{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix}$$

This $\bar{h}$ is the channel state information (CSI).

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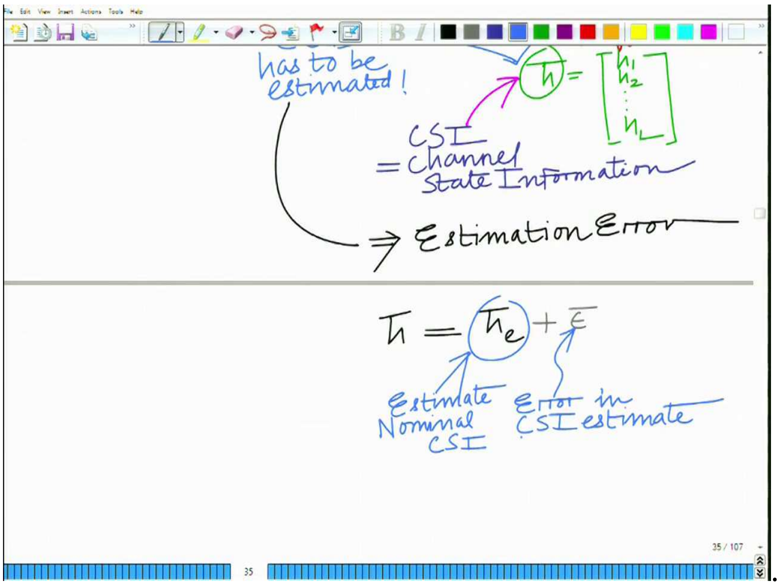

It is not available a priori but this has to be somehow obtained because the channel is varying with time as it depends on several things such as the scattering environment, the location of the base station, location of the user etc. In general, it depends on the environment and it is changing. So, this channel vector has to be estimated.

Also whenever there is an estimation process, there is always an Estimation Error that depends again on various factors such as SNR etc. therefore channel vector can be considered as

$$\bar{h} = \bar{h}_e + \bar{\varepsilon}$$

Here $\bar{h}_e$ is the Nominal CSI estimate and $\bar{\varepsilon}$ is the error in the CSI estimate. This is the true underlying channel which is unknown. Therefore

$$\bar{h} \in \bar{\varepsilon}$$

One of the ways to characterize this is to basically look at a region around the estimate. This CSI is an uncertainty ellipsoid. This ellipsoidal region is the uncertainty region. If the uncertainty is severe then the ellipsoid is larger. If the uncertainty is smaller then the ellipsoid shrinks which means the true channel vector is actually very close to the estimated channel. So this can be modelled as follows.

$$\bar{\varepsilon} = \left\{ \bar{h}_e + P\bar{u} \mid \|\bar{u}\| \le 1 \right\}$$

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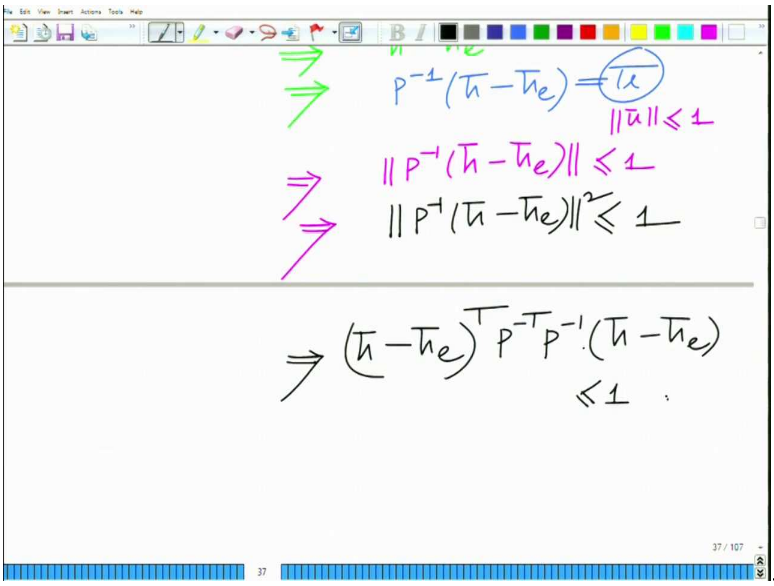

Therefore

$$\bar{h} = \bar{h}_e + P\bar{u}$$
$$\bar{u} = P^{-1}\left(\bar{h} - \bar{h}_e\right)$$

As $\|\bar{u}\| \le 1$ therefore

$$\left\| P^{-1}\left(\bar{h} - \bar{h}_e\right) \right\| \le 1$$

Also

$$\left\| P^{-1}\left(\bar{h} - \bar{h}_e\right) \right\|^2 \le 1$$
$$\left(\bar{h} - \bar{h}_e\right)^T P^{-T} P^{-1} \left(\bar{h} - \bar{h}_e\right) \le 1$$
$$\left(\bar{h} - \bar{h}_e\right)^T A^{-1} \left(\bar{h} - \bar{h}_e\right) \le 1$$

Here A is a positive definite matrix and is defined as follows.

$$A = PP^T$$

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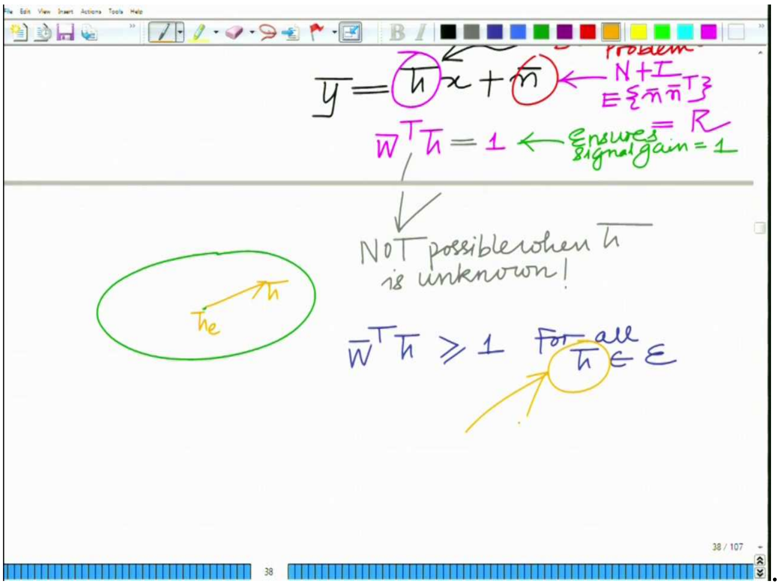

The original beamforming problem is

$$\bar{y} = \bar{h}x + \bar{n}$$

Where $\bar{y}$ is the received signal, $\bar{h}$ is the channel vector of the desired user, x is the transmitted signal and $\bar{n}$ is the additive white Gaussian noise plus interference term such that the covariance matrix of this is R.

$$E\left\{\bar{n}\bar{n}^T\right\} = R$$

For beamforming it must be ensured that

$$\bar{w}^T\bar{h} = 1$$

But it is not possible if CSI $\bar{h}$ is unknown. Therefore modify this constraint as follows.

$$\bar{w}^T\bar{h} \geq 1 \qquad \text{for all } \bar{h} \in \varepsilon$$

This means that weighted signal gain must be greater that unity for all CSI values those belong to the uncertainty ellipsoid. So, instead of just fixing the gain to unity for one particular channel vector, it is ensured that for all the channel vectors that belong to the uncertainty ellipsoid, the minimum gain is unity and therefore, it is robust. So the revised optimization problem is

$$
\begin{aligned}
&\min \ \bar{w}^T R \bar{w} \\
&\quad \text{such that } \ \bar{w}^T \bar{h} \geq 1 \qquad \text{for all } \bar{h} \in \varepsilon
\end{aligned}
$$

This robustness criterion makes sure that the designed beamformer is resilient or it can withstand the challenge of this uncertainty that is arising because of the estimation error.

So, this robust beamforming problem ensures that you are minimizing the noise plus interference power while at the same time ensuring a minimum signal gain for all channel vectors that belong to the Uncertainty set. It has significant practical utility because it takes into account the practical effects that arise in systems such as the channel estimation error.

After formulating the optimization problem, one has to solve the problem to derive the optimal beam form. So, in that sense, the problem has also become significantly more complicated.