Series 6

1. Bootstrap

In Series 5 we computed several confidence intervals for different parameters of interest. This was done based on the Wald-statistic or on the profile t-function. In this exercise, we will now construct confidence intervals using the bootstrap method.

a) Load the data `body.dat` used in Exercise 1 of Series 5 and redo the nonlinear regression with the same starting values as in Series 5.

b) We are still interested in the question whether θ_4 is significantly different from 1. Perform bootstrap with 999 bootstrap samples. Look at the distribution of the bootstrap replicates of θ_4. To do so, make a histogram of the values and add a line where the original estimate of θ_4 lies. Deciding by eye, would you say that θ_4 is significantly different from 1?

R-Hint:

```
> library(nlstools)
> D.Boot <- nlsBoot(..., niter = ...)
```

c) Now write down the 95%-confidence-interval using the bootstrap method and compare it to the approximate 95%-confidence-interval computed using the Wald-statistics which we obtained in Exercise **1c)** of Series 5 and the 95%-confidence-interval based on the profile t-function which we obtained in Exercise **1f)** of Series 5. Do you find any differences?

d) Load the data `isomer.dat` used in Exercises 2 and 3 of Series 5 and redo the nonlinear regression from Exercise 2 of Series 5 with the same starting values.

e) Perform bootstrap with 999 bootstrap samples and plot the result. Are the problems obtained in Exercise 2 of Series 5 visible?

R-Hint:

```
> D.Boot <- ...
> plot(D.Boot)
```

f) Do the same for the reparametrized model of Exercise 3 of Series 5. Does the plot look better?

g) Make a histogram of the bootstrap replicates of φ_1. Try to decide by eye what the upper and lower limit of the 95%-bootstrap confidence-interval is.

h) Calculate the bootstrap confidence interval for φ_1 "by hand" (i.e. without using the `summary()` command).

i) Look at the 95%-bootstrap confidence-intervals for all parameters $\varphi_1, \ldots, \varphi_4$. We are interested to know whether the variable x_1 has an influence on the response variable y. How can you decide this by looking at the bootstrap confidence-intervals?
Hint: Look at the equation of the reparametrized model.

2. Radioimmunoassay of Cortisol

Since the quantity of hormones contained in a compound cannot be measured directly, it is determined using a calibration curve that is estimated from a Radioimmunoassay (RIA). The assay is based on the fact that the hormone H and its radioactive Isotope H^* behave in a similar way with respect to their specific antibody. For a given quantity of antibodies A and a radioactive hormone H^*, the quantity of ligated complexes $[AH^*]$ will decrease the more the "cold" hormone H is present. This relationship is the basic concept for the calibration curve. For a given dose of pure hormone H, the number (per minute) of radioactive complexes $[AH^*]$ is counted. The functional relation between the logarithmized (base 10) dose (Variable `dose`) and the logarithmized number per minute (`y`) is given by the Richards-function:

$$y \approx h(x, \boldsymbol{\theta}) = \theta_1 + \frac{\theta_2 - \theta_1}{(1 + \exp(\theta_3 + \theta_4 x))^{\theta_5}}.$$

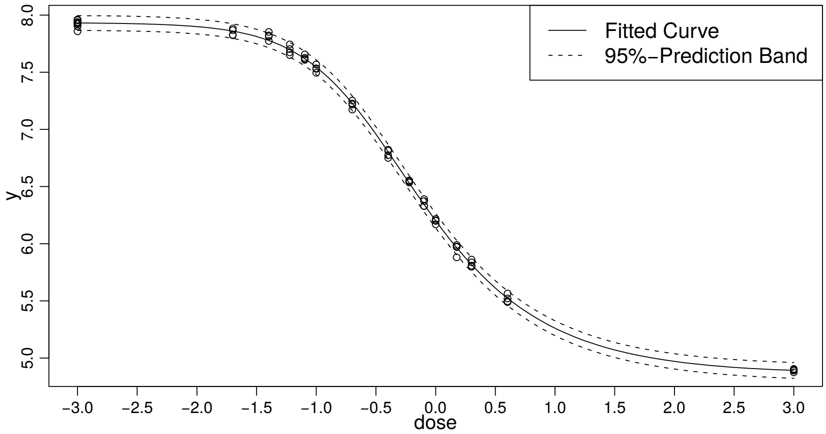
Load the data via
`read.table("http://stat.ethz.ch/Teaching/Datasets/cas-das/cortisol.dat",`
`header = TRUE, sep = ",").`

a) Estimate the parameters in the nonlinear model.
Use the starting values: $\theta_1^{(0)} = 5$, $\theta_2^{(0)} = 8$, $\theta_3^{(0)} = 3$, $\theta_4^{(0)} = 3$ and $\theta_5^{(0)} = 0.6$.

b) Take a look at the likelihood profile traces. How do they change if you set the parameter θ_5 to 0.5? In the following, set θ_5 to the fixed value of 0.5.

c) Calculate the 95%-prediction interval for $h\langle x_0, \boldsymbol{\theta}\rangle$ at $x_0 = -1.5, 0, 2$ using the fit of subtask 2 **b)** as well as the fit of subtask 2a. Compare the results. Which of the three prediction intervals is the longest?.

d) In the plot below, draw by hand the 95%-calibration interval at $y_0 = 7$ and $y_0 = 5.1$. Are there obvious differences?



e) Estimate with R the limits of the calibration intervals at $y_0 = 7$ and $y_0 = 5.1$.

f) Calculating the calibration intervals for $y0 = 7$ and $y0 = 5.1$ using the parametric bootstrap method. Are there any differences to the solution of 2 **e)**?

Exercise hour: Monday, June 24, afternoon.