

# Backpropagation

CSCI 601 471/671  
NLP: Self-Supervised Models

<https://self-supervised.cs.jhu.edu/sp2023/>

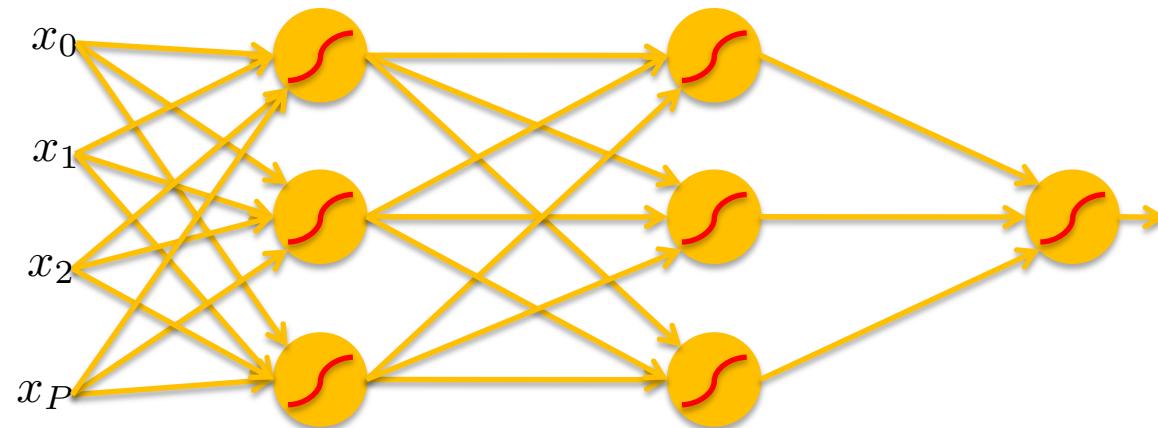


[Slide credit: Andrej Karpathy and many others ]

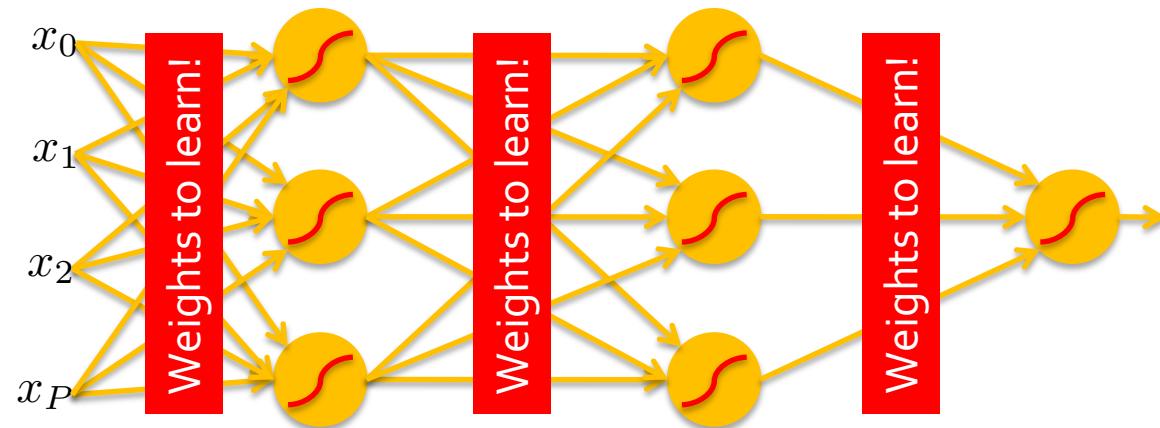
# HW update

- HW1 grades are up!
  - Stats: Mean: 93.1 (std: ~5)
  - There was a mistake in grading Q4.6, but should be corrected now.
- Regrade requests can be submitted via Gradescope.
  - Please don't spam us! 🙏
- HW3 is up!
  - Focus: training neural networks

# Recap: Feed Forward Neural Networks



# Recap: Feed Forward Neural Networks



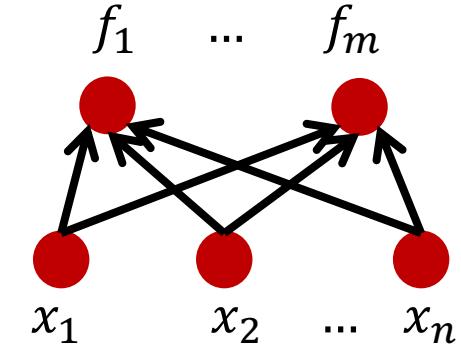
## Recap: Jacobian Matrix

- Generalization of gradients
- Given a function with ***m* outputs** and ***n* inputs**

$$\mathbf{f}(\mathbf{x}) = [f_1(x_1, x_2, \dots, x_n), \dots, f_m(x_1, x_2, \dots, x_n)] \in \mathbb{R}^m$$

- It's Jacobian is an ***m* x *n* matrix** of partial derivatives:

$$\mathbf{J}_{\mathbf{f}}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{m \times n}$$



## Recap: Chain Rule for Multivariable Functions

- Looks similar to the single-variable setup:



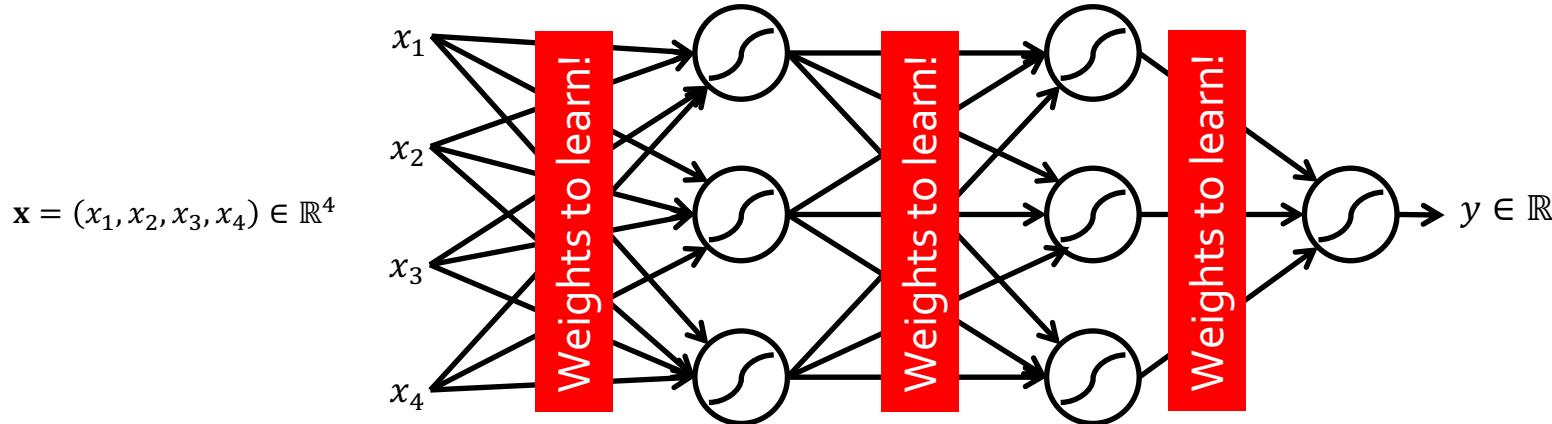
$$J_{f \circ g}(x) = J_f(g(x)) \ J_g(x)$$

Note, the above statement is a **matrix** multiplication!

Function  $f \circ g$  has  $m$  outputs and  $d$  inputs  $\rightarrow m$  by  $d$  Jacobian

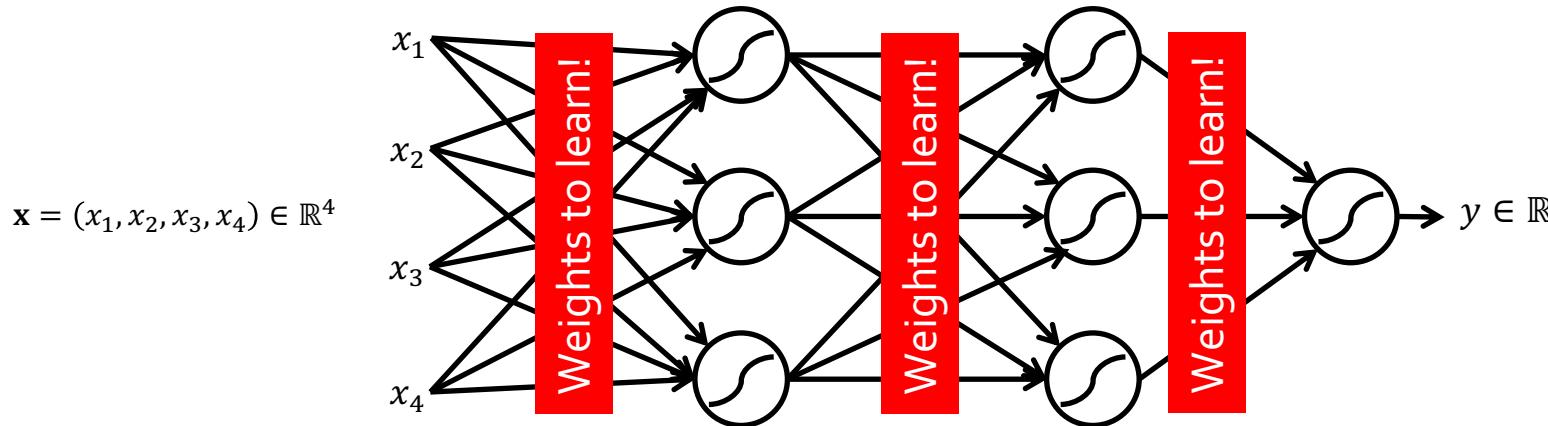
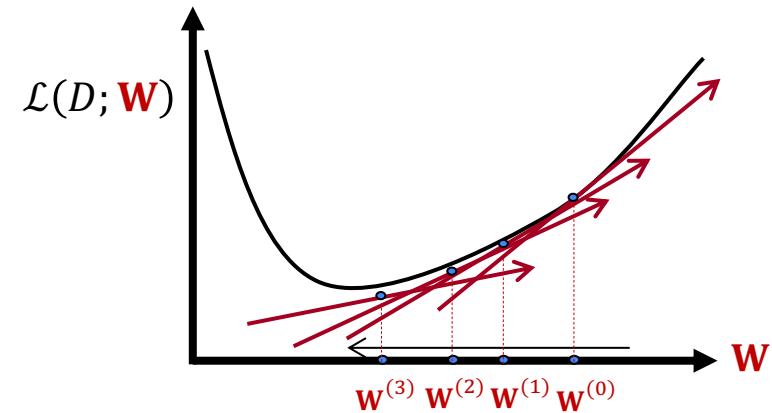
# Training Neural Networks: Setup

- We are given an architecture through its weights  $\mathbf{W}$ .
- We are given a loss function  $\ell: \mathbb{R} \times \mathbb{R} \rightarrow (0, 1)$ 
  - $\ell(y^*, y)$  quantifies distance between an answer  $y^*$  and prediction  $y = \text{NN}(\mathbf{x}; \mathbf{W})$  — lower is better
- Also given a training data  $D = \{(\mathbf{x}_i, y_i^*)\}$
- Overall objective to optimize:  $\mathcal{L}(D; \mathbf{W}) = \sum_{(\mathbf{x}_i, y_i^*) \in D} \ell(\text{NN}(\mathbf{x}_i; \mathbf{W}), y_i^*)$



# Training Neural Networks ~ Optimizing Parameters

- We can use **gradient descent** to minimizes the loss.
- At each step, the **weight vector** is modified in the **direction that produces the steepest descent** along the error surface.

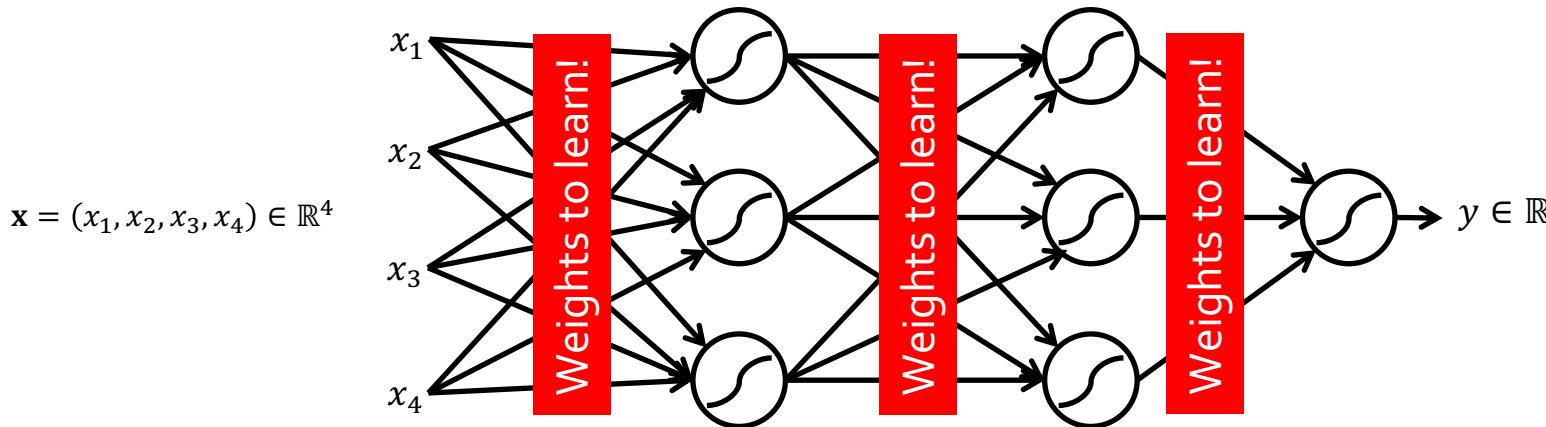
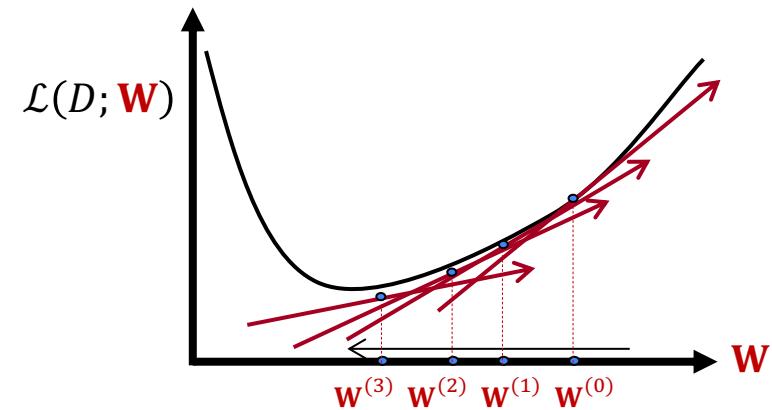


# Training Neural Networks ~ Optimizing Parameters

For each sub-parameter  $W_i \in \mathbf{W}$ :

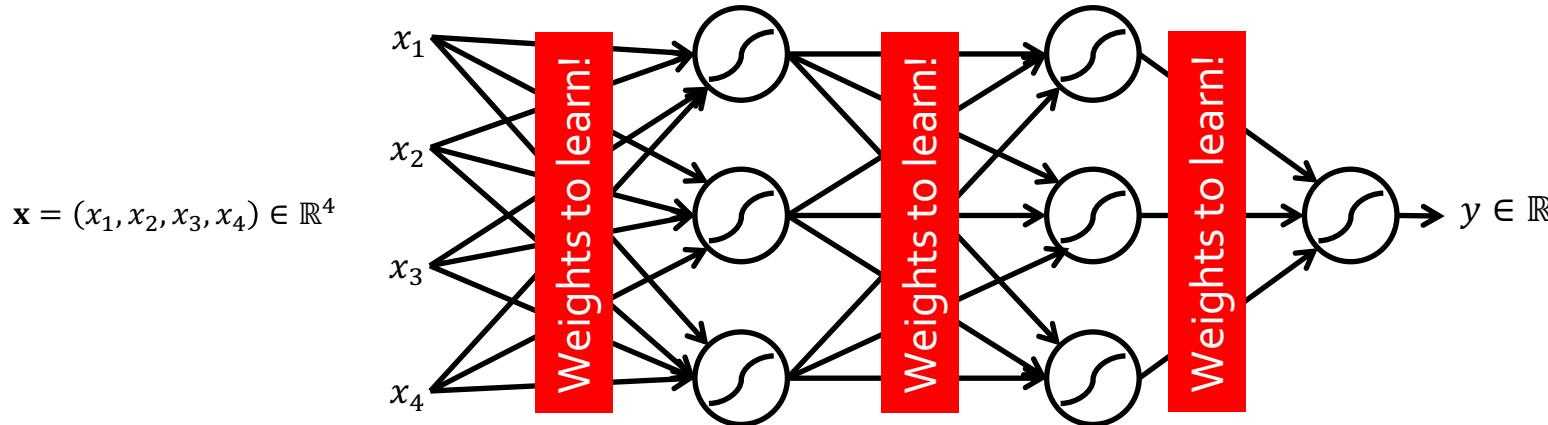
$$W_i^{(t+1)} = W_i^{(t)} - \alpha \frac{\partial \mathcal{L}}{\partial W_i}$$

It all comes down to effectively computing  $\frac{\partial \mathcal{L}}{\partial W_i}$



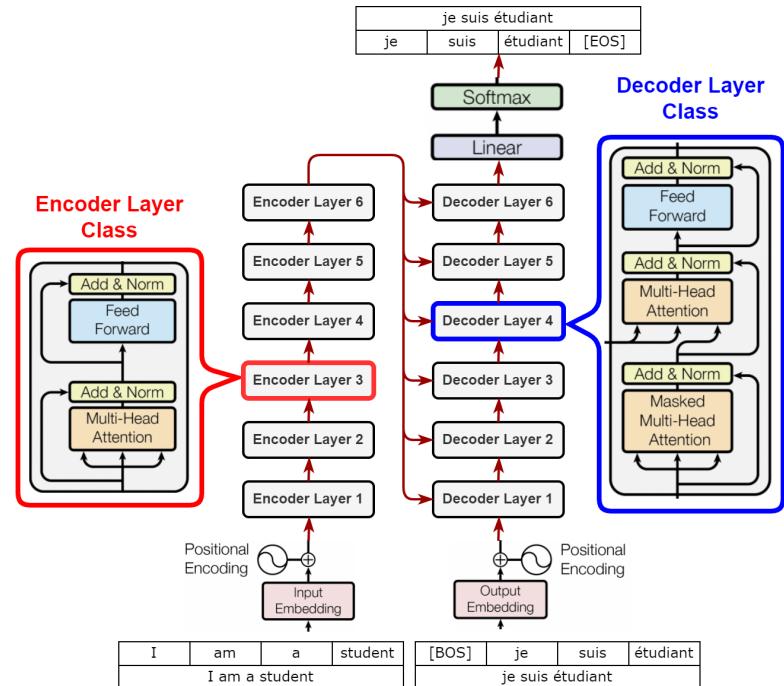
# Training Neural Networks ~ Computing the Gradients

- How do you **efficiently** compute  $\frac{\partial \mathcal{L}}{\partial w_i}$  for all parameters?
- It's easy to learn the final layer – it's just a linear unit.
- How about the weights in the earlier layers (i.e., before the final layer)?



# Necessity of a Principled Algorithm for Gradient Computation

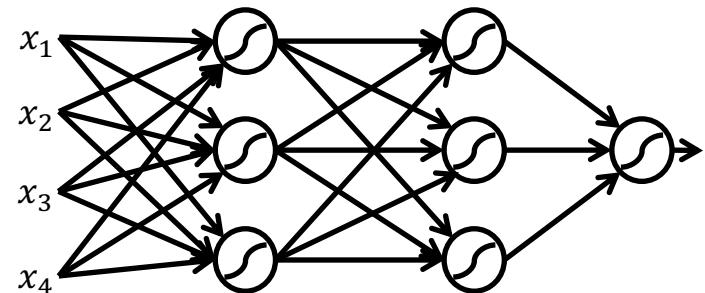
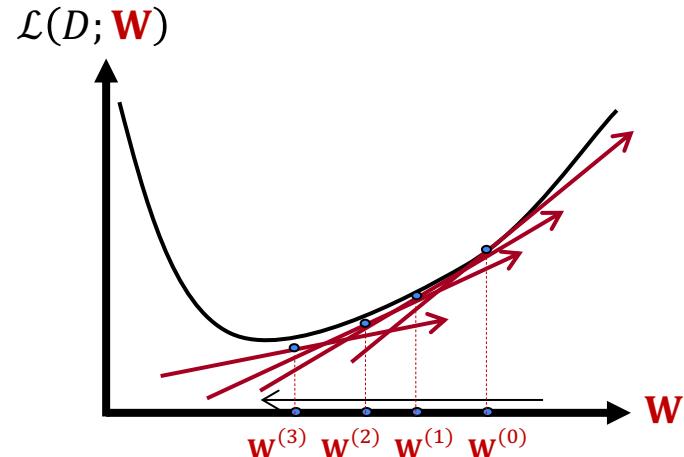
- Depth gives more representational capacity to neural networks.
- However, training deep nets is not trivial.
- Even if we have analytical formula for each gradient, they can be tedious and must be repeated for each new architecture.
- The solution is “Backpropagation” algorithm!



Architecture of the BERT model with over 24 layers and millions of parameters — we will study get to this model in a few weeks!

# Key Intuitions Required for BP

1. Gradient Descent
  - Change the weights  $\mathbf{W}$  in the direction of gradient to minimize the error function.
2. Chain Rule
  - Use the chain rule to calculate the weights of the intermediate weights
3. Dynamic Programming (Memoization)
  - Memoize the weight updates to make the updates faster.



# A Generic Neural Network

- Given the following definition:

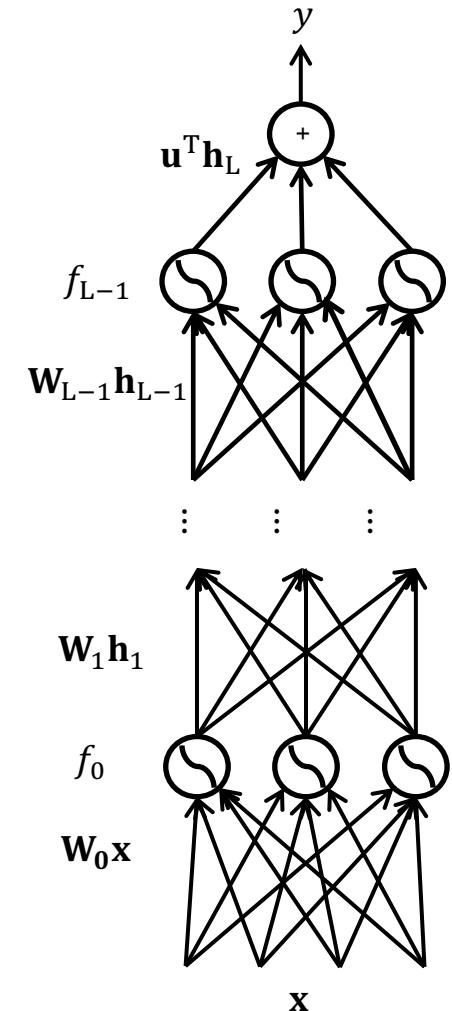
$$\mathbf{x} = \mathbf{h}_0 \in \mathbb{R}^{d_0} \text{ (input)}$$

$$\mathbf{h}_{i+1} = f_i(\mathbf{W}_i \mathbf{h}_i) \in \mathbb{R}^{d_i} \text{ (hidden layer } i, 0 \leq i \leq L - 1)$$

$$y = \mathbf{u}^T \mathbf{h}_L \in \mathbb{R} \text{ (output)}$$

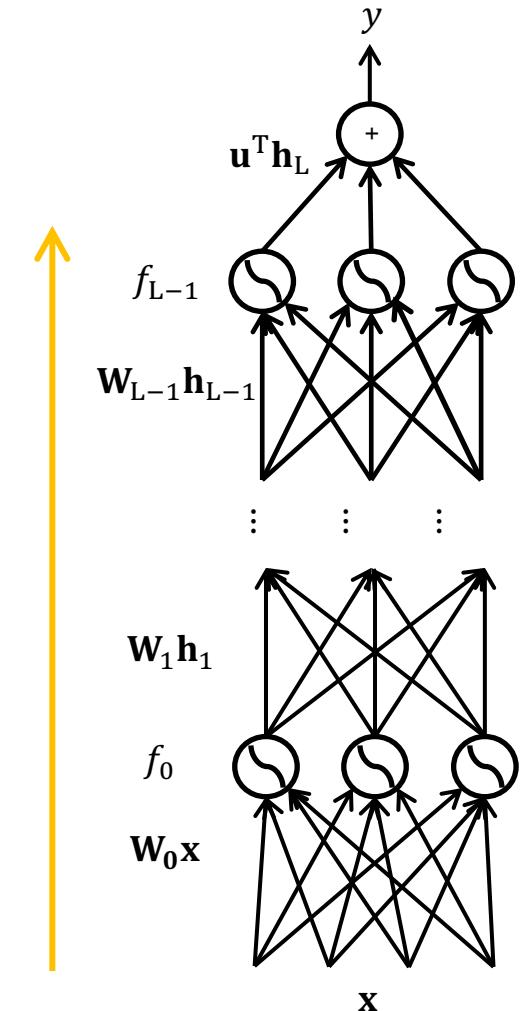
$$\mathcal{L} = \ell(y, y^*) \in \mathbb{R} \text{ (loss)}$$

- Trainable parameters:  $\mathbf{W} = \{\mathbf{W}_0, \mathbf{W}_1, \dots, \mathbf{W}_L, \mathbf{u}\}$



# A Generic Neural Network: Forward Step

- Given some [initial] values for the parameters, we can compute **the forward pass**, layer by layer.
- Forward pass is basically  **$L$  matrix multiplications**, each followed by an activation function.
- Matrix multiplication can be done efficiently with GPUs.
  - Therefore, **forward pass is somewhat fast**.
- Complexity of forward pass, **linear of depth  $O(L)$** .



# A Generic Neural Network: Direct Gradients

$$\mathbf{x} = \mathbf{h}_0 \in \mathbb{R}^{d_0} \text{ (input)}$$

$$\mathbf{h}_{i+1} = f_i(\mathbf{W}_i \mathbf{h}_i) \in \mathbb{R}^{d_i}$$

$$(0 \leq i \leq L - 1)$$

$$y = \mathbf{u}^T \mathbf{h}_L \in \mathbb{R} \text{ (output)}$$

$$\mathcal{L} = \ell(y, y^*) \in \mathbb{R} \text{ (loss)}$$

$$\mathbf{W} = \{\mathbf{W}_0, \mathbf{W}_1, \dots, \mathbf{W}_L, \mathbf{u}\}$$

We want the gradients of  $\mathcal{L}$  with respect to model parameters.

- $\nabla_{\mathcal{L}}(\mathbf{W}_{L-1}) = (\mathbf{J}_{\mathcal{L}}(\mathbf{W}_{L-1}))^T = (\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{W}_{L-1}))^T$
- $\nabla_{\mathcal{L}}(\mathbf{W}_{L-2}) = (\mathbf{J}_{\mathcal{L}}(\mathbf{W}_{L-2}))^T = (\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}))^T$
- ...
- $\nabla_{\mathcal{L}}(\mathbf{W}_0) = (\mathbf{J}_{\mathcal{L}}(\mathbf{W}_{L-3}))^T = (\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \dots \mathbf{J}_{\mathbf{h}_1}(\mathbf{W}_0))^T$

3 matrix multiplications

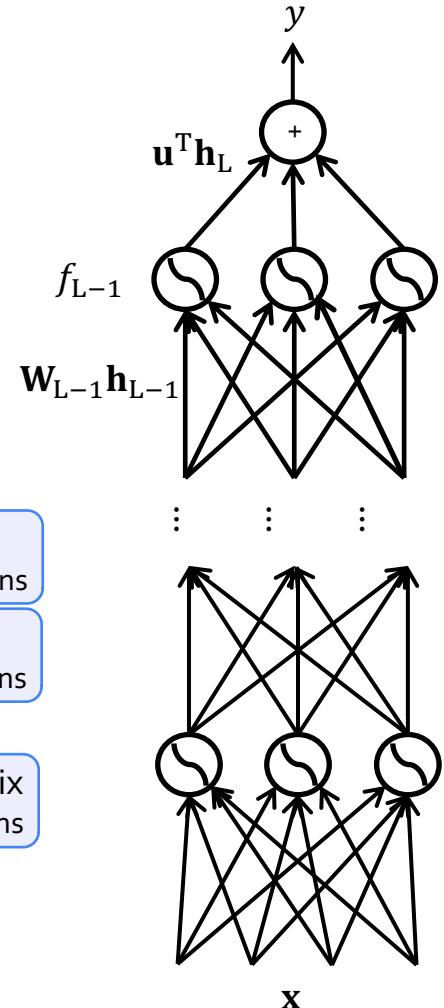
4 matrix multiplications

$L + 2$  matrix multiplications

In total, how many matrix multiplications are done here?

- (A)  $O(L)$    (B)  $O(L^2)$    (C)  $O(L^3)$    (D)  $O(\exp(L))$

Can we do better than this? 🤔



# A Generic Neural Network: Gradients

with Caching/Memoization

$$\nabla_{\mathcal{L}}(\mathbf{W}_{L-1}) = \left( \mathbf{J}_\ell(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{W}_{L-1}) \right)^T = \left( \delta_L \mathbf{J}_{\mathbf{h}_L}(\mathbf{W}_{L-1}) \right)^T$$

$$\nabla_{\mathcal{L}}(\mathbf{W}_{L-2}) = \left( \mathbf{J}_\ell(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}) \right)^T = \left( \delta_{L-1} \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}) \right)^T$$

...

$$\nabla_{\mathcal{L}}(\mathbf{W}_0) = \left( \mathbf{J}_\ell(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \dots \mathbf{J}_{\mathbf{h}_1}(\mathbf{W}_0) \right)^T = \left( \delta_1 \mathbf{J}_{\mathbf{h}_1}(\mathbf{W}_0) \right)^T$$

- Parameter gradients **depend on the gradients of the earlier layers!**
- So, when computing gradients at each layer, **we don't need to start from scratch!**
- I can **reuse gradients** computed for higher layers for lower layers (i.e., memoization).

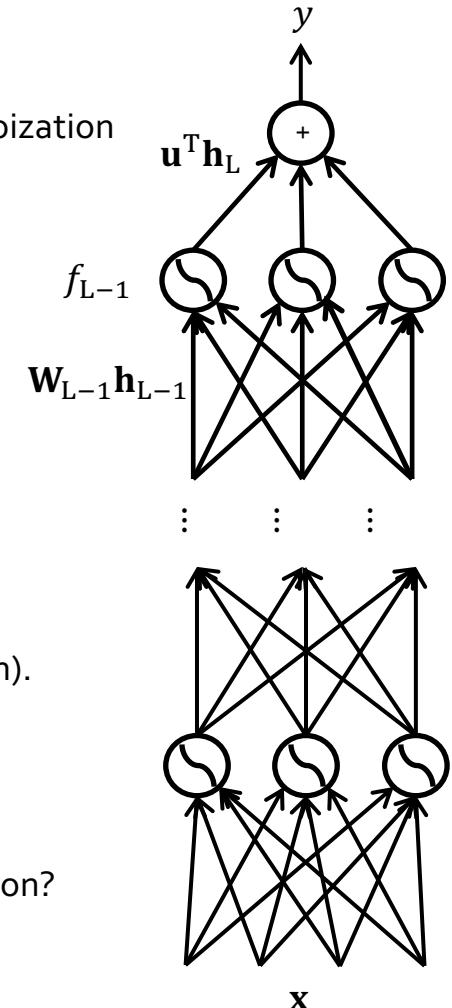
Let  $\delta_i$  denote Jacobian at the output of layer  $i$ :

$$\delta_i = \mathbf{J}_\ell(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \dots \mathbf{J}_{\mathbf{h}_i}(\mathbf{h}_{i-1})$$

$$\delta_i = \delta_{i+1} \mathbf{J}_{\mathbf{h}_i}(\mathbf{h}_{i-1})$$

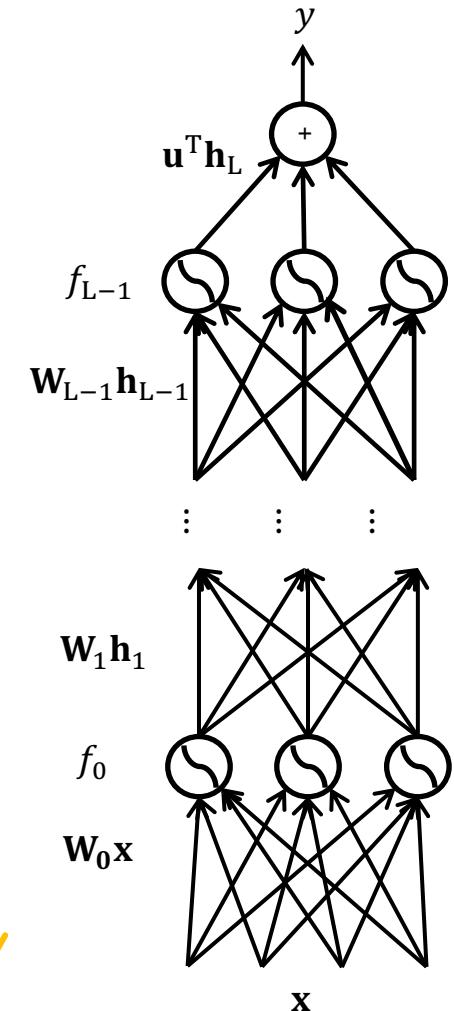
In total, how many matrix multiplications are done here when using caching/memoization?

- (A)  $O(L)$    (B)  $O(L^2)$    (C)  $O(L^3)$    (D)  $O(\exp(L))$



# A Generic Neural Network: Backward Step

- Backward step computes the gradients starting from the end to the beginning, layer by layer.
- Start by computing **local gradients**:  $\mathbf{J}_{\mathbf{h}_i}(\mathbf{h}_{i-1})$
- Use then to compute **upstream gradients**  $\delta_L$ , then  $\delta_{L-1}$ , then  $\delta_{L-2}$ , ....
- Use these to compute **global gradients**:  $\nabla_{\mathcal{L}}(\mathbf{W}_i)$
- Computational cost as a function of depth:
  - With memoization, gradient computation is a **linear** function of depth L
    - (same cost as the forward process!!)
  - Without memorization, gradients computation would grow **quadratic** with L



# A Generic Neural Network: Back Propagation

Initialize network parameters with random values

Loop until convergence

Loop over training instances

i. **Forward step:**

Start from the input and compute all the layers till the end (loss  $\mathcal{L}$ )

In practice, this step is done over **batches** of instances!

ii. **Backward step:**

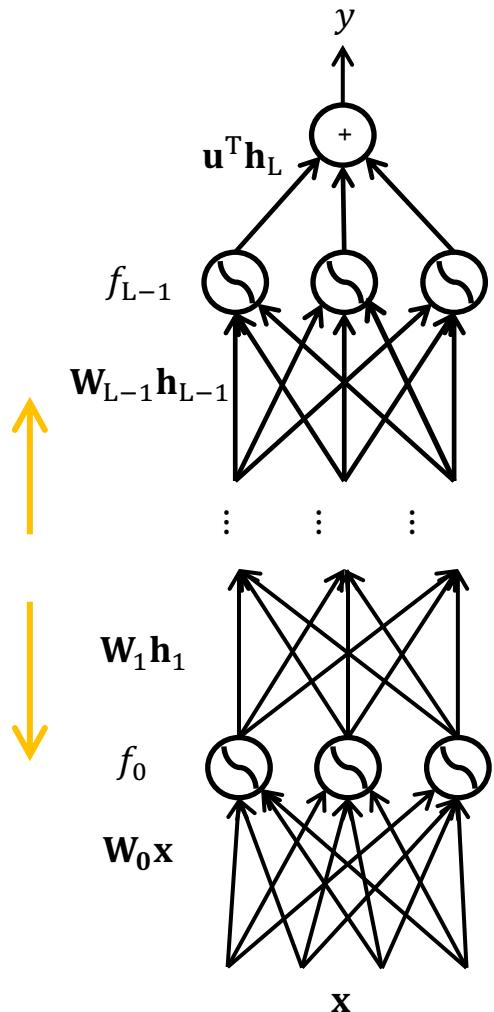
Compute **local gradients**, starting from the last layer

Compute **upstream gradients**  $\delta_i$  values, starting from the last layer

Use  $\delta_i$  values to compute global gradients  $\nabla_{\mathcal{L}}(\mathbf{W}_i)$  at each layer

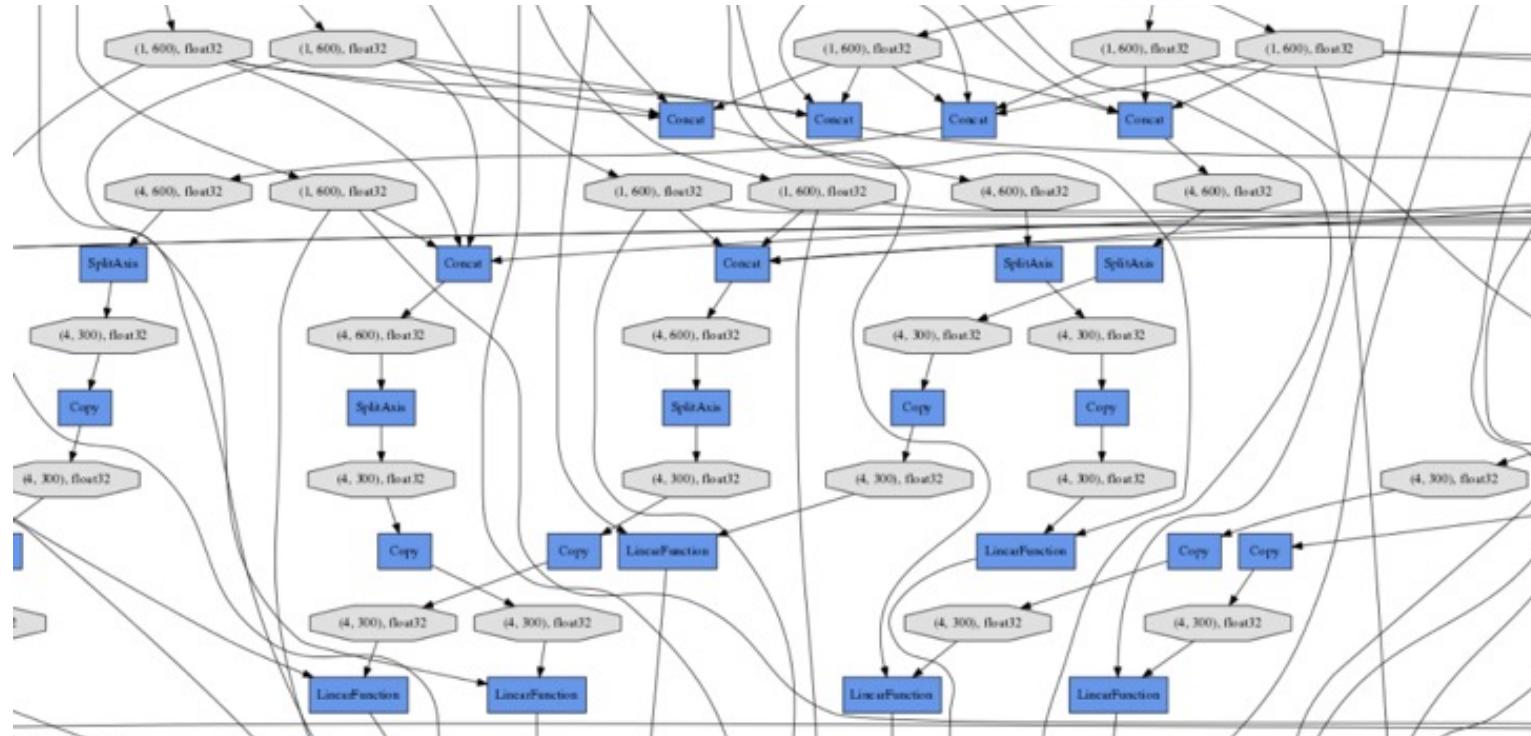
iii. **Gradient update:**

Update each parameter:  $\mathbf{W}_i^{(t+1)} \leftarrow \mathbf{W}_i^{(t)} - \alpha \nabla_{\mathcal{L}}(\mathbf{W}_i)$



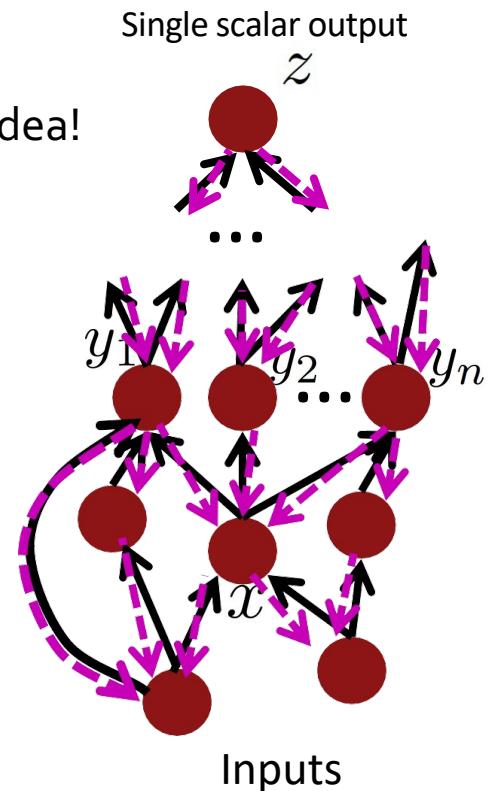
# Computation Graph: Example

- In reality, networks are not as regular as the previous example ...



# Back-Prop in General Computation Graph

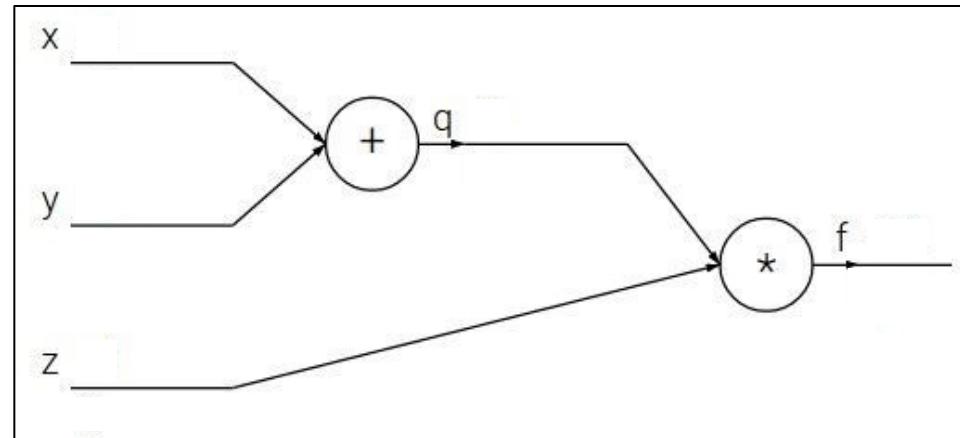
- What if the network does not have a regular structure? Same idea!
- Sort the nodes in **topological order** (what depends on what)
- Forward-Propagation:
  - Visit nodes in topological sort order and compute value of node given predecessors
- Backward-Propagation:
  - Compute **local gradients**
  - Visit nodes in reverse order and compute **global gradients** using gradients of successors



# Computation Graph: An Example

$$f(x, y, z) = (x + y)z$$

- Evaluated at:  $x = -2, y = 5, z = -4$

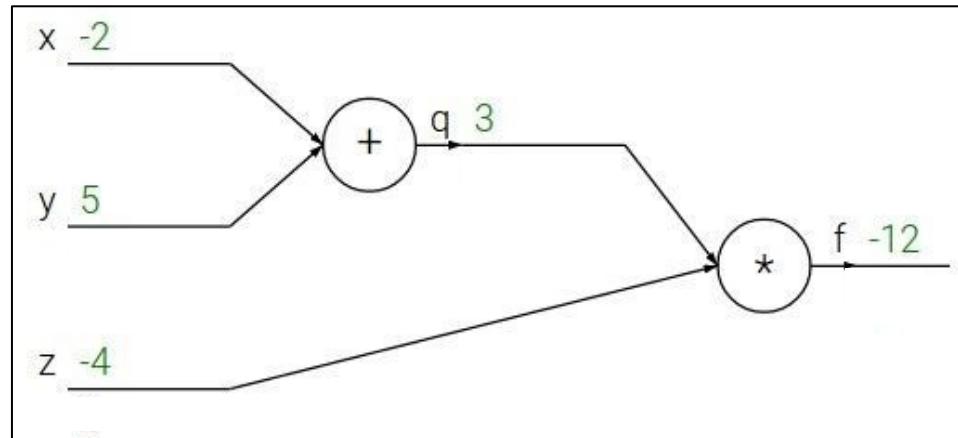


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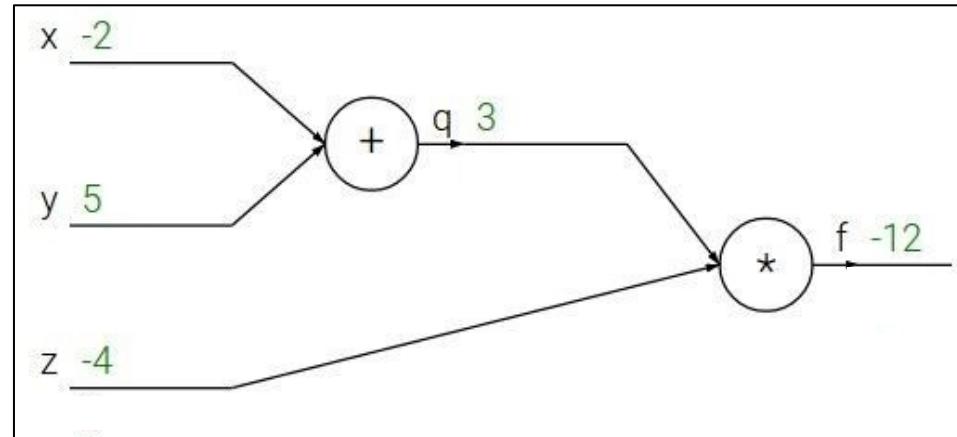
Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



# Computation Graph: An Example

$$f(x, y, z) = (x + y)z$$

- Evaluated at:  $x = -2, y = 5, z = -4$
- Start with local gradients!



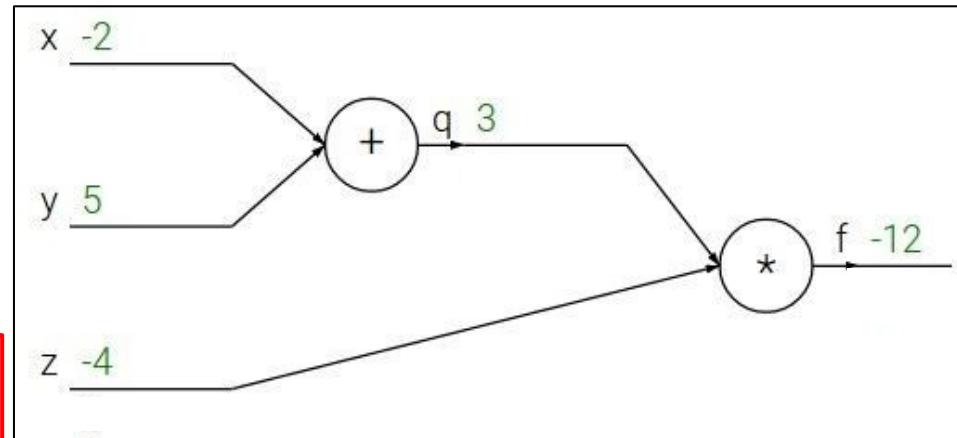
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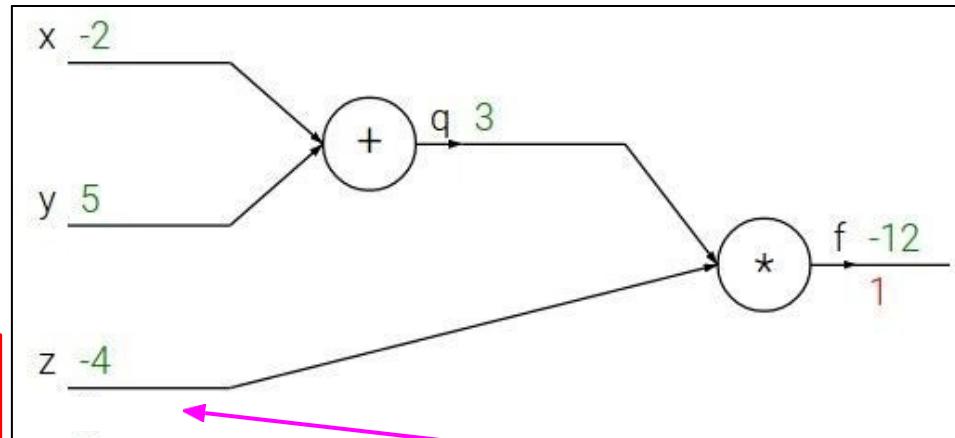
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$$\frac{\partial f}{\partial z}$$

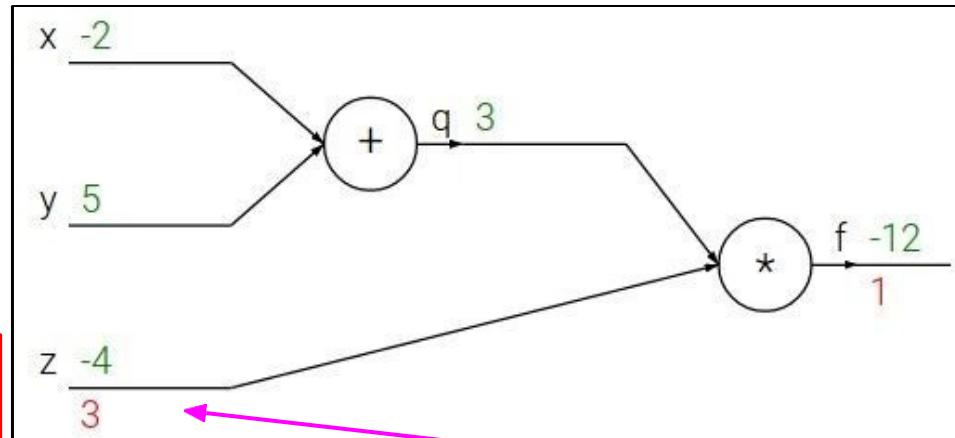
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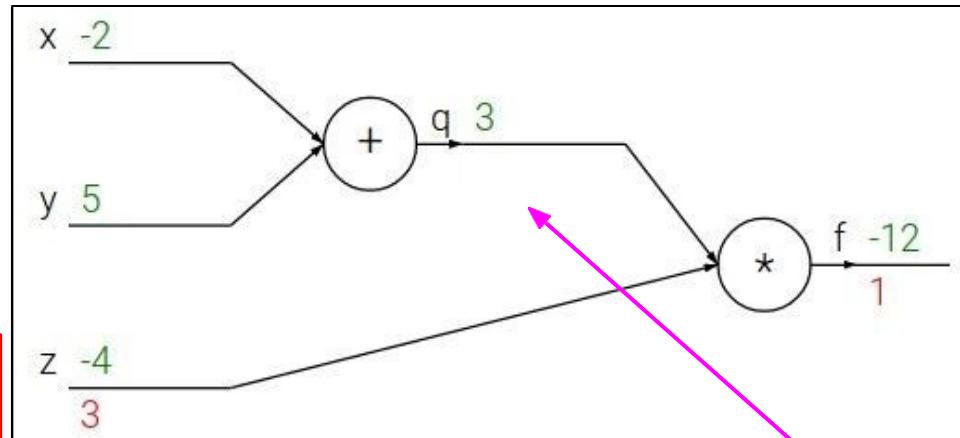
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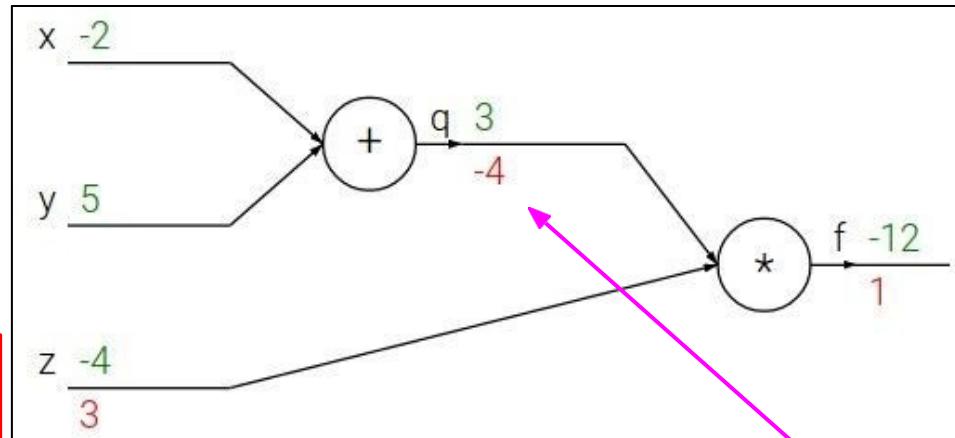
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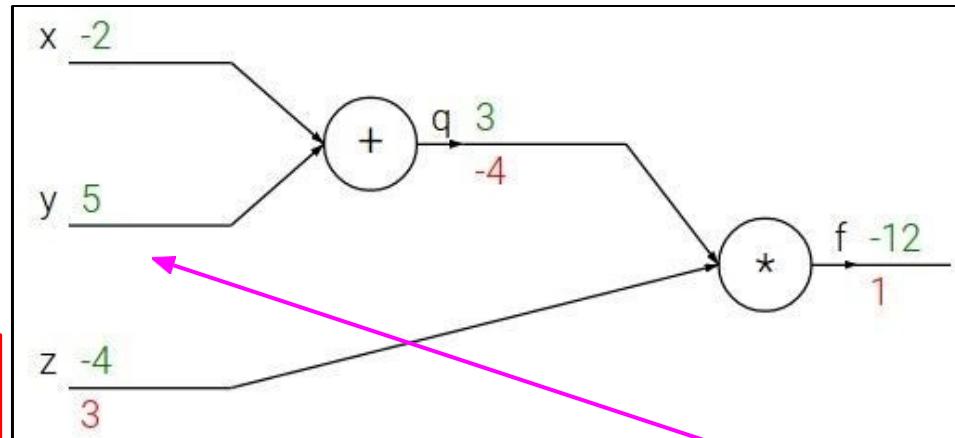
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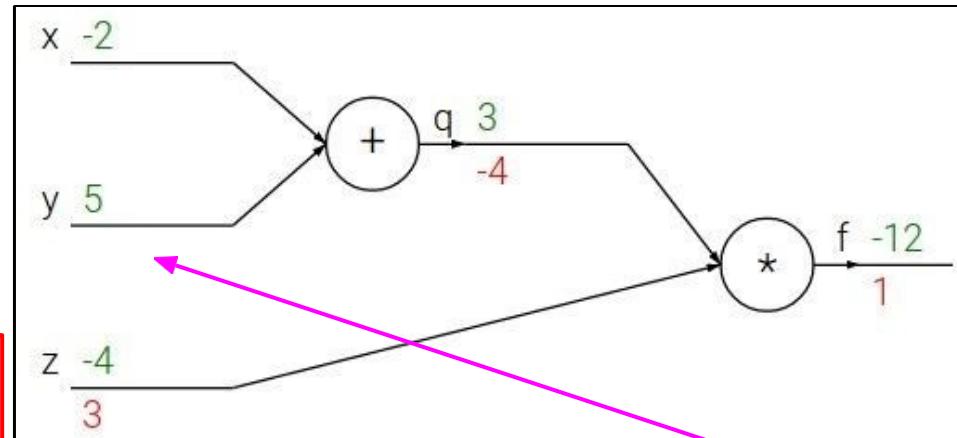
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Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

Upstream  
gradient

Local  
gradient

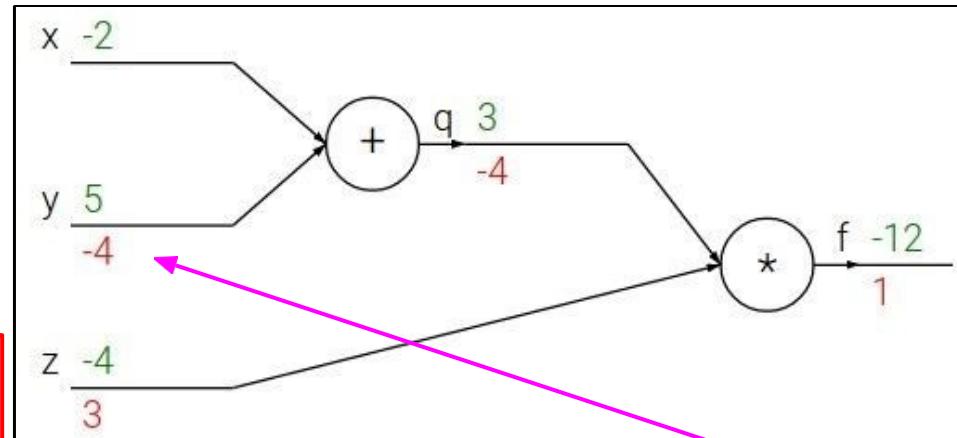
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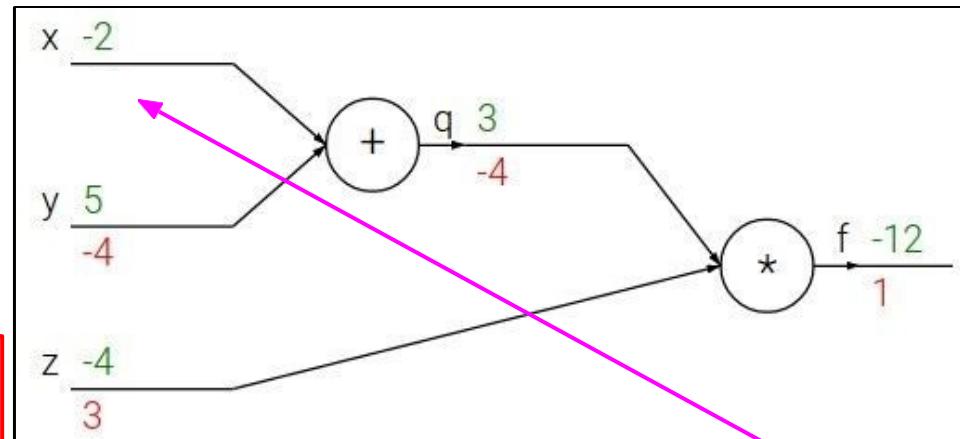
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$$\frac{\partial f}{\partial x}$$

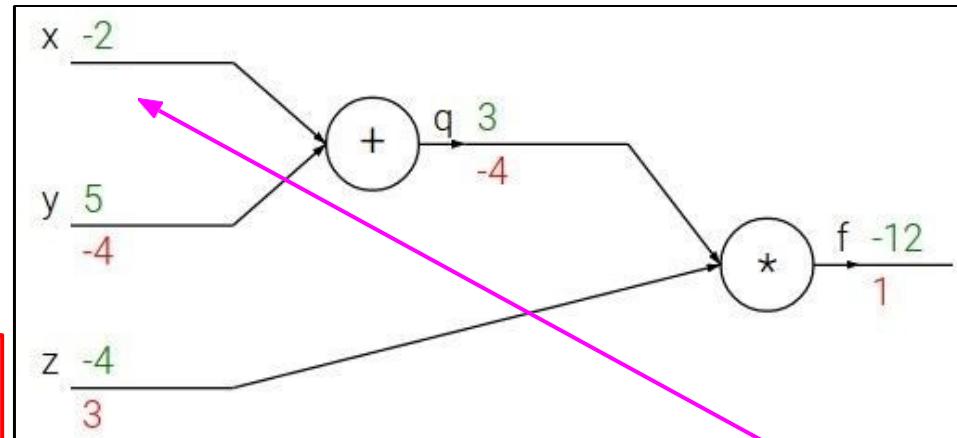
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Local  
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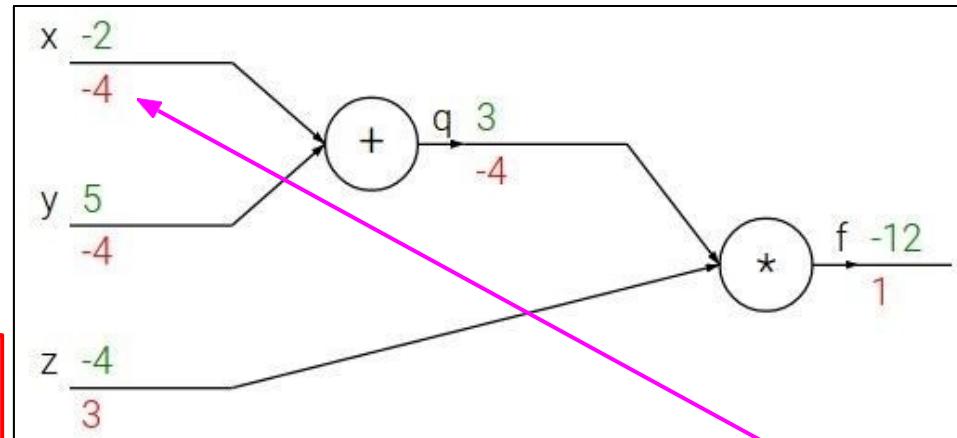
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Chain rule:

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Upstream  
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Local  
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# A Generic Example

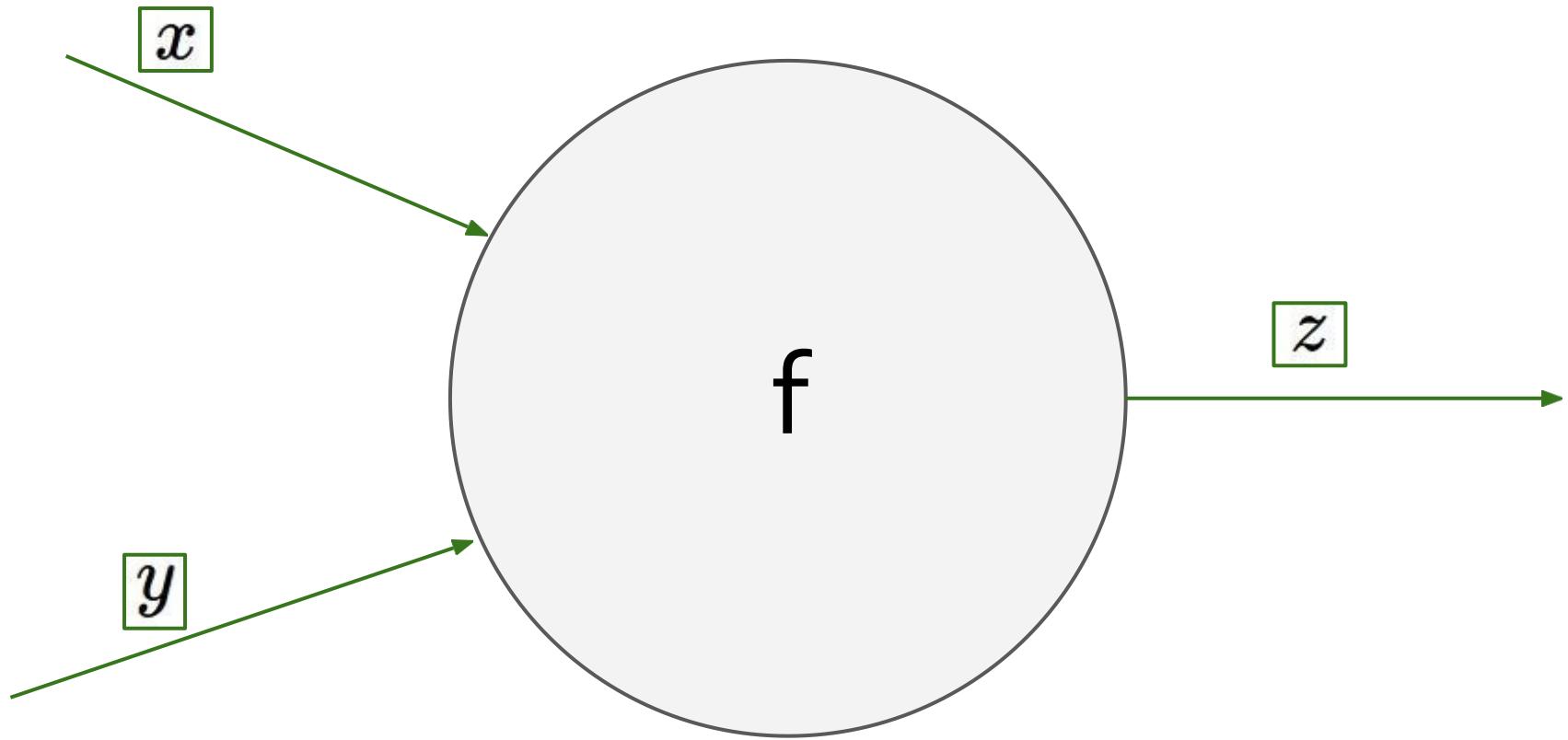


Figure from Andrej Karpathy

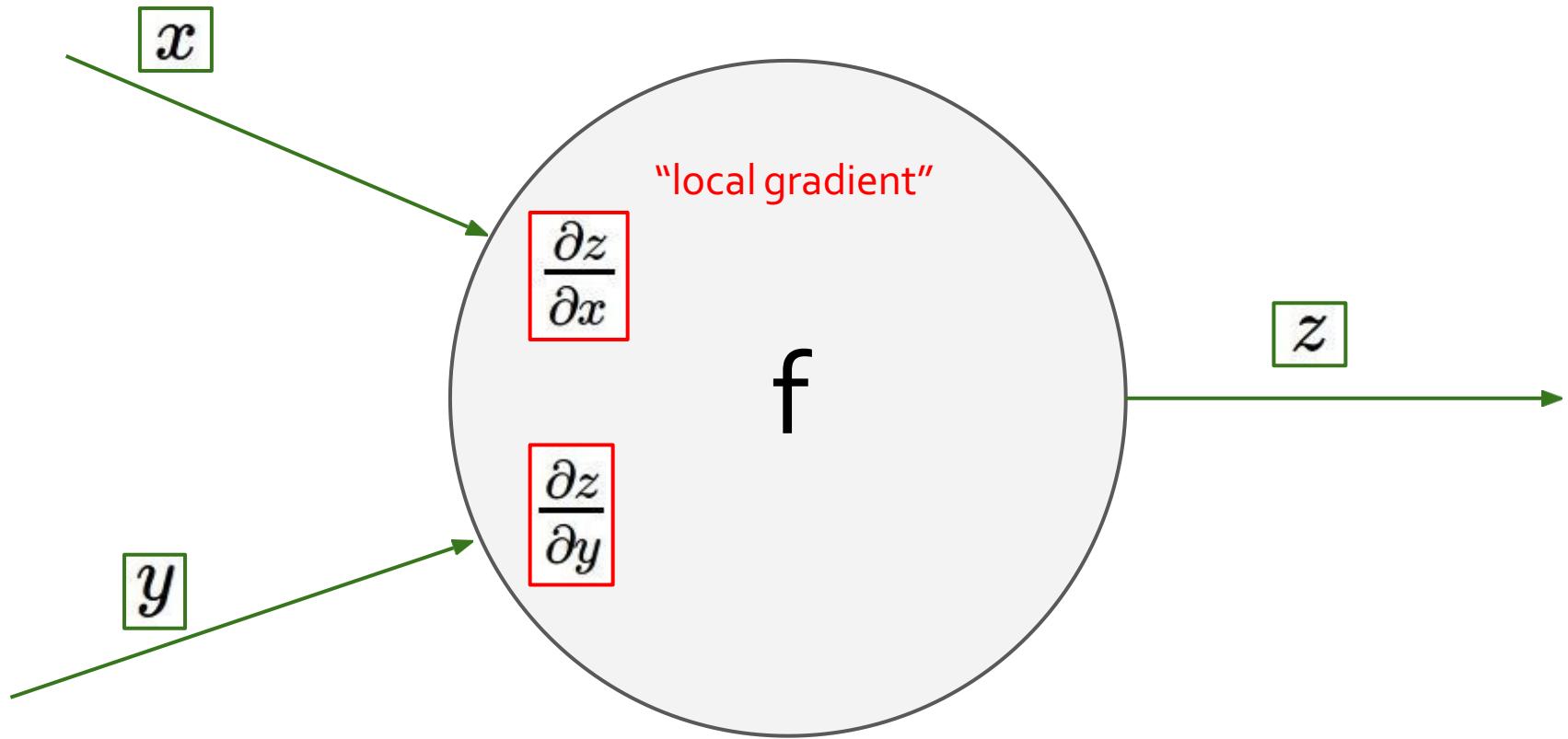
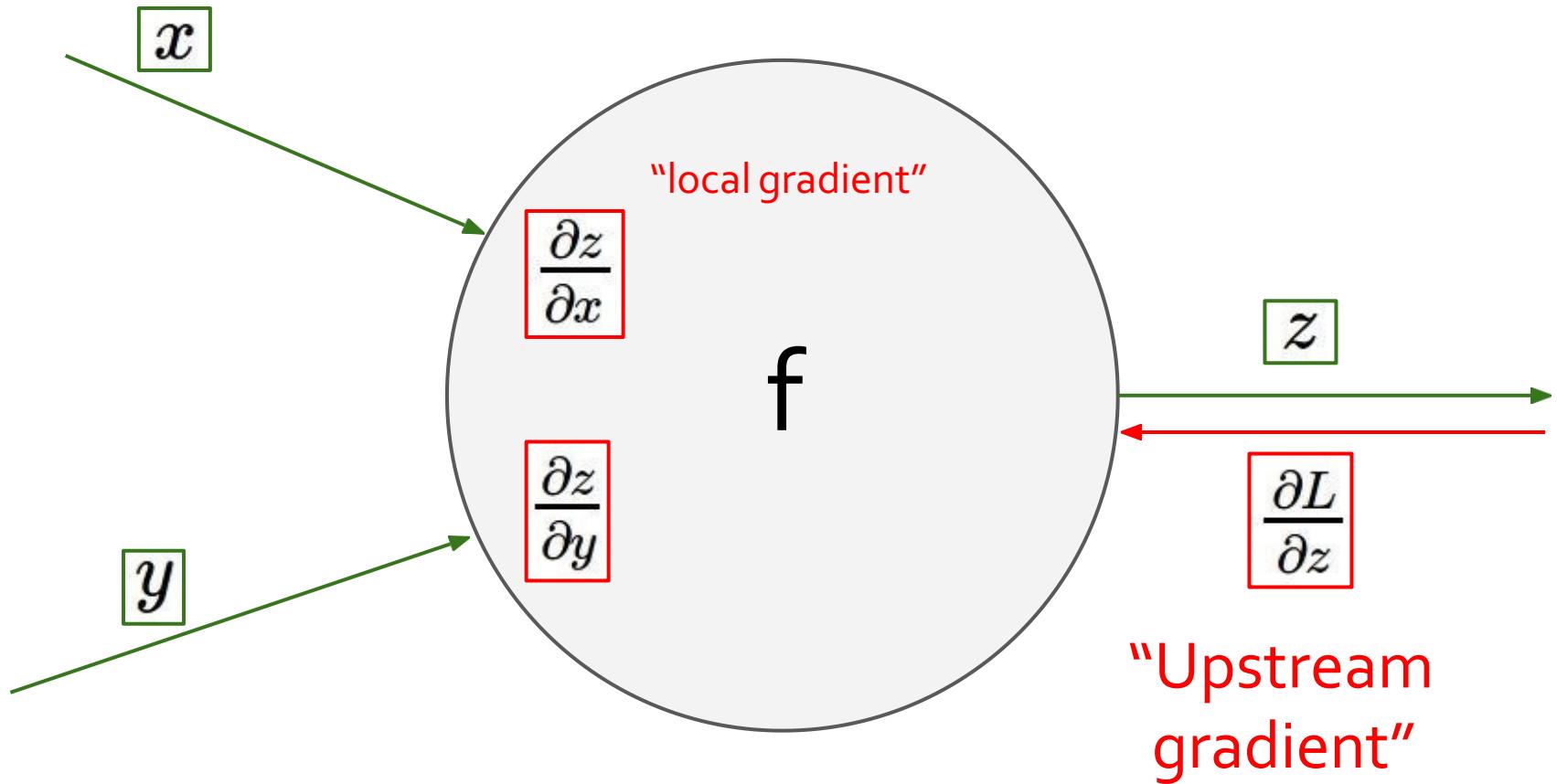
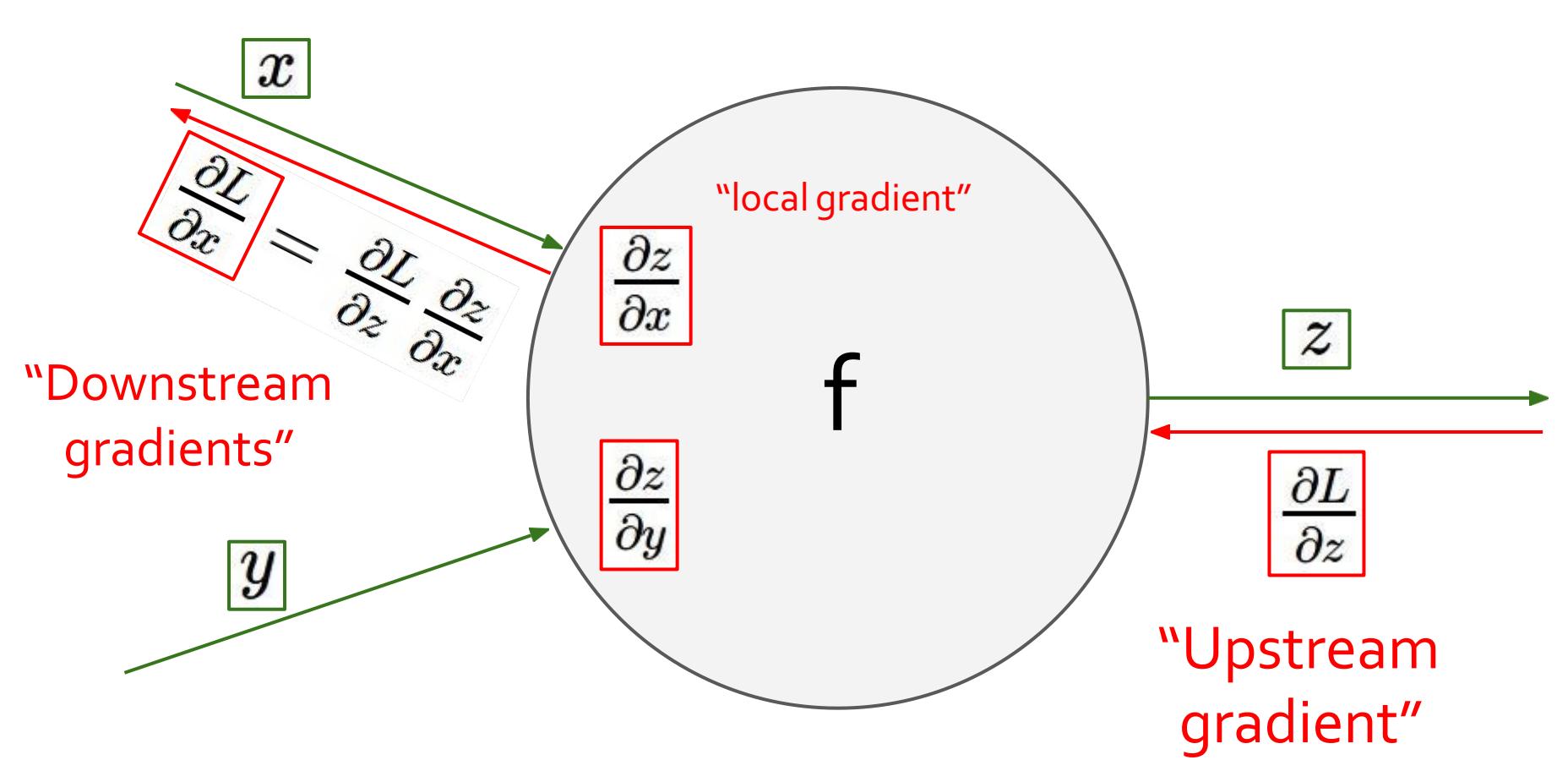
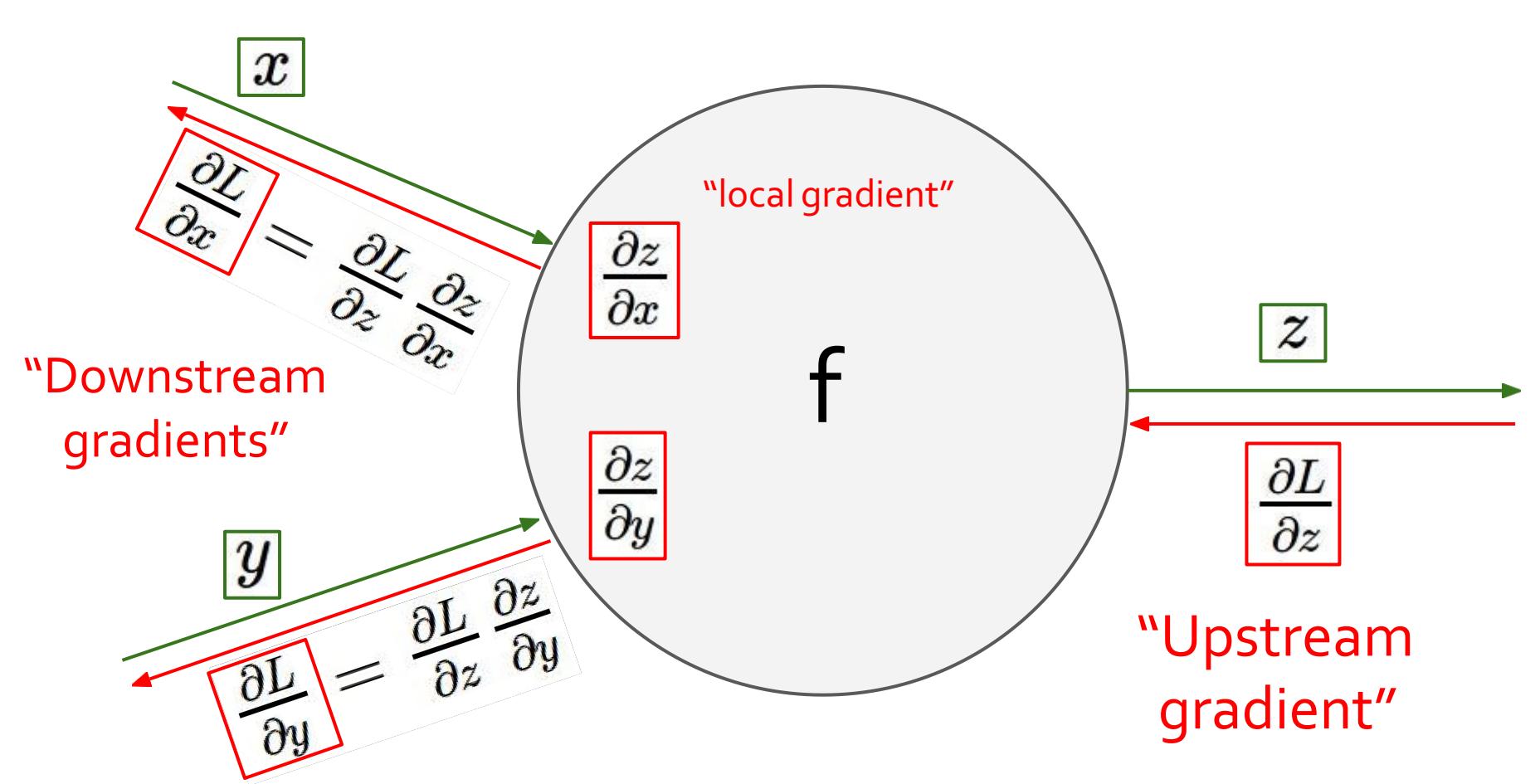


Figure from Andrej Karpathy







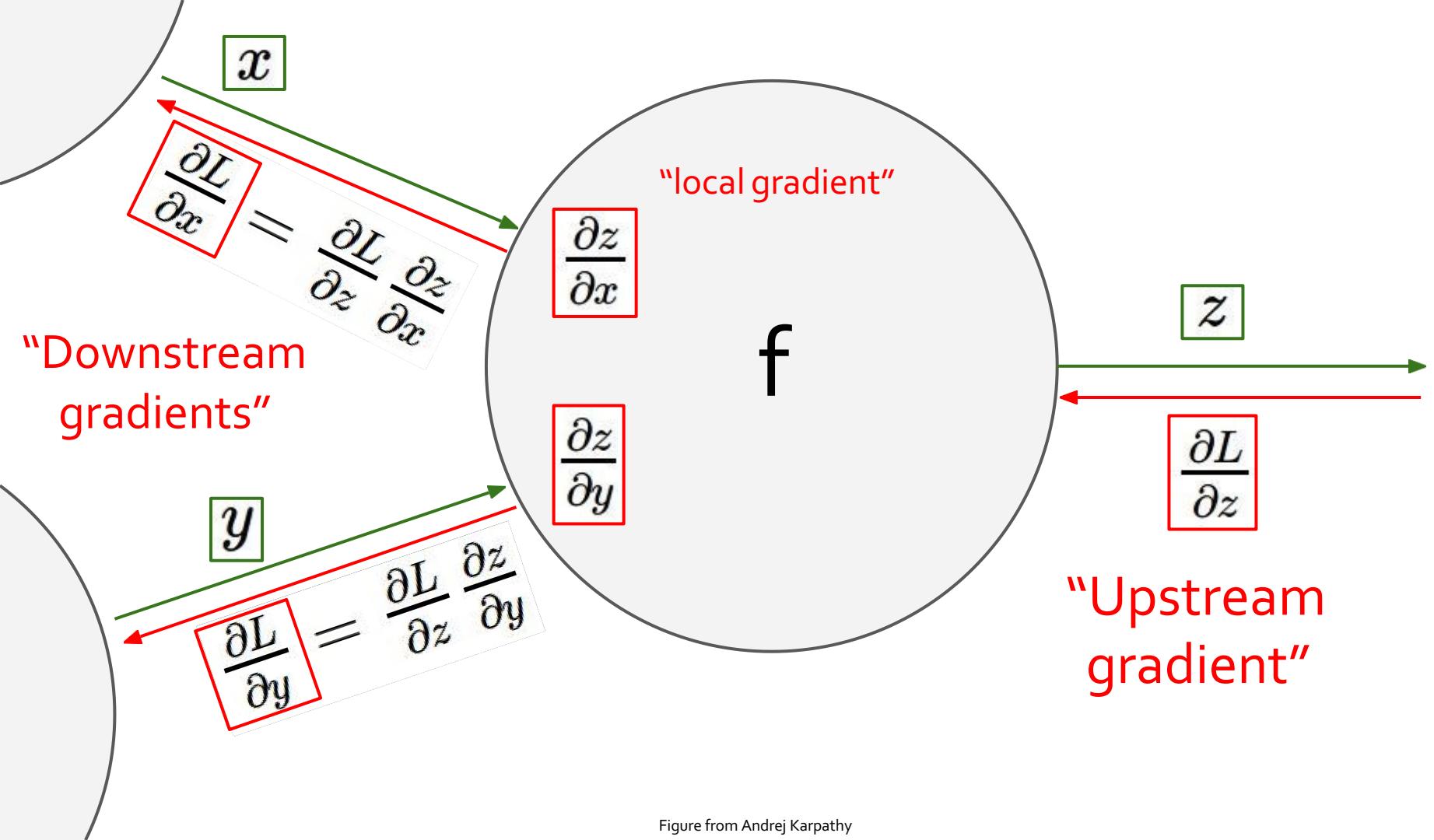


Figure from Andrej Karpathy

# Demo time!

- Link: <https://playground.tensorflow.org/>

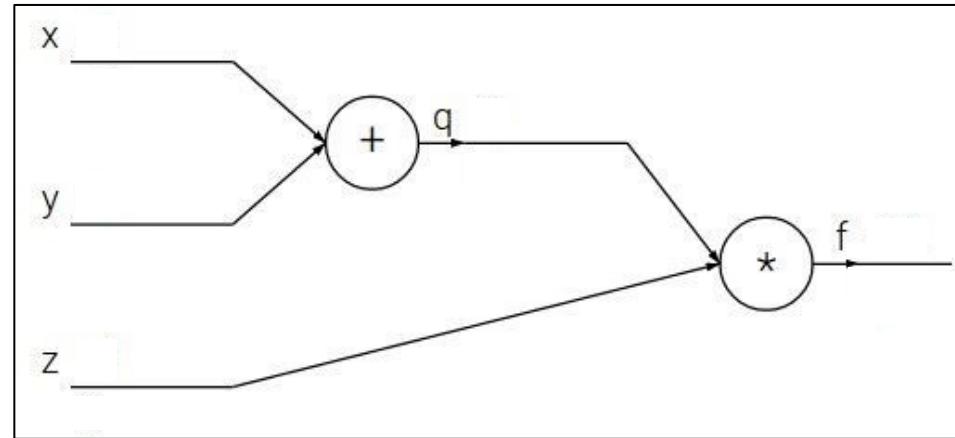
# Chapter Plan

1. Feed-forward networks
2. Neural nets: brief history
3. Word2Vec as a simple neural network
4. Training neural networks: back-propagation
5. Backprop in practice

# Backprop in PyTorch

$$f(x, y, z) = (x + y)z$$

Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



```
x = torch.tensor(-2.0, requires_grad=True)
y = torch.tensor(5.0, requires_grad=True)
z = torch.tensor(-4.0, requires_grad=True)

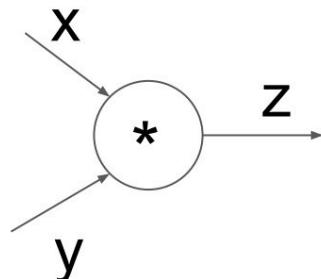
f = (x+y)*z # Define the computation graph

f.backward() # PyTorch's internal backward gradient computation

print('Gradients after backpropagation:', x.grad, y.grad, z.grad)
```

# PyTorch's Implementation: Forward/Backward API

- PyTorch has implementation of forward/backward operations for various operators.
- Example: multiplication operator



```
class Multiply(torch.autograd.Function):  
    @staticmethod  
    def forward(ctx, x, y):  
        ctx.save_for_backward(x, y) ← Need to cash some values for use in backward  
        z = x * y  
        return z  
    @staticmethod  
    def backward(ctx, grad_z): ← Upstream gradient  
        x, y = ctx.saved_tensors  
        grad_x = y * grad_z # dz/dx * dL/dz  
        grad_y = x * grad_z # dz/dy * dL/dz  
        return grad_x, grad_y ← Multiply upstream and local gradients
```

# PyTorch Operators

A screenshot of the PyTorch GitHub repository interface. The top navigation bar shows 'pytorch/pytorch' as the repository, with 'Public' selected. Below the bar are buttons for 'Code', 'Issues', 'Pull requests', 'Actions', 'Projects', 'Wiki', 'Security', and 'Insights'. The 'Code' tab is active. A search bar at the top right contains the query 'vulkan'. Below the search bar, a dropdown menu shows the current search term 'd0a4e2e782' and the full search path 'pytorch / aten / src / ATen / native / vulkan / glsl /'. The main content area displays a list of pull requests related to Vulkan support, with titles like 'manuelcandales and pytorchmergebot [Vulkan] Enable copying QInt8 and QInt32 tensors from cpu to vulkan.', 'adaptive\_avg\_pool2d.glsl', and 'add\_gisl'. Each entry includes a 'Go to file' button.

PyTorch's lower-level functions translate activities to graphics processor via libraries like OpenGL

mul_scalar_gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
nchw_to_image.gisl	[vulkan] Enable 2D texture types (#86971)	2 months ago
nchw_to_image2d.gisl	[vulkan] Enable 2D texture types (#86971)	2 months ago
nchw_to_image_int32.gisl	[Vulkan] Enable copying QInt8 and QInt32 tensors from cpu to vulkan. (#...)	last month
nchw_to_image_int8.gisl	[Vulkan] Enable copying QInt8 and QInt32 tensors from cpu to vulkan. (#...)	last month
nchw_to_image_uint8.gisl	[Vulkan] Enable copying QInt8 and QInt32 tensors from cpu to vulkan. (#...)	last month
permute_4d.gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
quantize_per_tensor_qint32.gisl	[Vulkan] Enable QInt8 and QInt32 quantization (#89788)	last month
quantize_per_tensor_qint8.gisl	[Vulkan] Enable QInt8 and QInt32 quantization (#89788)	last month
quantize_per_tensor_quint8.gisl	[Vulkan] Enable QInt8 and QInt32 quantization (#89788)	last month
quantized_add.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_conv2d.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_conv2d_dw.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_conv2d_pw_2x2.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_div.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_mul.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_sub.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
quantized_upsample_nearest2d.gisl	[Vulkan][TCC] Fix quantized shaders (#89456)	last month
reflection_pad2d.gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
replication_pad2d.gisl	[vulkan] replication_pad2d.gisl: use clamp() instead of min(max()) (#...)	7 months ago
select_depth.gisl	[Vulkan] Implement select.int operator (#81771)	5 months ago
sigmoid.gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
sigmoid_gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
slice_4d.gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
softmax.gisl	[vulkan] Add image format qualifier to glsl files (#69330)	last year
stack_feature.gisl	[Vulkan] Implement Stack operator (#81064)	5 months ago
sub.gisl	[Vulkan] Implement arithmetic ops where one of the arguments is a ten...	5 months ago
sub_gisl	[Vulkan] Implement arithmetic ops where one of the arguments is a ten...	5 months ago
tanh.gisl	[vulkan] Clamp tanh activation op input to preserve numerical stabili...	10 months ago
tanh_olsl	[vulkan] Clamp tanh activation op input to preserve numerical stabili...	10 months ago

# Example Activation Functions

master pytorch / aten / src / ATen / native / vulkan / [glsl](#) / sigmoid.glsl

SS-JIA [vulkan] Add image format qualifier to glsl files (#69330) ...

1 contributor

23 lines (17 sloc) | 710 Bytes

```
1 #version 450 core
2 #define PRECISION $precision
3 #define FORMAT    $format
4
5 layout(std430) buffer;
6
7 /* Qualifiers: layout - storage - precision - memory */
8
9 layout(set = 0, binding = 0, FORMAT) uniform PRECISION restrict writeonly image3D  uOutput;
10 layout(set = 0, binding = 1)          uniform PRECISION                         sampler3D uInput;
11 layout(set = 0, binding = 2)          uniform PRECISION restrict           Block {
12     ivec4 size;
13 } uBlock;
14
15 layout(local_size_x_id = 0, local_size_y_id = 1, local_size_z_id = 2) in;
16
17 void main() {
18     const ivec3 pos = ivec3(gl_GlobalInvocationID);
19
20     if (all(lessThan(pos, uBlock.size.xyz))) {
21         imageStore(uOutput, pos, 1/(1+exp(-1*texelFetch(uInput, pos, 0))));
22     }
23 }
```

master pytorch / aten / src / ATen / native / vulkan / [glsl](#) / tanh.glsl

SS-JIA [vulkan] Clamp tanh activation op input to preserve numerical stabili... ...

2 contributors

27 lines (21 sloc) | 777 Bytes

```
1 #version 450 core
2 #define PRECISION $precision
3 #define FORMAT    $format
4
5 layout(std430) buffer;
6
7 /* Qualifiers: layout - storage - precision - memory */
8
9 layout(set = 0, binding = 0, FORMAT) uniform PRECISION restrict writeonly image3D  uOutput;
10 layout(set = 0, binding = 1)          uniform PRECISION                         sampler3D uInput;
11 layout(set = 0, binding = 2)          uniform PRECISION restrict           Block {
12     ivec4 size;
13 } uBlock;
14
15 layout(local_size_x_id = 0, local_size_y_id = 1, local_size_z_id = 2) in;
16
17 void main() {
18     const ivec3 pos = ivec3(gl_GlobalInvocationID);
19
20     if (all(lessThan(pos, uBlock.size.xyz))) {
21         const vec4 intex = texelFetch(uInput, pos, 0);
22         imageStore(
23             uOutput,
24             pos,
25             tanh(clamp(intex, -15.0, 15.0)));
26     }
27 }
```

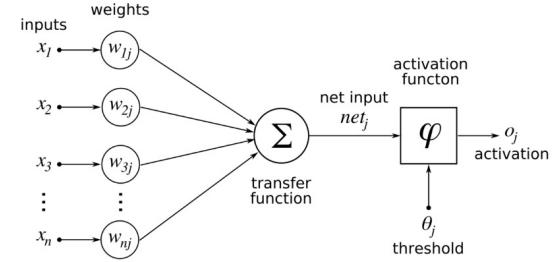
# Why Learn All These Details About Backprop?

- **Modern deep learning frameworks compute gradients for you!**
- But why take a class on compilers or systems when they are implemented for you?
  - Understanding what is going on under the hood is useful!
- Backpropagation doesn't always work perfectly out of the box
  - Understanding why is crucial for debugging and improving models

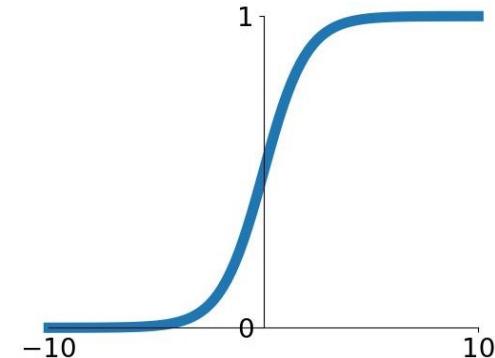
# Backprop in Practice

# Activation Functions

- How do you choose what activation function to use?
- In general, it is problem-specific and might require trial-and-error.
- Here are some tips about popular action functions.



# Activation Functions : Sigmoid

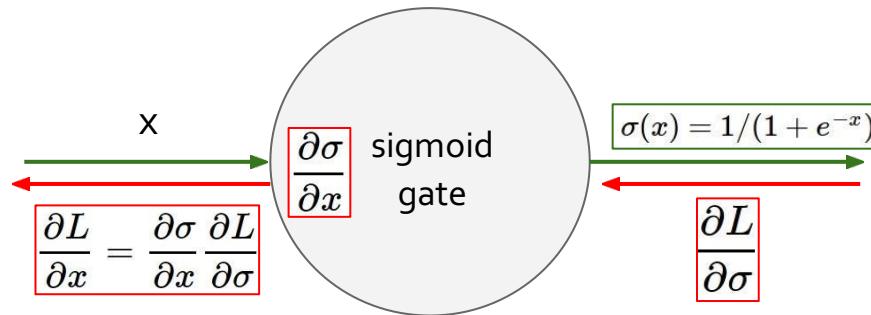


- Squashes numbers to range [0,1]
- Historically popular, interpretation as “firing rate” of a neuron
- Key limitation: Saturated neurons “kill” the gradients
- Whenever  $|x| > 5$ , the gradients are basically zero.

$$\sigma(x) = 1/(1 + e^{-x})$$

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

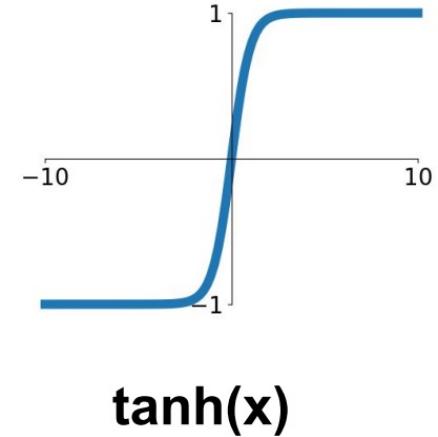
If all the gradients flowing back will be zero and weights will never change.



# Activation Functions : Tanh

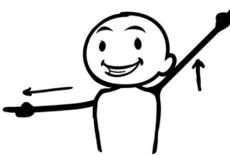


- Symmetric around  $[-1, 1]$
- Still saturates  $|x| > 3$  and “kill” the gradients
- Zero-centered — good for stacking hidden layers

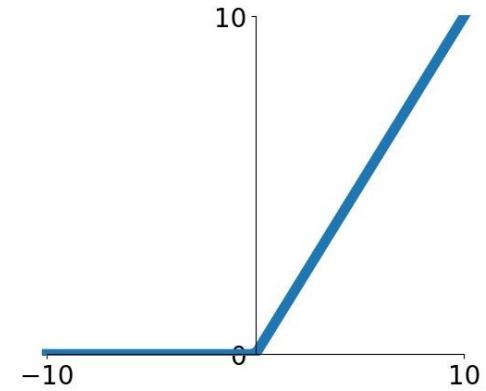


[LeCun et al., 1991]

# Activation Functions : ReLU



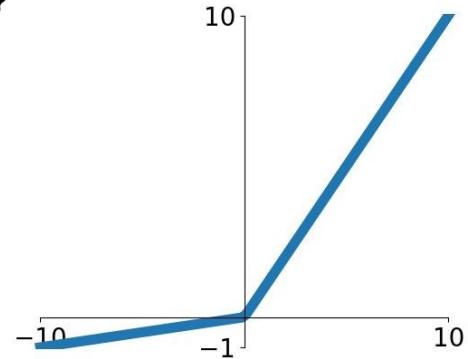
- Computationally efficient
- In practice, converges faster than sigmoid/tanh in practice
- Does not saturate (in +region) — will die less!



ReLU  
(Rectified Linear Unit)

[Krizhevsky et al., 2012]

# Activation Functions : Leaky ReLU



$$f(x) = \max(0.01x, x)$$

- Does not saturate — will not die.
- Computationally efficient
- In practice it converges faster than sigmoid/tanh in practice
- Other parametrized variants:
  - Parametric Rectifier (PReLU):  $f(x) = \max(\alpha x, x)$  [He et al., 2015]
  - Maxout:  $\max(w_1^T x + b_1, w_2^T x + b_2)$  [Goodfellow et al., 2013]
- Provide more flexibility, though at the cost of more learnable parameters.
  - For example, Maxout doubles the number of parameters.

# How do You Choose What Activation Function to Use?

- In general, it is problem-specific and might require trial-and-error.
- A useful recipe:
  1. Generally, ReLU is a good activation to start with.
  2. Time/compute permitting, you can try other activations to squeeze out more performance.

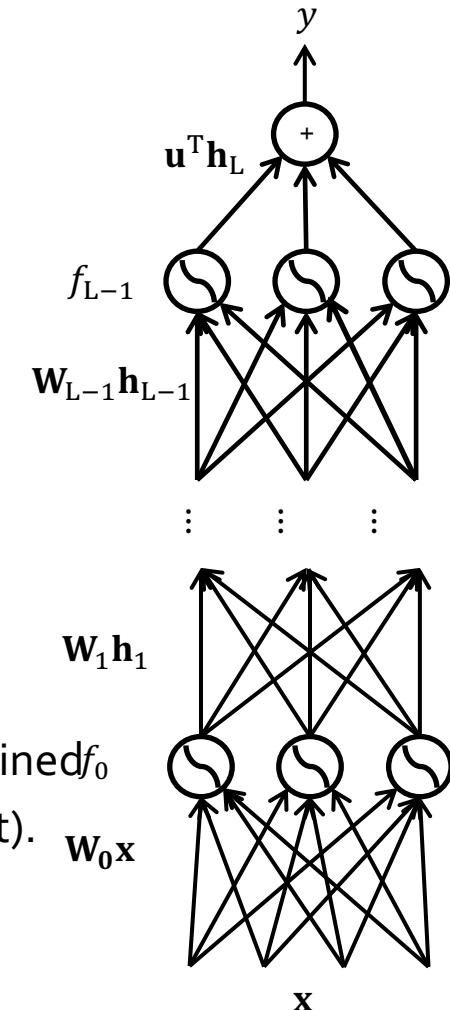
# Exploding/Vanishing Gradients

- Remember gradient computation at layer  $L - k$ :

$$\nabla_{\mathcal{L}}(\mathbf{W}_{L-k}) = \left( \mathbf{J}_\ell(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}) \dots \mathbf{J}_{\mathbf{h}_{L-k+1}}(\mathbf{W}_{L-k}) \right)^T$$

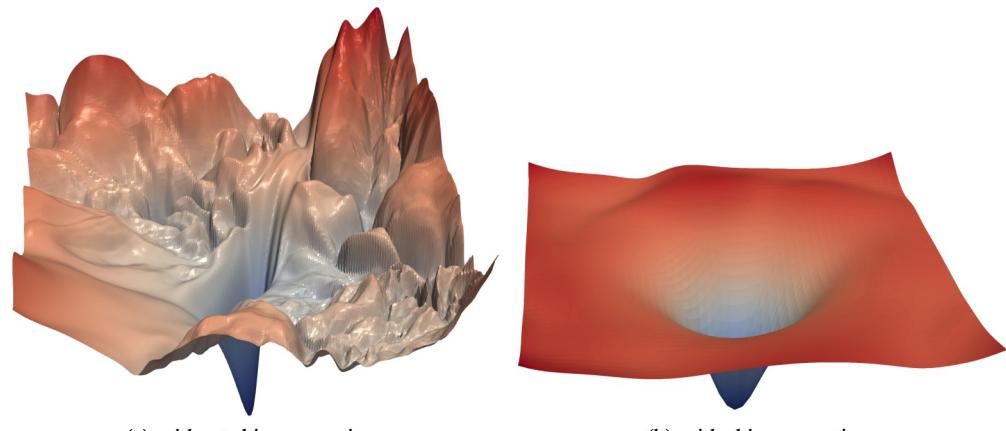
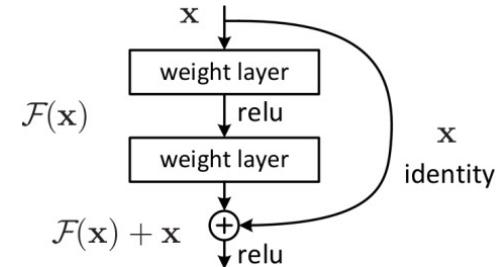
$\underbrace{\hspace{10em}}$   
 $O(k)$ -many matrix multiplication

- This matrix multiplication could quickly approach
  - $\infty$ , if the matrix elements are large — exploding gradients.
  - 0, if the matrix elements are small — vanishing gradients.
- For those interested, convergences of matrix powers is determined by its largest eigenvalue (out of scope for this class, extra credit).
- $\infty/0$  gradients would kill learning (no flow of information).



# Residual Connections/Blocks

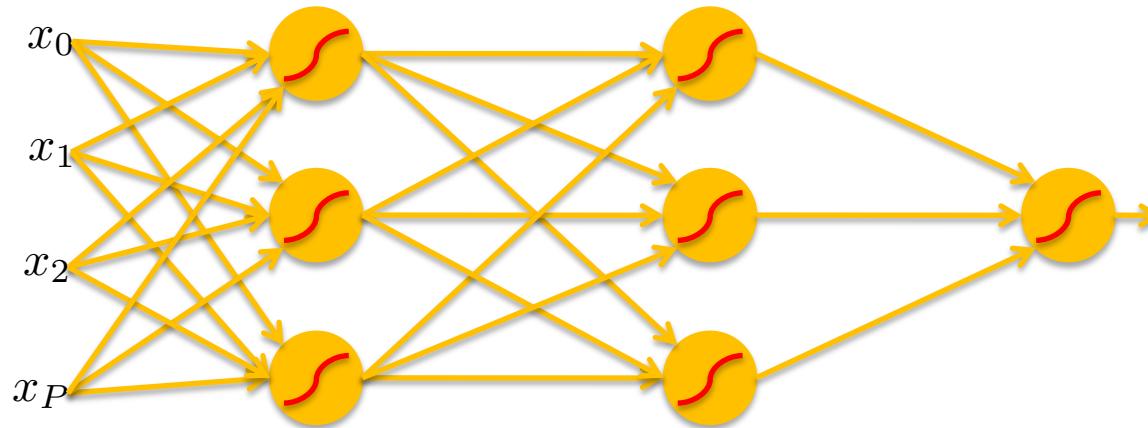
- Create direct “information highways” between layers.
- Shown to diminish the effect of vanishing/exploding gradients
  - Early in the training, there are fewer layers to propagate through.
  - The network would restore the skipped layers, as it learns richer features.
  - It is also shown to make the optimization objective smoother.
  - Fun fact: [the paper](#) introducing residual layers (He et al. 2015) is the most cited paper of century.



[Li et al. “Visualizing the Loss Landscape of Neural Nets”]

# Weight Initialization

- Initializing all weights with a fixed constant (e.g., 0) is a very bad idea! (why?)



- If the neurons start with the same weights, then all the neurons will follow the same gradient, and will always end up doing the same thing as one another.

# Weight Initialization

- Better idea: initialize weights with random Gaussian noise.

```
x = torch.tensor.empty(3, 5)
nn.init.normal_(w)
```

- There are fancier initializations (Xavier, Kaiming, etc.) that we won't get into.

# Comments on Training

- No guarantee of convergence; neural networks form non-convex functions with multiple local minima
- In practice, many large networks can be trained on large amounts of data for realistic problems.
- May be hard to set learning rate and to select number of hidden units and layers.
- Many steps (tens of thousands) may be needed for adequate training. Large data sets may require many hours of CPU
- Termination criteria: Number of epochs; Increased error on a validation set.
- To avoid local minima: several trials with different random initial weights with majority or voting techniques

# Over-training Prevention

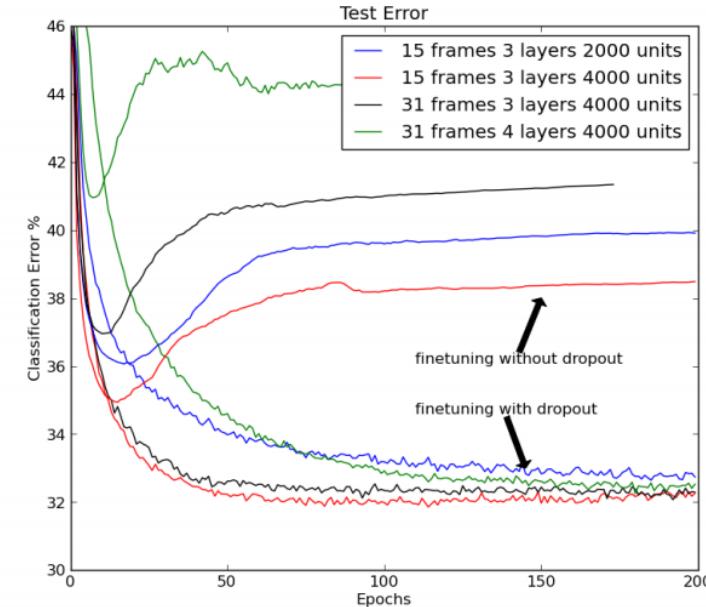
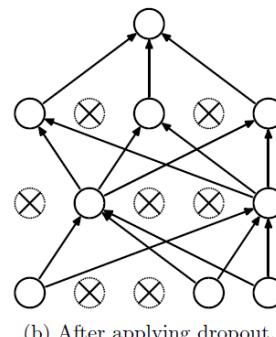
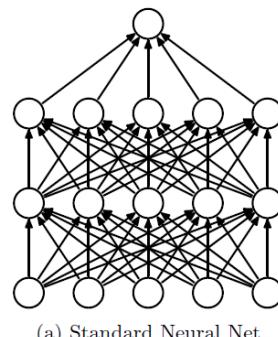
- Running too many epochs and/or a NN with many hidden layers may lead to an **overfit** network
- Keep a **held-out validation** set and evaluate accuracy after every epoch
- Early stopping: maintain weights for best performing network on the validation set and return it when performance decreases significantly beyond that.
- To avoid losing training data to validation:
  - Use 10-fold cross-validation to determine the average number of epochs that optimizes validation performance
  - Train on the full data set using this many epochs to produce the final results

# Over-fitting prevention

- Too few hidden units prevent the system from adequately fitting the data and learning the concept.
- Using too many hidden units leads to over-fitting.
- Similar cross-validation method can be used to determine an appropriate number of hidden units. (general)
- Another approach to prevent over-fitting is weight-decay: all weights are multiplied by some fraction in  $(0,1)$  after every epoch.
  - Encourages smaller weights and less complex hypothesis
  - Equivalently: change Error function to include a term for the sum of the squares of the weights in the network. (general)

# Dropout training

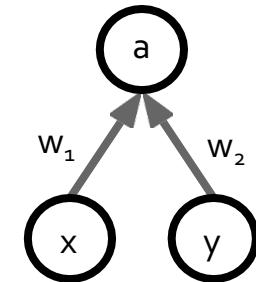
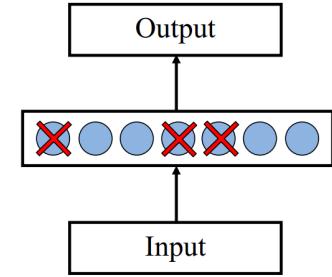
- In each forward pass, randomly set some neurons to zero
- Probability of dropping is a hyperparameter; 0.5 is common
- Dropout is implicitly a ensemble (average) of model that share parameters.
  - Each binary mask is one model
  - For example, an FC layer with 4096 units has  $2^{4096} \sim 10^{1233}$  possible masks!
  - Only  $\sim 10^{82}$  atoms in the universe ...



[Hinton et al, 2012; Srivastava et al, "Dropout: A simple way to prevent neural networks from overfitting", JMLR 2014]

# Dropout During Test Time

- The issue for test time is that Dropout adds randomization.
  - Each dropout mask would lead to a slightly different outcome.
- In ideal world, we would like to “average out” the outcome across all the possible random masks:
  - Not feasible.
$$y = f(x) = E_z[f(x, z)] = \int p(z)f(x, z)dz$$
- The alternative is to not apply dropout. Without dropout, the input values to each neuron would be higher than what was seen during the training (mismatch between train/test).
  - **Example:** Input to activation during:
    - training time:  $E[a] = \frac{1}{4}(w_1x_1 + w_2x_2) + \frac{1}{4}(0 + 0)$   
 $+ \frac{1}{4}(0 + w_2x_2) + \frac{1}{4}(w_1x_1 + 0) = \frac{1}{2}(w_1x_1 + w_2x_2)$
    - test time:  $E[a] = w_1x_1 + w_2x_2$
- **Solution:** scale the values proportional to dropout probability.
  - Can be applied in either testing (scaling down) or training (scaling up).



# Dropout in Practice

Just call the PyTorch function!

```
dropout = nn.Dropout(p=0.2)
x = torch.randn(20, 16)
y = dropout(x)
```

It automatically

- activates the dropout for **training**.
- deactivates it during **evaluations** and scales the values according to its parameter.

```
# training step
...
model.train()
...
```

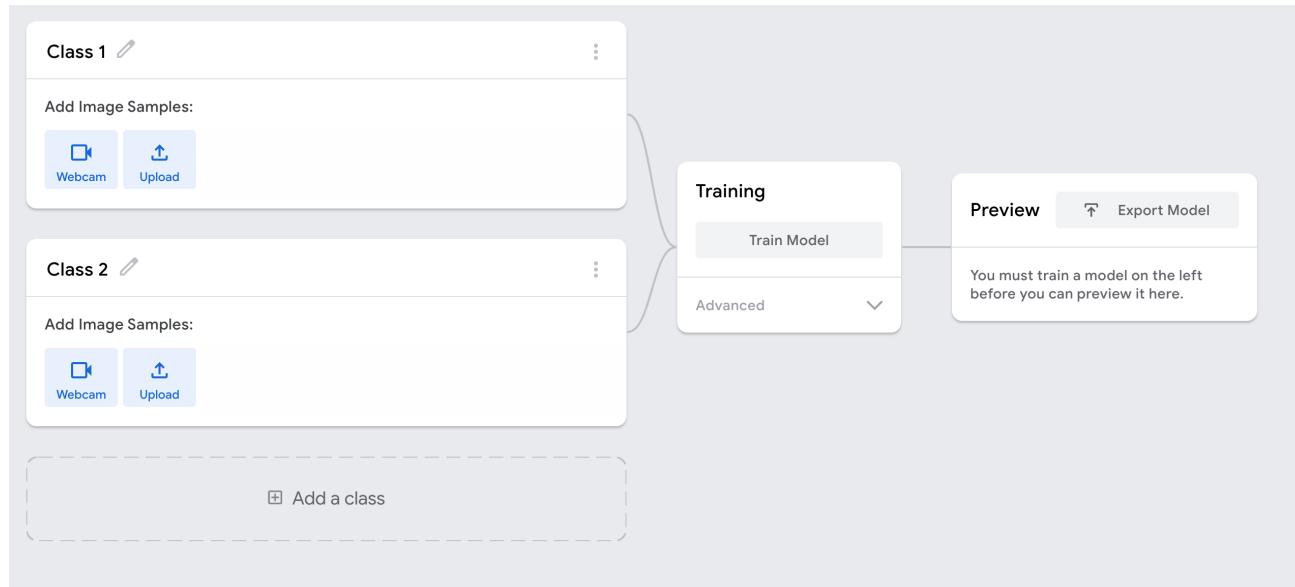
```
# evaluate model:
...
model.eval()
...
```

# The Only Time You Want to Overfit: The First Tryout

- A model with buggy implementation (e.g., incorrect gradient calculations or updates) cannot learn anything.
- Therefore, a good and easy sanity check is to see if you can overfit few examples.
  - This is really the first test you should do, before any hyperparameter tuning.
- Try to train to 100% training accuracy/performance on a small sample (<30) of training data and monitor the **training** loss trends.
  - Does it down? If not, something must be wrong.
  - Try checking the **learning rate** or modifying the initialization.
  - If those don't help, check the gradients.
    - If they're **Nan** or **Inf**, might indicate **exploding gradients**.
    - If they're **zeros**, might indicate **vanishing gradients**.

# Demo Time!

- <https://teachablemachine.withgoogle.com/>



# Chapter Summary

- Feed-forward network architecture
- Word2Vec is just a feedforward net!
  - And we can easily extend it!
- We learned Back-Prop, the most important algorithm in neural networks! 
  - Recursively (and hence efficiently) apply the chain rule along computation graph
- Lots of empirical tricks for training neural networks:
  - First test: check if you can overfit.
  - Dropout
  - Be mindful of activations
  - Careful of exploding/vanishing gradients

