

Language Modeling

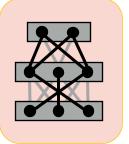
CSCI 601-471/671 (NLP: Self-Supervised Models)

https://self-supervised.cs.jhu.edu/sp2025/

Recap: Self-Supervised Models

Earlier we define Self-Supervised models as as predictive models of the world!







"Wings over Kansas is an aviation website founded in 1998 by Carl Chance owned by Chance Communications, Inc."



Language Modeling: Motivation

- Earlier we define Self-Supervised models as as predictive models of the world!
- Language models are self-supervised, or predictive models of language.

• How do you formulate? How do you build them?



Language Modeling: Chapter Plan

- 1. Language modeling: definitions and history
- 2. Language modeling with counting
- 3. Measuring language modeling quality
- 4. Language Modeling as a Machine Learning problem

Chapter goal — getting comfortable with the concept of "language modeling."



Language Modeling: Definitions and History



The

The cat

The cat sat

The cat sat on

The cat sat on ___?___

The cat sat on the mat.

P(mat | The cat sat on the)

next word

context or prefix

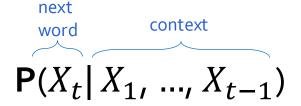
Probability of Upcoming Word

$$P(X_t | X_1, ..., X_{t-1})$$
next word context or prefix



LMs as a Marginal Distribution

Directly we train models on "marginals":



Prob

"The cat sat on the [MASK]"

Language Model

bed desk chair



LMs as Implicit Joint Distribution over Language

- While language modeling involves learning the marginals, we are implicitly learning the full/joint distribution of language.
 - Remember the chain rule:

$$P(X_1, ..., X_t) = P(X_1) \prod_{i=1}^t P(X_i | X_1, X_2 ..., X_i)$$



Doing Things with Language Model

What is the probability of

"I like Johns Hopkins University"

"like Hopkins I University Johns"



Doing Things with Language Model

What is the probability of

"I like Johns Hopkins University"

"like Hopkins I University Johns"

LMs assign a probability to every sentence (or any string of words).

P("I like Johns Hopkins University EOS") = 10^{-5}

P("like Hopkins I University Johns EOS") = 10^{-15}



Doing Things with Language Model (2)

We can rank sentences.

P(
$$X_t | X_1, ..., X_{t-1}$$
)

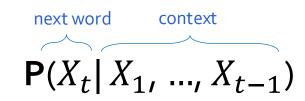
- While LMs show "typicality", this may be a proxy indicator to other properties:
 - Grammaticality, fluency, factuality, etc.

P("I like Johns Hopkins University. EOS") > P("I like John Hopkins University EOS") P("I like Johns Hopkins University. EOS") > P("University. I Johns EOS Hopkins like")P("JHU is located in Baltimore. EOS") > P("JHU is located in Virginia. EOS")



Doing Things with Language Model (3)

Can also generate strings!



- Let's say we start "Johns Hopkins is "
- Using this prompt as an initial condition, recursively sample from an LM:
 - 1. Sample from $P(X | "Johns Hopkins is") \rightarrow "located"$
 - 2. Sample from $P(X | "Johns Hopkins is located") \rightarrow "at"$
 - 3. Sample from $P(X| "Johns Hopkins is located at") \rightarrow "the"$
 - 4. Sample from $P(X | "Johns Hopkins is located at the") \rightarrow "state"$
 - 5. Sample from $P(X | "Johns Hopkins is located at the state") <math>\rightarrow$ "of"
 - 6. Sample from $P(X | "Johns Hopkins is located at the state of") <math>\rightarrow$ "Maryland"
 - 7. Sample from $P(X | "Johns Hopkins is located at the state of Maryland") <math>\rightarrow$ "EOS"

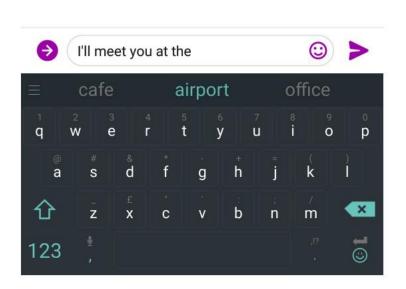


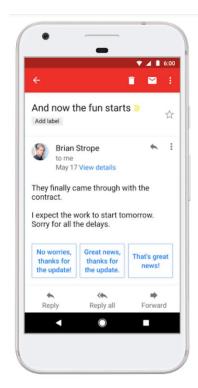
Why Care About Language Modeling?

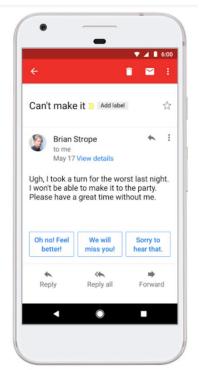
- Language Modeling is a subcomponent superset of many tasks:
 - Summarization
 - Machine translation
 - Spelling correction
 - Dialogue etc.
- Language Modeling is an effective proxy for language understanding.
 - Effective ability to predict forthcoming words requires on understanding of context/prefix.



You use Language Models every day!

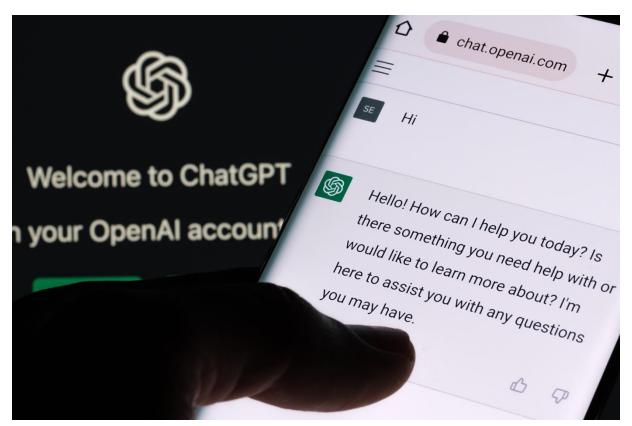








You use Language Models every day!





Summary

Language modeling: building probabilistic distribution over language.

 An accurate distribution of language enables us to solve many important tasks that involve language communication.

• Next question: how do you actually estimate this distribution?



Language Modeling with Counting



LMs as a Marginal Distribution

$$P(X_t \mid X_1, ..., X_{t-1})$$

Now the question is, how to estimate this distribution.



$$P(X_t | X_1, ..., X_{t-1})$$

How do we estimate these probabilities? Let's just count!

P("mat" | "the cat sat on the") >

count("the cat sat on the mat")
count("the cat sat on the")

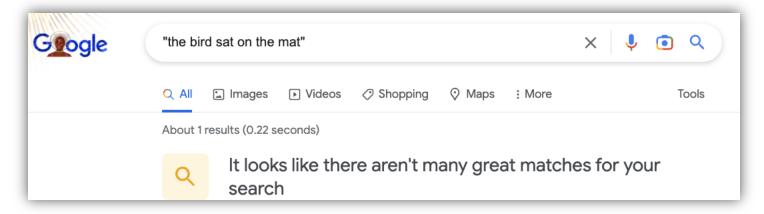
Count how often
"the cat sat on the mat"
has appeared in the world (internet)!

Divide that by, the count of "the cat sat on the" in the world (internet)!

$$P(X_t | X_1, ..., X_{t-1})$$

How do we estimate these probabilities? Let's just count!

P("mat" | "the cat sat on the") $\approx \frac{\text{count}(\text{"the cat sat on the mat"})}{\text{count}(\text{"the cat sat on the"})}$



$$P(X_t | X_1, ..., X_{t-1})$$

How do we estimate these probabilities? Let's just count!

P("mat" | "the cat sat on the")
$$\approx \frac{\text{count}(\text{"the cat sat on the mat"})}{\text{count}(\text{"the cat sat on the"})}$$

Challenge: Increasing n makes sparsity problems worse. Typically, we can't have n bigger than 5.

Some partial solutions (e.g., smoothing and backoffs) though still an open problem.

Understanding Sparsity: A Thought Experiment

How common are zero-probabilities?



- **Example:** Shakespeare as a text corpus
 - The size vocab used by Shakespeare: |V|=29,066
 - Shakespeare produced: ~300,000 bigrams
 - Out of |V|^2= 844 million possible bigrams
 - (some of them don't make sense, but ok!)
- So, 99.96% of the possible bigrams are never seen (hence, have zero) entries for bigram counts).



Language Models: A History

Shannon (1950): The redundancy and predictability (entropy) of English.



Prediction and Entropy of Printed English

By C. E. SHANNON

(ManuscriptReceived Sept. 15, 1950)

A new method of estimating the entropy and redundancy of a language is described. This method exploits the knowledge of the language statistics possessed by those who speak the language, and depends on experimental results in prediction of the next letter when the preceding text is known. Results of experiments in prediction are given, and some properties of an ideal predictor are developed.





$P(X_t | X_1, ..., X_{t-1})$



Shannon (1950) built an approximate language model with word cooccurrences.

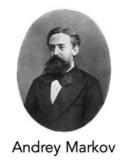
Markov assumptions: every node in a Bayesian network is conditionally independent of its non-descendants, given its parents.

1st order approximation:

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{the})$



$P(X_t | X_1, ..., X_{t-1})$



2nd order approximation:

2 elements

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{on the})$

1st order approximation:

1 element

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{the})$



$P(X_t | X_1, ..., X_{t-1})$



3rd order approximation:

3 elements

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{sat on the})$

2nd order approximation:

2 elements

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{on the})$

1st order approximation:

1 element

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{the})$







3rd order approximation:

 $P(\text{mat} | \text{the cat sat on the}) \approx P(\text{mat} | \text{sat on the})$

Then, we can use counts of approximate conditional probability. Using the 3rd order approximation, we can:

$$P(\text{mat} \mid \text{the cat sat on the}) \approx P(\text{mat} \mid \text{sat on the}) = \frac{\text{count("sat on the mat")}}{\text{count("on the mat")}}$$

N-gram Language Models

- **Terminology:** *n*-gram is a chunk of *n* consecutive words:
 - o unigrams: "cat", "mat", "sat", ...
 - bigrams: "the cat", "cat sat", "sat on", ...
 - trigrams: "the cat sat", "cat sat on", "sat on the", ...
 - four-grams: "the cat sat on", "cat sat on the", "sat on the mat", ...
- n-gram language model:

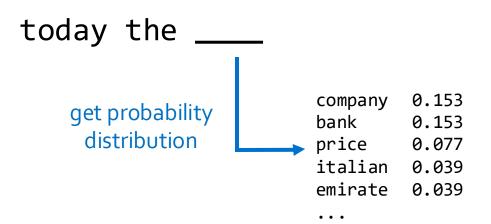
$$p(X_t | X_1, ..., X_{t-1}) \approx P(X_t | X_{t-n+1}, ..., X_{t-1})$$



Generation from N-Gram Models

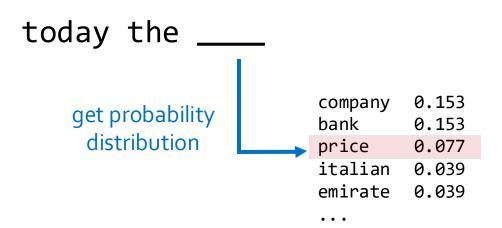
You can build a simple trigram Language Model over a 1.7 million words corpus in a few seconds on your laptop*

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Sparsity problem: not much granularity in the probability distribution

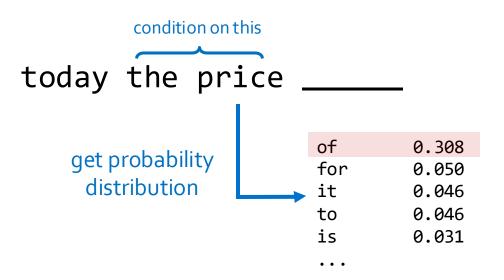
Now we can sample from this mode:



Sparsity problem: not much granularity in the probability distribution

^{*} Try for yourself: https://nlpforhackers.io/language-models/

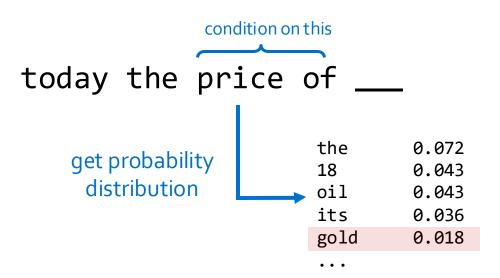
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Sparsity problem: not much granularity in the probability distribution

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Now we can sample from this mode:



Sparsity problem: not much granularity in the probability distribution

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N-Gram Models in Practice

Now we can sample from this mode:

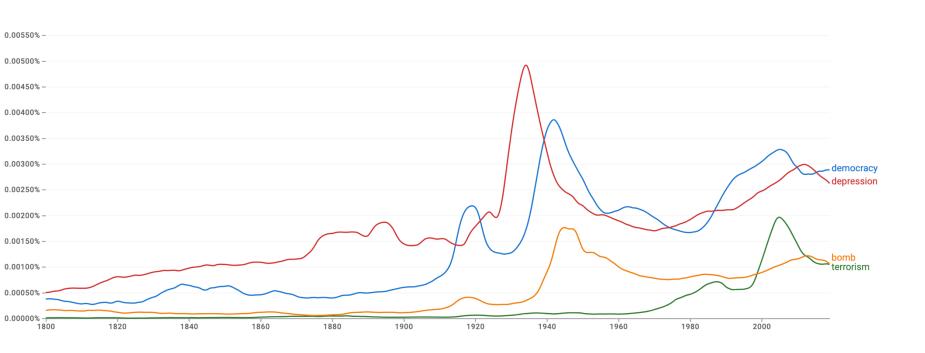
today the price of gold per ton, while production of shoe lasts and shoe industry, the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .

Surprisingly grammatical!

But quite incoherent! To improve coherence, one may consider increasing larger than 3-grams, but that would worsen the sparsity problem!

Pre-Computed N-Grams

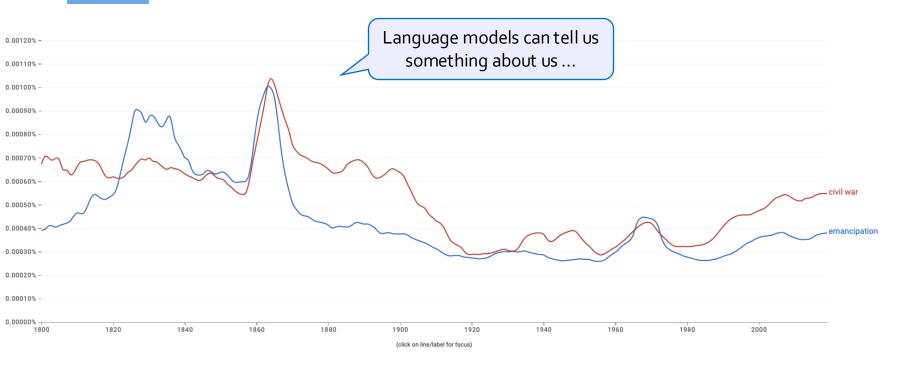
Google Books Ngram Viewer





Pre-Computed N-Grams

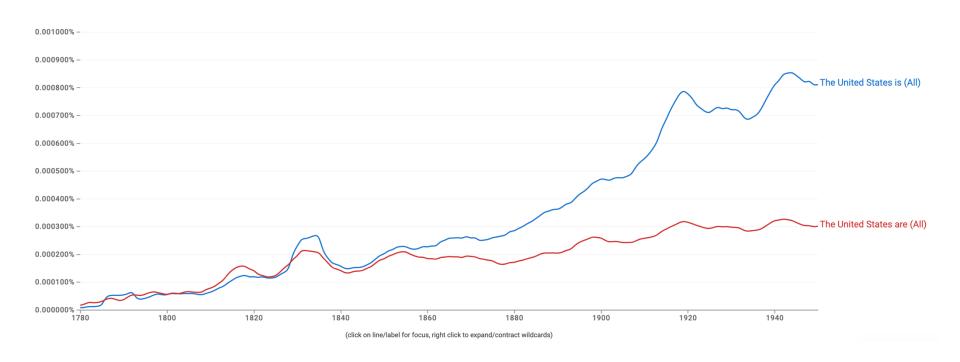
Google Books Ngram Viewer





Pre-Computed N-Grams

Google Books Ngram Viewer





Limits of N-Grams LMs: Long-range Dependencies

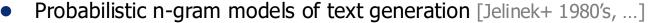
 In general, count-based LMs are insufficient models of language because language has long-distance dependencies:

"The computer which I had just put into the machine room on the fifth floor crashed."



N-Gram Language Models, A Historical Highlight "Every time I fire a linguist, the performance of

"Every time I fire a linguist, the performance of the speech recognizer goes up"!!



Applications: Speech Recognition, Machine Translation



Fred Jelinek (1932-2010)

532

PROCEEDINGS OF THE IEEE, VOL. 64, NO. 4, APRIL 1976

Continuous Speech Recognition by Statistical Methods

FREDERICK JELINEK, FELLOW, IEEE

Abstract—Statistical methods useful in automatic recognition of continuous speech are described. They concern modeling of a speaker and of an acoustic processor, extraction of the models' statistical parameters, and hypothesis search procedures and likelihood computations of linguistic decoding. Experimental results are presented that indicate the power of the methods.

utterance models used will incorporate more grammatical features, and statistics will have been grafted onto grammatical models. Most methods presented here concern modeling of the speaker's and acoustic processor's performance and should, therefore, be universally useful.

Automatic recognition of continuous (English) speech is an



Summary

- Learning a language model ~ learning conditional probabilities over language.
- One approach to estimating these probabilities: counting word co-occurrences.
- Challenges:
 - Word co-occurrences become rare for long sequences. (the sparsity issue)
 - But language understanding requires long-range dependencies.
- We need a better alternative! 🚱
- Next: Measuring quality of language models.

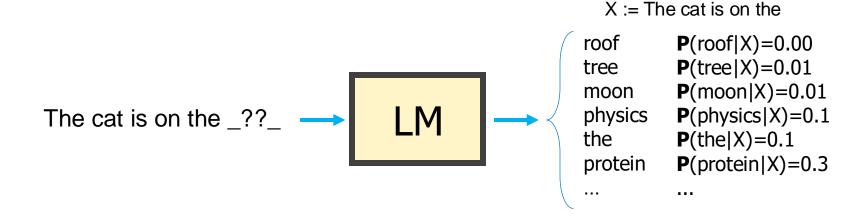


How Good are Language Models?



Large Language Models

 A language model can predict the next word based on the given context.





Large Language Models

X :=The cat is on the A language model can predict the next word roof P(roof|X) = 0.00based on the given context. P(tree|X) = 0.01tree P(moon|X)=0.01moon P(physics|X)=0.1physics LM1 the P(the|X)=0.1protein P(protein|X)=0.3The cat is on the ?? P(roof|X)=0.14roof P(tree|X)=0.13tree P(moon|X)=0.001moon Which LM is better? P(physics|X)=0.0LM₂ physics P(the|X) = 0.000the P(protein|X)=0.00protein **52**

Evaluating Language Models

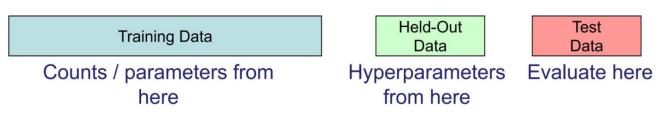
- Does our language model prefer good sentences to bad ones?
 - Assign higher probability to "real" or "frequently observed" sentences
 - Than "ungrammatical" or "rarely observed" sentences?
- We test the model's performance on data we haven't seen.



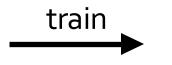
Evaluating Language Models

Setup:

- Train it on a suitable training documents.
- Evaluate their predictions on different, unseen documents.
- An evaluation metric tells us how well our model does on the test set.







count("on the mat")

Evaluating Language Models: Example

Setup:

- Train it on a suitable training documents.
- Evaluate their predictions on different, unseen documents.
- An evaluation metric tells us how well our model does on the test set.

Example: I use a bunch of New York Times articles to build a bigram probability table



A good language model should assign a high probability to held-out text!

count("on the mat") eval

Now I'm going to evaluate the probability of some heldout data using our bigram table



ACENCENTERED

Be Careful About Data Leakage!

Advice from a grandpa ::

- Don't allow test sentences to leak into into training set.
- Otherwise, you will assign it an artificially high probability (==cheating).

Example: I use a bunch of New York Times articles to build a bigram probability table



A good language model should assign a high probability to held-out text!

eval

Now I'm going to evaluate the probability of some heldout data using our bigram table



count("on the mat")

Evaluating Language Models: Intrinsic vs Extrinsic

- Intrinsic: measure how good we are at modeling language
- Extrinsic: build a new language model, use it for some task (MT, ASR, etc.)

Example: I use a bunch of New York Times articles to build a bigram probability table



train



Now I'm going to evaluate the probability of some heldout data using our bigram table



eval

 Perplexity is the inverse probability of the test set, normalized by the number of words:

$$ppl(w_1, ..., w_n) = \mathbf{P}(w_1, w_2, ..., w_n)^{-\frac{1}{n}}$$

- A measure of predictive quality of a language model.
- Minimizing perplexity is the same as maximizing probability



 Perplexity is the inverse probability of the test set, normalized by the number of words:

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$$= \sqrt[n]{\frac{1}{\mathbf{P}(w_1, w_2, ..., w_n)}} = \sqrt[n]{\frac{1}{\mathbf{P}(w_i | w_{< i})}}$$
chain rule
$$= 2^H, \text{ where}$$



$$H = -\frac{1}{n} \sum_{i=1}^{n} \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$$

- In practice, we prefer to use log-probabilities (also known as "logits")
- We can rewrite perplexity formula in terms of log-probs:

$$ppl(w_1, ..., w_n) = 2^H$$
, where $H = -\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$

Recap: Definition of cross-entropy between two distributions:

$$H(p,q) = -\sum_{x \in \mathcal{X}} p(x) \, \log q(x)$$

Can be interpreted as cross-entropy between LM prob and language prob. Why?



- In practice, we prefer to use log-probabilities (also known as "logits")
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$$ppl(w_1, ..., w_n) = 2^H$$
, where $H = -\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$

Perplexity for n-grams:

o Unigrams: $H = -\frac{1}{n} \sum_{i=1}^{n} \log_2 \mathbf{P}(w_i)$

o Bigrams: $H = -\frac{1}{n} \sum_{i=1}^{n} \log_2 \mathbf{P}(w_i | w_{i-1})$

o Trigrams: $H = -\frac{1}{n} \sum_{i=1}^{n} \log_2 \mathbf{P}(w_i | w_{i-2}, w_{i-1})$

0 ...



Intuition-building Quizzes (1)

- In practice, we prefer to use log-probabilities (also known as "logits")
- We can rewrite perplexity formula in terms of log-probs:

$$ppl(w_1, ..., w_n) = 2^H$$
, where $H = -\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$

• **Quiz:** let's suppose we have a sentence $w_1, ..., w_n$ and it's fixed. Our model will correctly guess each word with probability 1/5. What is perplexity of our model?

$$H = -\frac{1}{n} \left[\log_2 \left(\frac{1}{5} \right) + \dots + \log_2 \left(\frac{1}{5} \right) \right] = -\log \left(\frac{1}{5} \right) \Rightarrow \text{ppl}(D) = 5$$



Intuition: the model is indecisive among 5 choices.

Intuition-building Quizzes (2)

- In practice, we prefer to use log-probabilities (also known as "logits")
- We can rewrite perplexity formula in terms of log-probs:

$$ppl(w_1, ..., w_n) = 2^H$$
, where $H = -\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$

• Quiz: let's we evaluate an exact (!!) model of language, i.e., our model always knows what exact word should follow a given context. What is the perplexity of this model?

$$\forall w \in V : \mathbf{P}(w_i | w_{1:i-1}) = 1 \implies \text{ppl}(D) = 2^{-\frac{1}{2}n \log_2 1} = 1$$



Intuition: the model is indecisive among 1 (the right!) choice!

Intuition-building Quizzes (3)

- In practice, we prefer to use log-probabilities (also known as "logits")
- We can rewrite perplexity formula in terms of log-probs:

$$ppl(w_1, ..., w_n) = 2^H$$
, where $H = -\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$

• **Quiz:** let's we evaluate a confused (!!) model of language, i.e., our model has no idea what word should follow each context—it always chooses a uniformly random word. What is the perplexity of this model?

$$\forall w \in V : \mathbf{P}(w|w_{1:i-1}) = \frac{1}{|V|} \implies \text{ppl}(D) = 2^{-\frac{1}{n}n\log_2\frac{1}{|V|}} = |V|$$



Intuition: the model is indecisive among all the vocabulary terms.

Perplexity: Summary

$$ppl(w_1, ..., w_n) = 2^H$$
, where $H = -\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})$

- Perplexity is a measure of model's uncertainty about next word (aka "average branching factor").
 - The larger the number of vocabulary, the more options there to choose from.
 - o (the choice of atomic units of language impacts PPL more on this later)
- Perplexity ranges between 1 and |V|.
- We prefer LMs with lower perplexity.



Lower perplexity == Better Model

Training on 38 million words, test 1.5 million words, Wall Street Journal

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

Lower is better

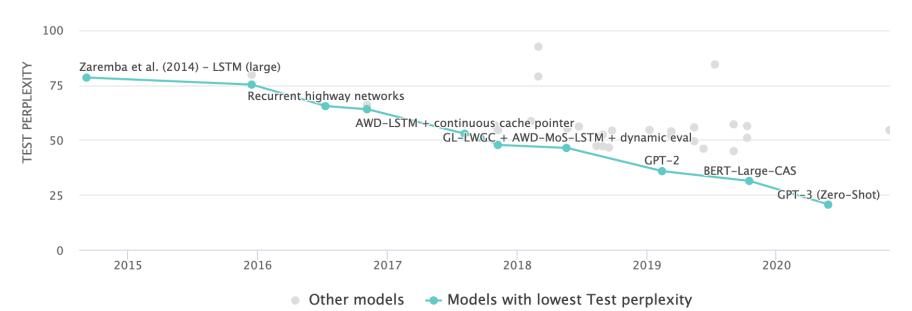
Note these evaluations are done on data that was not used for "counting." (no cheating!!)



[Mohit lyyer] 67

Lower perplexity == Better Model

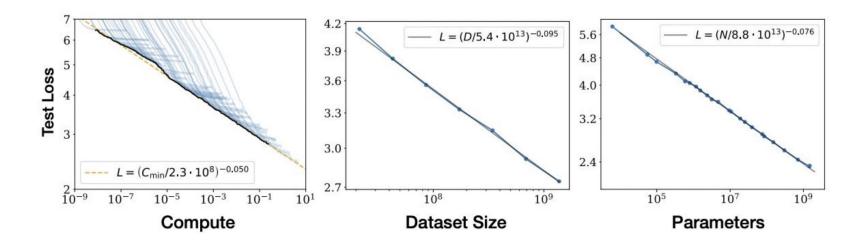
The PPL of modern language models have consistently been going down.





Lower perplexity == Better Model

The PPL of modern language models have consistently been going down.

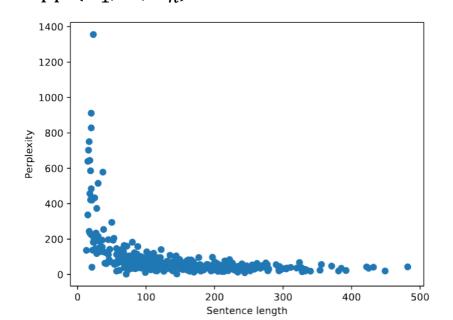




Quiz:

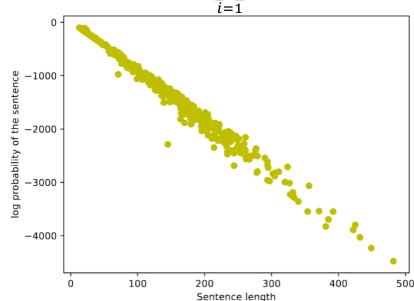
• Would you expect higher PPL for longer sentences?

$$ppl(w_1, ..., w_n) = 2^{-\frac{1}{n} \sum_{i=1}^n \log_2 \mathbf{P}(w_i | w_1, ..., w_{i-1})}$$



Would you expect a higher probability for longer sentences?

$$\mathbf{P}(w_1, ..., w_n) = \prod_{i=1}^{n} \mathbf{P}(w_i | w_1, ..., w_{i-1})$$



Summary

- Language Models (LM): distributions over language
- Measuring LM quality: use perplexity on held-out data.
- Count-based LMs have limitations.
 - Challenge with large N's: sparsity problem many zero counts/probs.
 - Challenge with small N's: lack of long-range dependencies.
- Next: Rethinking language modeling as a statistical learning problem.

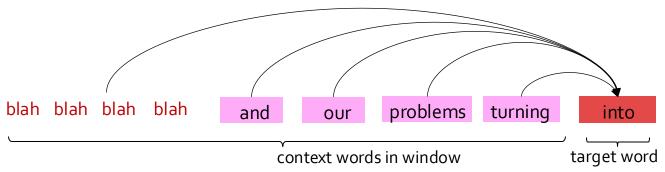


Beyond Counting: Language Models as a Learning Problem



LM as a Machine Learning Problem

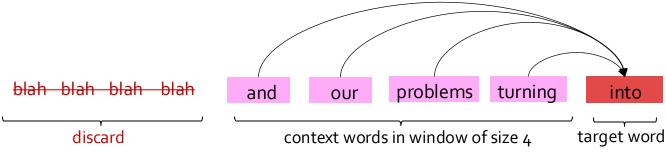
- Given the embeddings of the context, predict the word on the right side.
 - Dropping the right context for simplicity -- not a fundamental limitation.
- Discard anything beyond its context window





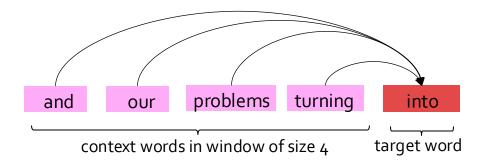
LM as a Machine Learning Problem

- Given the embeddings of the context, predict the word on the right side.
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LM as a Machine Learning Problem

- Given the embeddings of the context, predict the word on the right side.
 - Dropping the right context for simplicity -- not a fundamental limitation.
- Discard anything beyond its context window





A Fixed-Window Neural LM

- Given the embeddings of the context, predict a target word on the right side.
 - Dropping the right context for simplicity -- not a fundamental limitation.

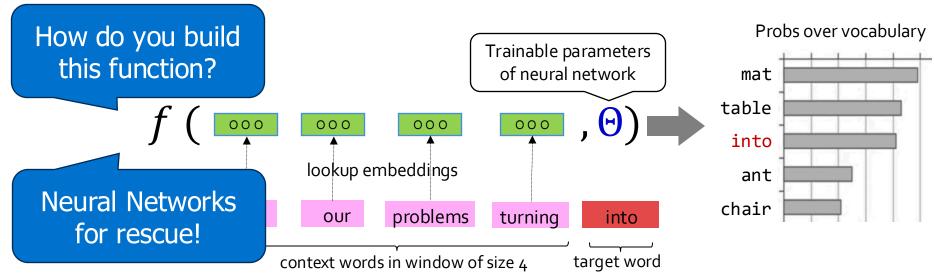
■ Training this model is basically optimizing its parameters ⊕ such that it assigns

high probability to the target word. Probs over vocabulary Trainable parameters of neural network mat table 000 000 into lookup embeddings ant chair problems turning and our into target word context words in window of size 4



A Fixed-Window Neural LM

- It will also lay the foundation for the future models (recurrent nets, transformers, ...)
- But first we need to figure out how to train neural networks!





80

From Counting (N-Gram) to Neural Models

- n-gram models of text generation [Jelinek+ 1980's, ...]
 - Applications: Speech Recognition, Machine Translation

"Shallow" statistical/neural language models (2000's) [Bengio+ 1999 & 2001, ...]

NeurIPS 2000

A Neural Probabilistic Language Model

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Summary

- Language Modeling (LM), a useful predictive objective for language
- Perplexity, a measure of an LM's predictive ability
- N-gram models (~1980 to early 2000's),
 - Early instances of LMs
 - Difficult to scale to large window sizes
- Shallow neural LMs (early and mid-2000's),
 - We will need in coming sessions that one can build these models with neural networks.
 - These will be effective predictive models based on feed-forward networks

