

Problem Set 1

Intertemporal Choice

1. **Friedman (1957)'s Permanent Income Hypothesis.** Answer the following questions under the assumption that Friedman's Permanent Income Hypothesis $c_i = \mathbf{p}_i$ is true. Assume that income in each period for each household is $y_{t,i} = \mathbf{p}_i + \xi_{t,i}$ where $\xi_{t,i}$ is a mean-0 random ('white noise') transitory shock to income.

- a) Suppose that you have consumption and income data for a cross-section of households in a particular year. Suppose further that you know that the variance of transitory income is higher for farmers than it is for members of other occupational groups. Now imagine estimating a Keynesian consumption function $c_i = \alpha_0 + \alpha_1 y_i + \epsilon_i$. How would you expect the estimated coefficients α_1 to differ between the farmers in your sample and the rest of the population? Do you need to know any other information to answer the question definitively?

Use the programming software (Matlab, Python, or Mathematica) you have been assigned to use for this first assignment, simulate 50 observations of y_i for each farmers and non-farmers group, with:

- i. $\sigma_{p,\text{farmers}}^2 = \sigma_{p,\text{workers}}^2, \sigma_{\xi,\text{farmers}}^2 = 3\sigma_{\xi,\text{workers}}^2$
- ii. $\sigma_{p,\text{farmers}}^2 = 10\sigma_{p,\text{workers}}^2, \sigma_{\xi,\text{farmers}}^2 = 3\sigma_{\xi,\text{workers}}^2$
- iii. $\sigma_{p,\text{farmers}}^2 = 0.1\sigma_{p,\text{workers}}^2, \sigma_{\xi,\text{farmers}}^2 = 3\sigma_{\xi,\text{workers}}^2$

For purposes of these simulations, assume that $\{\sigma_{p,\text{workers}}^2, \sigma_{\xi,\text{workers}}^2\} = 0.01, 0.02$. For each scenario, estimate coefficient α_1 of the Keynesian consumption function, draw the observations and estimated consumption function on a graph with y on the horizontal axis and \mathbf{p} on the vertical axis (use different colors for different groups). Make sure you include a 45-degree line (corresponding to $c = y$) on each graph. Relate your results to the first part of this question, and briefly discuss what these graphs tell you. Do the graphs support your answers?

- b) Imagine that you observe a set of households for two consecutive periods, t and $t + 1$. What relationship should you find between the saving rate at time t and income growth between t and $t + 1$?

Using the programming software (Matlab, Python, or Mathematica) you are assigned in class, simulate 50 observations of y_i for each of two consecutive periods t and $t+1$. Assume that any individual household's permanent income does not change from period t to $t + 1$.

For each household, calculate the simulated saving rate in period t , the growth rate of income between t and $t + 1$, and draw them on a graph (Hint: Put the saving rate in period t on the x-axis, and income growth between t and $t + 1$ on the y-axis. Your graph should consist of 50 points). Relate your results to

the first part of this question, and briefly discuss what these graphs tell you. Do the graphs support your answers?

- c) In his 1992 State of the Union address, President George H.W. Bush said that, in order to spur consumer spending, he was instructing the IRS to reduce the rate at which income taxes were withheld from taxpayers' wages. Income tax rates were not changed, only the timing of when consumers would pay those taxes. What effect might you expect this change in withholding to have had on consumption?

2. The Income, Substitution, and Human Wealth Effects in a Two Period Lifetime (Fisher (1930)). For a consumer who solves the following maximization problem,

$$\begin{aligned} v(y_1) &= \max_{c_1} u(c_1) + \beta u(c_2) \\ \text{s.t.} \\ c_2 &= (y_1 - c_1)R + y_2 \\ u(c) &= \frac{c^{1-\rho}}{1-\rho}. \end{aligned}$$

- a) Solve for consumption and saving in the first period as a function of R, y_1, y_2, ρ , and β .
- b) Suppose that all noncapital income is earned in the first period of life, i.e. $y_2 = 0$. How does the sign of the derivative of saving with respect to the interest rate depend on the value of ρ (consider the cases of $\rho \rightarrow 0, 0 < \rho < 1, \rho = 1, \rho > 1$, and $\rho \rightarrow \infty$)? Give a verbal explanation.
- c) Now suppose that all noncapital income is earned in the second period of life, i.e. $y_1 = 0, y_2 > 0$. For a given value of ρ , how does the responsiveness of consumption and saving to the interest rate compare to the case when all noncapital income was earned in the first period? What is the name of the additional effect on saving? (Hint: There is a particular value of ρ which makes this question easy.)
- d) Now suppose that noncapital income is earned equally in both periods, $y_1 = y_2$. How does the response of consumption and saving to interest rates depend on ρ ? Describe and explain the special results obtained as ρ approaches infinity and as it approaches zero.

3. Income Growth Over the Lifetime Versus Between Generations (Modigliani (1986), Carroll and Summers (1991)). This question concerns the effects on aggregate saving of income growth over the lifetime versus income growth between generations. Consider an overlapping generations economy in which each individual lives for two periods. Population is constant, so that the population growth factor between generations is $\Xi = 1$; normalize population itself to $N = 1$ per generation. The individuals' noncapital incomes in each

period are exogenous. The first period noncapital income of an individual born at time t is $y_{t,1}$, and the second-period noncapital income of the same individual is $y_{t+1,2} = Xy_{t,1}$ where X can be greater than or less than one. The consumer solves the optimization problem:

$$\max \quad \log(c_{t,1}) + \beta \log(c_{t+1,2}) \quad (1)$$

$$\text{s.t.} \quad (2)$$

$$c_{t+1,2} = (y_{t,1} - c_{t,1})R + y_{t+1,2}. \quad (3)$$

Finally, between generations the first period noncapital incomes grow by a factor $G = (1 + g)$ so that:

$$y_{t+1,1} = Gy_{t,1}$$

For the purposes of this question, consider this to be an open economy so that the aggregate interest rate R and noncapital incomes y are fixed (that is, don't try to derive their values from an aggregate production function).

- a) How does an increase in the growth rate of noncapital income over the lifetime, X , affect the saving of young households? Explain.
- b) Calculate the level of aggregate saving $S_t = K_{t+1} - K_t$ as a function of $y_{t,1}$, and then calculate the aggregate saving rate out of noncapital income $\sigma_t = S_t/Y_t = S_t/(y_{t,1} + y_{t,2})$. How is the aggregate saving rate related to the growth rate of income between generations, G ? How and why does the answer depend on the relationship between β and X/R ?
- c) Thus far in the problem, we have assumed that X is independent of G ; that is, we have assumed that the rate at which income grows during your lifetime is unrelated to the rate at which each generation's income exceeds the income of the previous generation. Now assume that $X = \gamma G$ for some constant γ . Also assume (for simplicity) that $\beta = 1/R$ (qualitative results are the same when $\beta \neq 1/R$, but the analysis is messier). Now when does an increase in G increase or reduce the aggregate saving rate?
- d) Empirical evidence shows that the ratio of the income of the old (people aged 55-85) to income of the young (people aged 25-55) is about 0.7 in both the US and in Japan. From the late 1940s to the late 1980s, Japan's economic growth rate was about 4 percent per year in per-capita terms. Over the same period income growth in the US was about 1 percent per capita. Japan's aggregate saving rate was also much higher than the US saving rate during this period. Discuss whether the overlapping generations model can explain Japan's high saving rate as being the result of its rapid growth rate (continue to assume $\beta = 1/R$). (Hint: start by figuring out the OLG model's implications for the ratio of the income of the old to income of the young.)

References

- CARROLL, CHRISTOPHER D., AND LAWRENCE H. SUMMERS (1991): “Consumption Growth Parallels Income Growth: Some New Evidence,” in *National Saving and Economic Performance*, ed. by B. Douglas Bernheim, and John B. Shoven. Chicago University Press, Chicago, <http://econ.jhu.edu/people/ccarroll/papers/CParallelsY.pdf>.
- FISHER, IRVING (1930): *The Theory of Interest*. MacMillan, New York.
- FRIEDMAN, MILTON A. (1957): *A Theory of the Consumption Function*. Princeton University Press.
- MODIGLIANI, FRANCO (1986): “Life Cycle, Individual Thrift, and the Wealth of Nations,” *American Economic Review*, 3(76), 297–313, Available at <http://www.jstor.org/stable/1813352>.