1. Buffer stock saving

a).

i.

If the lender lent money less than $\underline{H}_{-1} = N_0/r$ in period t, we have $A_t > -N_0/r$. ①

Thus the interest payment in period $t+1$ is $-r \cdot A_t$ and by ① we get: $-r \cdot A_t < N_0$.

Since the lender can at most seize N_0 in period $t+1$, which is larger than the original interest payment, the consumer can pay their interest on loan until forever. Thus even an infinitely risk-averse lender is willing to lend the amount less than N_0/r.

ii.

By equation (5) in Buffer stock model slides, we have:

$$C_t^u = (1 - R^{-1} \cdot (R\beta)^{1/\rho}) \cdot b_t.$$

Now since the unemployed consumer is receiving N_0 for each period, we have:

$$C_t^u = (1 - R^{-1} \cdot (R\beta)^{1/\rho}) \cdot \left(b_t + \frac{N_0}{1-1/R}\right)$$

$$= (\underbrace{1 - R^{-1} \cdot (R\beta)^{1/\rho}}_{\kappa}) \cdot \left(b_t + \frac{R}{r} \cdot N_0\right)$$

By imposing $R^{-1} \cdot (R\beta)^{1/\rho} < 1$, we make sure the MPC κ is positive. Thus when the market resource is positive, we will get positive consumption.

Notice that $M_0 = b_0 + \frac{R}{r} \cdot N_0$, where $b_0 = R \cdot A_{-1} > -RN_0/r$.

Thus $M_0 > -R \cdot N_0 / r + \frac{R}{r} \cdot N_0 = 0$. $\Rightarrow$ M_0 is positive.

Then C_0^u is positive, so is C_t^u for $t > 0$.

iii.

Since $C_t^u = (1 - R^{-1} \cdot (R\beta)^{1/\rho}) \cdot (b_t + \frac{R}{r} \cdot N_0)$, only the PDV of the future payment of N_0 matters to determine C_t^u, as we have the intertemporal budget constraint is:

$P_t^T(c) = b_t + P_t^T(p)$, where in this case $P_t^T(p) = \frac{R}{r} \cdot N_0$.

Thus the unemployment benefit system imposes the same constraint as the one-time lump-sum payment, and therefore results in the same consumption behavior.

iv.

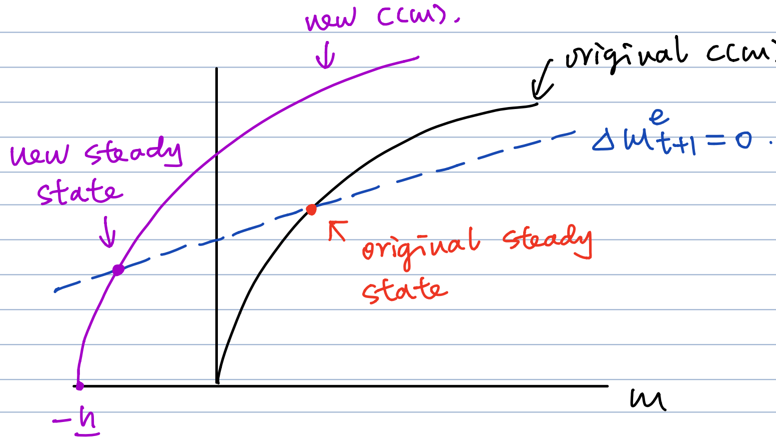
By definition $\underline{h}_{-1} = \frac{H_{-1}}{\ell_{-1} \cdot W_{-1}} = \frac{N_0}{r \cdot \ell_{-1} \cdot W_{-1}} = \frac{\eta \cdot \ell_0 \cdot W_0}{r \cdot \ell_{-1} \cdot W_{-1}}$, since the labor income grows by factor Γ, we have:

$\ell_0 \cdot W_0 = \Gamma \cdot \ell_{-1} \cdot W_{-1}$. Thus $\underline{h}_{-1} = \frac{\eta \cdot \ell_0 \cdot W_0}{r \cdot \ell_{-1} \cdot W_{-1}} = \frac{\Gamma \cdot \eta}{r}$.

v.

If we consider the consumption function for employed consumer at period t, the introduction of unemployment benefit is the same as increasing human wealth if the consumer becomes unemployed in period $t+1$, with the amount of $\frac{R}{r} \cdot N_0 = \frac{R}{r} \cdot \eta \cdot W_{t+1} \cdot \ell_{t+1}$.

Then the normalized human wealth discounted to today should be $\frac{1}{R} \cdot \frac{R \cdot \eta \cdot W_{t+1} \cdot \ell_{t+1}}{r \cdot W_t \cdot \ell_t} = \frac{\eta \cdot \Gamma}{r}$. Then this should be the left shift of the consumption function since the extra human wealth will relax the borrowing constraint so that when $b > -\underline{h}$, the consumer will be able to borrow and thus have a positive consumption. Therefore the new consumption curve will intersect the horizontal axis at $-\underline{h}$.



vi.

The "natural borrowing constraint" is the maximum amount that the consumer would choose to borrow.

The introduction of unemployment benefits brings the increased resources available for the consumer in the future with certainty. Since the intertemporal budget constraint has been relaxed by the increased human wealth, and since the consumer is impatient and would like to take advantage of the borrowing capacity, the natural borrowing constraint will be relaxed with an expansion of benefits.

b). Experiments

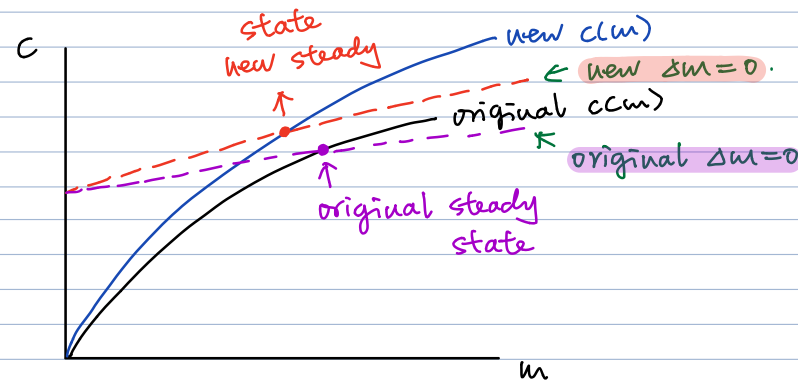
i.

Since the expected unemployment probability $\mho$ will be the lower, the precautionary saving motive will be decreased. Thus the MPC should become higher, shifting the optimal consumption curve counter-clockwise.

Notice that by equation (51) in Buffer-Stock, the $\Delta m=0$ locus satisfies: $c^e = (1-\mathcal{R}^{-1})m^e + \mathcal{R}^{-1}$, where $\mathcal{R} = \frac{R}{W_{t+1} \cdot \ell_{t+1}}$

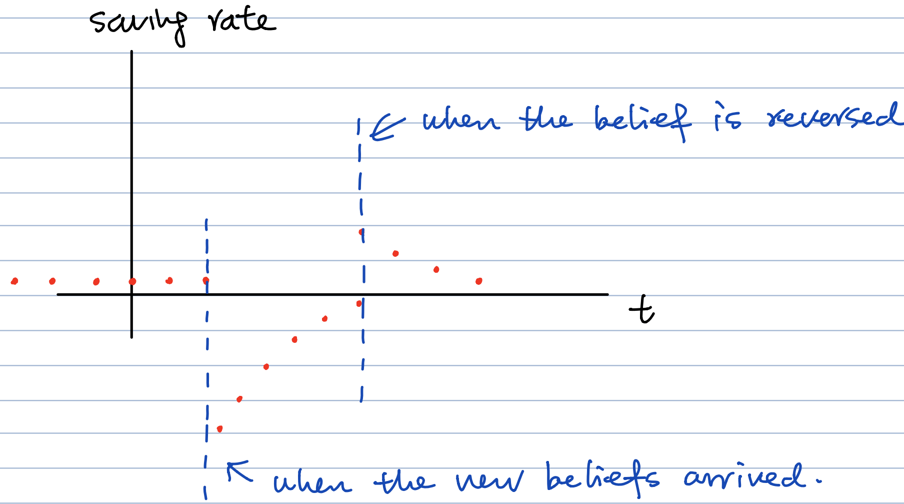
Since $\ell_{t+1} = \ell_t/(1-\mho)$, when $\mho \downarrow$, $\ell_{t+1} \downarrow$, $\mathcal{R} \uparrow$. $(1-\mathcal{R}^{-1}) \uparrow$.

Thus the slope of Δm locus will become larger.



Since the relax of precautionary saving should dominate the decrease in consumption due to lower income growth, a reduction in unemployment risk will result in a new steady state as shown in the graph above. The new target wealth will be lower, while the new target consumption level will be higher, resulting in lower saving rate.

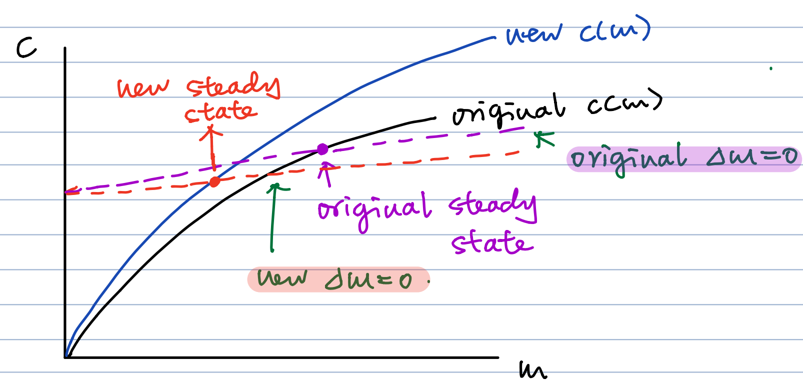
The saving rate will first drop sharply due to a sudden increase in consumption and then go up again when the wealth level has been decreasing.



ii.

The expectation of a higher income growth rate will make consumers more impatient and thus draw up the MPC. Therefore the consumption curve will again shift counter-clockwise.

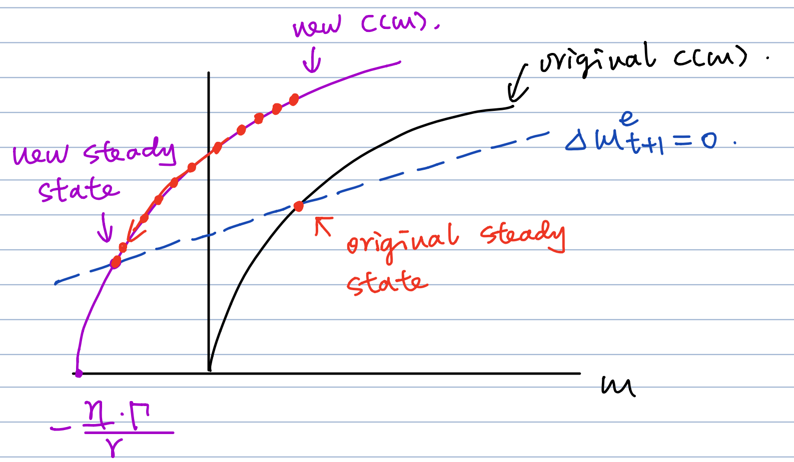
Because the slope of $\Delta m = 0$ locus is determined by the growth rate, and as the growth rate increases the slope will become smaller. Thus the phase diagram will look like:



An increase in growth rate will result in a even lower level of target wealth and target consumption. The trace plot of saving rate is similarly to the first one.

iii.

When η increases to $\underline{\eta}$, the consumption curve shifts further to the left since we have proved in 1(a)V. that the consumption will shift to the left by $\frac{\eta \cdot \Gamma}{\gamma}$. As $\eta \uparrow$, the shifting amount becomes larger.



The new steady state will result in lower level of target wealth and target consumption. Thus the trace plot of consumption will still be similar to the one in i.

2. Asset pricing

(a).

By Lucas tree model,

$$\frac{N \cdot P_t^o + P_t^s}{N d_t^o + d_t^s} = \frac{1}{\theta},$$ and we have $N >> 1$.

When there is a one-time permanent drop in d_t^s such that $d_t^{s'} = \frac{1}{2} d_t^s$, to hold the equation to be still true, P_t^s should also experience a permanent drop.

Since $N >> 1$, we have $\frac{N \cdot P_t^o + P_t^s}{N d_t^o + d_t^s} \approx \frac{P_t^o}{d_t^o} = \frac{1}{\theta}$.

Thus neither P_t^o nor $\frac{P_t^o}{d_t^o}$ will change in this case.

(b).

When the variance of the dividends associated with holding S becomes larger, the covariance between the return and consumption will also become higher. Then according to C-CAPM model, the price of the assets that have a higher positive correlation with consumption will have lower price today, since it covary negatively with the marginal utility. Thus an increase in $Var(d_t^s)$ will lead to a decrease in both P_t^s and P_t^s/d_t^s, while P_t^o and P_t^o/d_t^o should not be affected.

(c).

According to $E[R_{t+1,i}] - R \approx \rho \, cov(\Delta \log C_{t+1}, R_{t+1,i})$, when ρ increases, even when the covariance between $\Delta \log C_{t+1}$ and $R_{t+1,i}$ stays the same, $E[R_{t+1,i}]$ needs to increase to make the equation hold.

When $E[R_{t+1,i}] \uparrow$, we know the average price should fall, since if average return is high, average price will be low.

Thus $P_t^s \downarrow$, $P_t^s/d_t^s \downarrow$, $P_t^o \downarrow$, $P_t^o/d_t^o \downarrow$.

P_t^s will decrease even more since the covariance is higher.