Macroeconomics Problem Set 4

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1 Buffer Stock Saving and Balance Sheets

a)

i. Consider a potential lender deciding whether to lend a consumer $-A_t$ in period t. If the lender decides not to lend, she will get interest revenue $-rA_t$ in period $t+1$ for sure. Otherwise, suppose the consumer cannot make the interest payment $-rA_t$ in period $t+1$, then the lender can seize any amount from the consumer not exceeding N_0. When $N_0 \geq -rA_t$, the lender will not be worse off even if the consumer failed to pay the interest. The analysis above does not depend on whether the consumer is employed or unemployed in period $t+1$, therefore, for any private lender regardless of her degree of risk aversion, she will be willing to lend an employed consumer any amount not exceeding $\underline{H}_{-1} \equiv N_0/r$.

ii. Once unemployed in period $t+1$, the consumer's optimization problem becomes

$$\begin{aligned}\mathbf{c}^u_{t+1} &= (1 - R^{-1}(R\beta)^{1/\rho})(\mathbf{b}_{t+1} + \mathbf{h}_{t+1}) \\ &= (1 - R^{-1}(R\beta)^{1/\rho})(A_t R + \mathbf{h}_{t+1}) \qquad (1)\end{aligned}$$

where $\mathbf{h}_{t+1}$ is the human wealth term, which is no longer zero after the introduction of unemployed benefits. Here,

$$\mathbf{h}_{t+1} = N_0 + N_0/R + ... = N_0(\frac{R}{r}) \qquad (2)$$

So,

$$\begin{aligned}\mathbf{c}^u_{t+1} &= (1 - R^{-1}(R\beta)^{1/\rho})(A_t R + N_0(\frac{R}{r})) \qquad (3) \\ &= (1 - R^{-1}(R\beta)^{1/\rho})(A_t + N_0(\frac{1}{r}))R \qquad (4)\end{aligned}$$

As long as RIC holds and $A_{-1} > -N_0/r$, the unemployed consumer will have strictly positive consumption.

iii. This is obvious from equation (3). Suppose now the unemployed consumer's bank balance is $\tilde{\mathbf{b}}_{t+1} = \mathbf{b}_{t+1} + N_0(R/r)$, then we can also get the same result as (3).

iv.

$$\underline{h}_{-1} = \frac{\underline{H}_{-1}}{l_{-1}W_{-1}} = \frac{N_0}{rl_{-1}W_{-1}} = \frac{\eta W_0 l_0}{rl_{-1}W_{-1}} = \frac{\eta\Gamma}{r} \tag{5}$$

v. The unemployed consumer's problem is, by equation (4)

$$\mathbf{c}^u_{t+1} = \kappa(A_t + \frac{\eta W_{t+1} l_{t+1}}{r})R \tag{6}$$

After normalization,

$$\begin{aligned} c^u_{t+1} &= \kappa(\frac{a_t}{\Gamma} + \frac{\eta}{r})R \\ &= \kappa\mathcal{R}a_t + \frac{\kappa\eta\Gamma\mathcal{R}}{r} \\ &= \kappa\mathcal{R}(m^e_t - c^e_t + \frac{\eta\Gamma}{r}) \end{aligned} \tag{7}$$

Here, $\mathcal{R} = R/\Gamma$. Therefore, the new $\Delta c^e_t = 0$ locus becomes

$$\begin{aligned} c^e_t &= \kappa\Pi\mathcal{R}(m^e_t - c^e_t + \frac{\eta\Gamma}{r}) \\ c^e_t &= \frac{\kappa\Pi\mathcal{R}(m^e_t + \frac{\eta\Gamma}{r})}{1 + \kappa\Pi\mathcal{R}} \end{aligned} \tag{8}$$

which means the Δc^e_t locus shifts to the left by the amount $\eta\Gamma/r$. Since the dynamic budget constraint does not change if the consumer stayed employed, the consumption function, which is the saddle path in the phase diagram, also shifts to the left by the same amount, $\eta\Gamma/r$.

One possible case where the new steady state (target) level of m^e is negative is provided using the Jupyter notebook.

vi. The expansion of unemployment benefits increased the human wealth for the unemployed consumer. It serves as a commitment for their borrowing behavior; they now can borrow an amount of money not exceeding the benefits and make the payment in the next period. The larger the benifits, the larger $\underline{H}_{-1}$ is, allowing the unemployed consumer to borrow more.

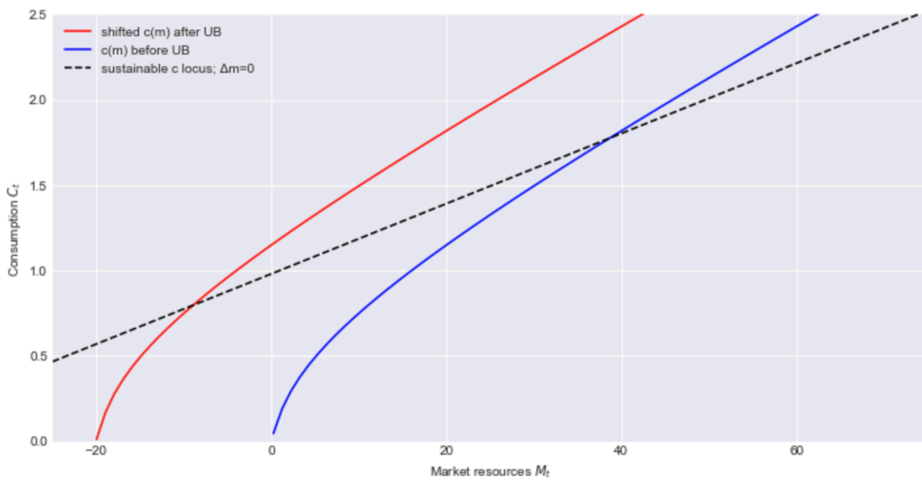

Figure 1: Case where target m is negative

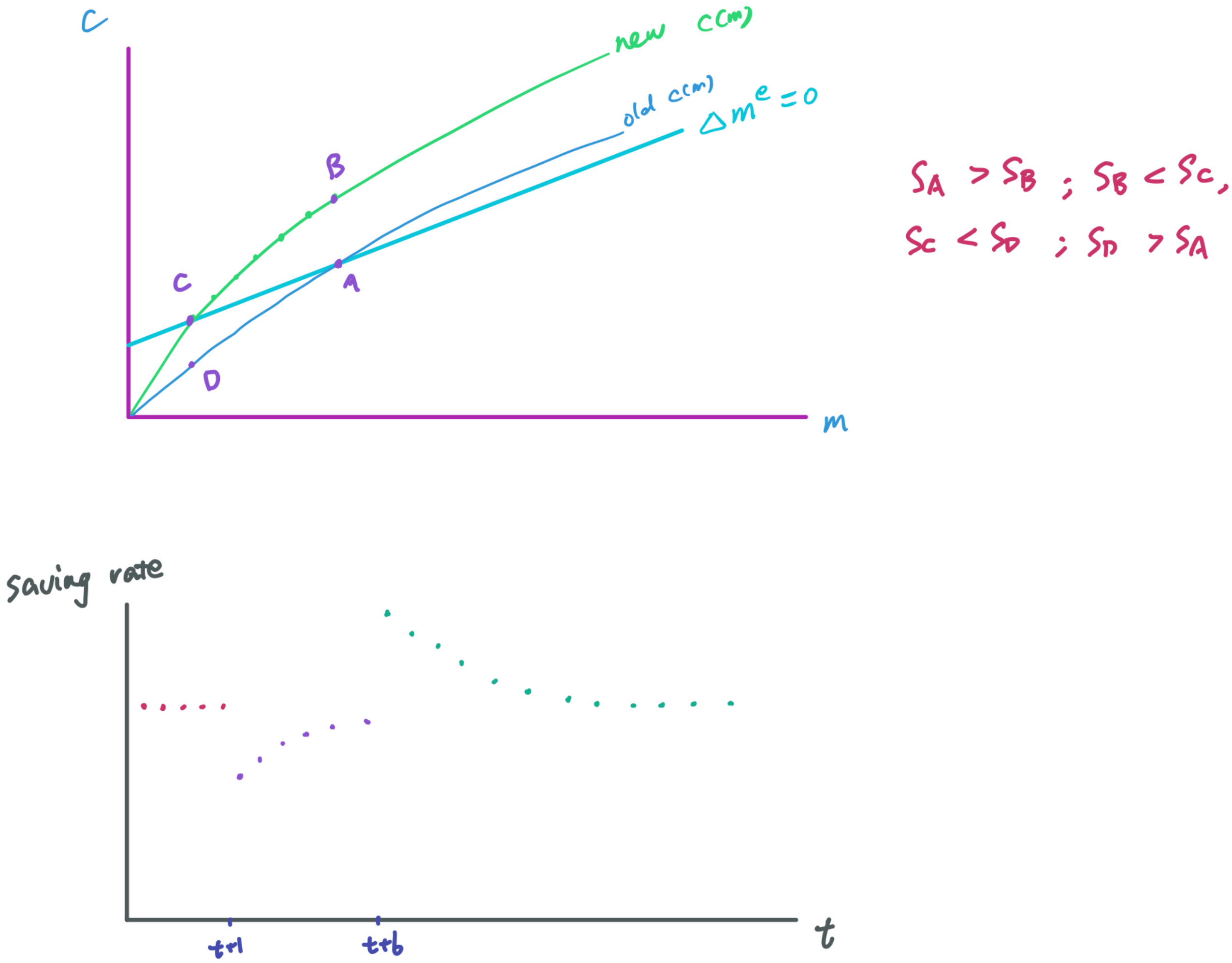

Figure 2: Effect of Decreasing Unemployment Probability

1.1 b)

i. Here I provide a manual graph. A more accurate phase diagram drawn by Jupyter notebook is also attached.

Consider equation (8), an expected decrease in the unemployment probability U will lead to an increase in Π, therefore, the new $\Delta c^e = 0$ locus will be steeper, and the new consumption function will also be higher. Intuitively, the new consumption function now approaches to the perfect foresight case.

As a result, the consumption first jumps from the old steady point A to B, then moves gradually towards C, the new steady state. After six years, the consumption function reverts itself to the original position. Therefore, consumption again jumps from C to D, and then moves gradually to A.

In this process, first we note that the personal saving rate is unambiguously lower in B compared with that in A; moving from B to A, as m decreases, the saving rate increases. Then jumping from C to D, the saving rate suddenly increases, and then gradually decreases as consumption reverts to A.

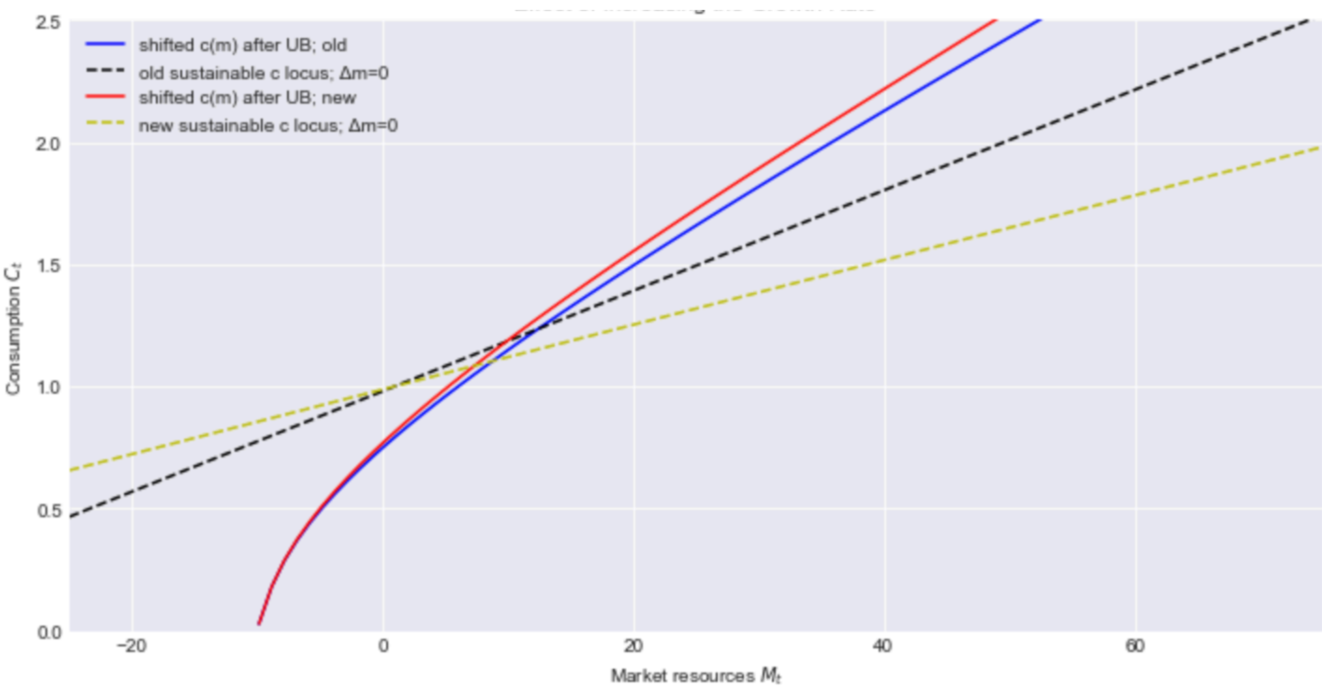

Figure 3: Effect of Increasing Growth Rate

ii. In this case, the effect on the $\Delta c^e = 0$ locus is similar to the above case, as an increase in Γ will also increase Π. Moreover, the $\Delta m^e = 0$ curve will also be flatter, since the slope $1 - \mathcal{R} = 1 - \dfrac{\Gamma}{R}$. The adjustment patterns of consumption and saving rate, however, are similar to case (i).

iii. Given that the new consumption function shifts to the left, the patterns of the adjustments of consumption and saving rate are the same as the previous two cases.

2 Asset Pricing in a Financial Crisis

i. In a Lucas Model, P_t/d_t is a constant which equals ϑ. If there is a permanent drop in the dividends of S, the price of S will also drop to half the original level. Since the subprime market is small, P_o and D_o will not change.

ii. Suppose that the increase of the variance of dividends of S will lead to an increase in the covariance of S and the consumption. Therefore, according to C-CAPM, the rate of return will be higher for asset S. Equaivalently, P_S will be lower. P_S/D_S will be lower since the mean of D_S does not change.

iii. An increase in the degree of risk aversion will increase the right hand side of equation (12) in the lecture notes C-CAPM. The rate of return of both assets will increase, suggesting that both prices will drop. The ratio, P_S/D_S and P_O/D_O, will remain constant.