Problem Set 2

Differential Equations

Fall 2024

Now, as we have dived into the contents of first order linear differential equations, you should have noticed many techniques, such as method of separation, integrating factors, or exactness. Meanwhile, we are about the see how these methods can be applied into various system, of how we can model by ODEs.

1. (Linearity of Solutions.) Let $y = y_1(t)$ be a solution to $y' + p(t)y = 0$, and let $y = y_2(t)$ be a solution to $y' + p(t)y = q(t)$. Show that $y = y_1(t) + y_2(t)$ is then also a solution to $y' + p(t)y = q(t)$.

2. (Integrating Factor.) Solve for the general solution to the following ODE with $y = y(t)$:
$$2y' + y = 3t.$$

3. (Integrating Factor or Exactness?) Let a differential equation be defined as follows:
$$\frac{dy}{dx} = e^{2x} + y - 1.$$
   (a) What is the integrating factor ($\mu(x)$) for the equation? Solve for the general solution.

   (b) Is the equation *exact*? If not, make it exact, then find the general solution.

   (c) Do solutions from part (a) and (b) agree?

4. (Decay and Dating.) Carbon-14, a radioactive isotope of carbon, is an effective tool in dating the age of organic compounds, as it decays with a relatively long period. Let $Q(t)$ denote the amount of carbon-14 at time t, we suppose that the decay of $Q(t)$ satisfies the following differential equation:
$$\frac{dQ}{dt} = -\lambda Q \text{ where } \lambda \text{ is the rate of decay constant.}$$
   (a) Let the half-life of carbon-14 be τ, find the rate of decay, λ.

   (b) Suppose that a piece of remain is discovered to have 10% of the original amount of carbon-14, find the age of the remain.

PILOT Tip of the Week

Friday, 9/6, is the last day to add courses and waitlists at Homewood. Sunday, 10/6, is the last day to add independent academic work and the last day to drop courses via self service. Friday, 11/8, is the last day to withdraw via Case Management System.