Problem Set 1

Differential Equations

Fall 2024

Welcome to the PILOT Learning for Differential Equations and Applications. Differential Equations study about the *dynamics* of system(s), while serve as foundations of many mathematical models. Moreover, we want to obtain maturity in mathematical logics and maturity. While the PILOT program cultivates the mastering of knowledge, please also seek for comprehension through collaborations.

1. (Review: Integration.) As one of the most important skills of differential equations, the study requires proficiency in integration. By the *Fundamental Theorem of Calculus*, the basics of most calculations are on finding antiderivatives. Please evaluate the following indefinite integrals:

   (a) $$\int e^{1/x} \cdot \frac{1}{x^2} dx.$$

   (b) $$\int \sin(5x) e^{-x} dx.$$

   (c) $$\int \cos(2t) \tan(t) dt.$$

2. (Separable ODE.) Solve the following initial value problem (IVP) on $y = y(x)$, and specify the domain for your solution:

$$\begin{cases} y' = (x \log x)^{-1}, \\ y(e) = -6. \end{cases}$$

3. (Direction Field.) Let a differential equation be defined as follows:

$$\frac{dy}{dx} = y^3 - 7y^2 + 16y - 12 \text{ where } x \geq 0 \text{ and } y \geq 0.$$

   (a) Classify the above differential equation.

   (b) Sketch a direction field on the differential equation, and state the equilibriums of y, interpret their stability.

4. (Constructing Solutions.) Let $x(t) = t^2 e^t$. Construct a second order ODE that has $x(t)$ as a solution and includes all of $x(t)$, $x'(t)$ and $x''(t)$, along with maybe some leftover stuff.
   Hint: Take the first and second derivative of $x(t)$ and fit them together into some linear combinations.

PILOT Tip of the Week

Friday, 9/6, is the last day to add courses and waitlists at Homewood. Sunday, 10/6, is the last day to add independent academic work and the last day to drop courses via self service. Friday, 11/8, is the last day to withdraw via Case Management System.