Problem Set 1

Differential Equations

Fall 2024

Welcome to the PILOT Learning for Differential Equations and Applications. Differential Equations study about the *dynamics* of system(s), while serve as foundations of many mathematical models. Moreover, we want to obtain maturity in mathematical logics and maturity. While the PILOT program cultivates the mastering of knowledge, please also seek for comprehension through collaborations.

1. (Review: Integration.) As one of the most important skills of differential equations, the study requires proficiency in integration. By the *Fundamental Theorem of Calculus*, the basics of most calculations are on finding antiderivatives. Please evaluate the following indefinite integrals:

   (a) $$\int e^{1/x} \cdot \frac{1}{x^2} dx.$$

   (b) $$\int \sin(5x) e^{-x} dx.$$

   (c) $$\int \cos(2t) \tan(t) dt.$$

2. (Separable ODE.) Solve the following initial value problem (IVP) on $y = y(x)$, and specify the domain for your solution:

$$\begin{cases} y' = (x \log x)^{-1}, \\ y(e) = -6. \end{cases}$$

   Note: Here $\log(x) := \log_e(x)$ is the natural logarithm function, which may be written as $\ln(x)$.

3. (Direction Field.) Let a differential equation be defined as follows:

$$\frac{dy}{dx} = y^3 - 7y^2 + 16y - 12 \text{ where } x \geq 0 \text{ and } y \geq 0.$$

   (a) Classify the above differential equation.

   (b) Sketch a direction field on the differential equation, and state the equilibriums of y, interpret their stability.

4. (Constructing Solutions.) Let $x(t) = t^2e^t$. Construct a second order ODE that has $x(t)$ as a solution and includes all of $x(t)$, $x'(t)$ and $x''(t)$, along with maybe some leftover stuff.
   Hint: Take the first and second derivative of $x(t)$ and fit them together into some linear combinations.

Clubs & Orgs Bulletin

Promote your club! https://forms.gle/V19BipzLyuAaWMyz8

GreenHacks: *Interested in sustainability and innovation? Apply to be on the GreenHacks team this year to help organize our innovation challenge in the spring semester. Looking for students of all disciplines and skill levels! Applications due Sept. 7* https://forms.gle/SfQofxjJUxXiDBHG7

KPM (Korean Pop Motion): *Are you interested in Kpop dance? KPM is the one and only award-winning kpop dance club at JHU and we are having auditions every fall semester. No previous dance experience is required, and we welcome everyone to join our audition workshop (9/4). Check out our Instagram @jhukpm for details/updates!*

Advocates for Baltimore Community (ABC) Health: *Are you looking for a way to serve our surrounding Baltimore community meaningfully? ABC Health members engage in community service, clinical volunteering, and various training opportunities such as for CPR and naloxone! Apply from our website* https://jhu.campusgroups.com/abchealth/join-us/

Tip of the Week

The Hopkins Food Pantry is a free resource for JHU affiliates facing food insecurity. Located at the LaB (right next to Homewood Apartments), it's open weekly on Mondays and Tuesdays to registered shoppers. Click here to learn more: https://studentaffairs.jhu.edu/student-life/student-outreach-support/hopkins-food-pantry/

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