PILOT

Problem Set 4

Differential Equations

Fall 2024

As we wrapped up the first order differential equation, we have encountered a larger class of differential equations, the differential equations of higher orders. As of right now, we should have seen the theorems behind the higher order ODEs as well as the method to tackle the most trivial case. Let us combine these methods and concepts to work on the problem set this week.

1. (Second Order Differential Equation.) Let an initial value problem for $y = y(t)$ be defined as follows:

$$\begin{cases} 4y'' - y = 0, \\ y(0) = 2,\ y'(0) = \beta, \end{cases}$$

where β is a real constant.

   (a) Find the specific solution to the initial value problem. Express your solution with constant β.

   (b) Find the value of β such that the solution *converges* to 0 as t tends to infinity.

2. (LI Set of Solutions.) Find the general solution to the following differential equation, and verify that your solution is a linearly independent set of solutions.

$$y^{(3)}(x) - 6y''(x) + 11y'(x) - 6y(x) = 0.$$

3. (A Missing Solution.) Let a third order differential equation of $y = y(x)$ be defined as below:

$$y''' - y'' + y' - y = 0.$$

   (a) Verify that $\sin(x)$ and $\cos(x)$ are two solutions to the above differential equation. Can you explain how we can find these two solutions?

   (b) Is the set $\{\sin(x), \cos(x)\}$ linearly independent?

   (c) Does $\{\sin(x), \cos(x)\}$ constitute a full set of solution to the differential equation?

   (d) Give the general solution to the differential equation.

4.* (Repeated and Complex Root.) Let a six order differential equation of $y = y(t)$ be defined as follows:

$$y^{(6)} - 2y^{(3)} + y = 0.$$

Find a set of real-valued function being the general solution to the above differential equation.

5.* (A Symmetric Solution.) Given the following second order initial value problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + \sin^2(1-x)y = \cosh(x-1), \\ y(1) = e, \ \dfrac{dy}{dx}(1) = 0. \end{cases}$$

Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.
Hint: Consider the interval in which the solution is unique.
Also, note that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

Clubs & Orgs Bulletin

Promote your club! `https://forms.gle/V19BipzLyuAaWMyz8`

Seedling: *Seedling is a nonprofit that teaches Baltimore youth about agriculture, nutrition, sustainability, and technology. We're looking for teaching volunteers, curriculum developers, and leadership team members. You can learn more and apply at* `https://bit.ly/seedlinginterest` *by September 30th!*

Hopkins Emergency Response Organization: *Applications are now open for both licensed EMTs and Sophomores and Freshmen with no EMS experience to join Hopkins Emergency Response Organization! We provide 24/7 emergency response for medical emergencies on campus and in the surrounding area. We provide an accelerated EMT course for non-priors!*

Phi Delta Epsilon: *Interested in being a pre-med and joining a community in Hopkins? Join PhiDE, we are a new co-ed medical fraternity that practices philanthropy, deity, and education. We have several events and opportunities to help people learn more about the medical journey.* `https://forms.gle/smLCyeAAmT6t2y4v9`

Tip of the Week

Interested in getting involved in research and don't know where to start? The Hopkins Office of Undergraduate Research (HOUR) connects students with open opportunities based on their interests and area of study. They also manage competitive grants like PURA and the BDP Summer Program. Check out their office at the Imagine Center or click here for more: `https://hour.jhu.edu/`.