PILOT

Problem Set 5

Differential Equations

Fall 2024

Second order differential equations are very similar to higher order differential equations. As of right now, we are manipulating the case of constant coefficients, which is the most simplified case overall. Before we get to the more complicated cases, *i.e.*, non-homogeneous cases, let us explore more to the canonicals behind them.

1. (Constructing Solutions, Again.) Construct an initial value problem for the following solutions:
   (a) $y(t) = 4e^{3t} - e^{-2t}$.
   (b) $y(t) = e^{2t}\cos t + e^{2t}\sin t + e^{2t}$.

2.* ($L^2([0, 2\pi])$ Space.) Recall that we have defined linear independence of functions, we define *orthogonality* of two real-valued, "square-integrable" functions over $[0, 2\pi]$, f and g, as:

$$\int_0^{2\pi} f(x)g(x)dx = 0.$$

   (a) Show that the set $\{\sin x, \cos x\}$ is linearly independent and orthogonal.
   (b) Show that if $\{f(x), g(x)\}$ is orthogonal, then $C_1 f(x)$ and $C_2 g(x)$ is orthogonal.
   (c) Note that $\{x, x^2\}$ are linearly independent, construct a basis that is orthogonal.

3.* (Repeated and Complex Root.) Let a six order differential equation of $y = y(t)$ be defined as follows:

$$y^{(6)} - 2y^{(3)} + y = 0.$$

   Find a set of real-valued function being the general solution to the above differential equation.

4. (Reduction of Order.) Let a ODE be defined as follows:

$$t^2y'' + 2ty' = 2y,\ t > 0.$$

   Given a solution is $y_1(t) = t$, find the full fundamental set of solution.

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Tip of the Week

Did you know that Hopkins has a planetary observatory right on Homewood campus? Every Friday night the Maryland Space Grant Observatory located on the roof of Bloomberg hosts a free open house for the public after sunset, weather permitting. Check here to see updates on hours and visibility status https://md.spacegrant.org/observatory-open-house/.