



Problem Set 1
Differential Equations
Fall 2025

Welcome to the PILOT Learning for AS.110.302 Differential Equations and Applications. This course studies the *dynamics* of system(s), described as ordinary differential equations, which are foundations of many more advanced mathematical models. While the PILOT program cultivates the mastering of knowledge, please also seek for comprehension through collaboration.

Prior to entering this class, let's remark on the key components that you might be familiar with:

- Pre-Calculus:
 - Polynomials and roots
 - Trigonometry
 - Complex Numbers
- Basic Calculus:
 - Differentiation
 - Antiderivatives
 - Integration Techniques
- Sequences and Series:
 - Power Series
 - Convergence & Divergence
 - Taylor Expansion
- Basic Linear Algebra Concepts:
 - Linear Independence
 - Eigenspace (optional)
 - Determinants

Clubs & Orgs Bulletin

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To be updated.

Tip of the Week

To be updated.

1. (Indefinite Integrals). As one of the most important skills of differential equations, the study requires proficiency in integration. By the *Fundamental Theorem of Calculus*, the basics of most computations are on finding antiderivatives. Please evaluate the following indefinite integrals:

(a) $\int e^{1/x} \cdot \frac{1}{x^2} dx.$

(b) $\int \sin(5x)e^{-x} dx.$

(c) $\int \cos(2t) \tan(t) dt.$

2. (Separable ODE.) Solve the following initial value problem (IVP) on $y = y(x)$, and specify the domain for your solution:

$$\begin{cases} y' = (x \log x)^{-1}, \\ y(e) = -6. \end{cases}$$

Note: Here $\log(x) := \log_e(x)$ is the natural logarithm function, which may be written as $\ln(x)$.

3. (Direction Field). Let a differential equation be defined as follows:

$$\frac{dy}{dx} = y^3 - 7y^2 + 16y - 12 \text{ where } x \geq 0 \text{ and } y \geq 0.$$

Recall that a differential equation is:

- **ordinary** if it is composed of only ordinary derivatives ($d^n y/dx^n$ or $y^{(n)}$), and **partial** if it contains any partial derivatives ($\partial^n y/\partial x^n$ or $\partial_x^n y$);
- **linear** if the operator $L[y] = f(x)$ is linear with respect to y , and **nonlinear** otherwise.
- of **order** k if the highest order of derivative is k .

- (a) Classify the above differential equation.
- (b) Sketch a direction field on the differential equation, and state the equilibriums of y , interpret their stability.

4. (Constructing ODEs from Solutions). Let $x(t) = t^2 e^t$.
- (a) Construct a second order ODE such that $x(t)$ is a solution and the differential equation includes all terms of $x(t)$, $x'(t)$ and $x''(t)$, along with some leftover terms independent of x .
Hint: Take the derivative of $x(t)$ and fit them together into some linear combinations.
- (b) Are the ODEs satisfying (a) unique?