PILOT

Problem Set 2

Differential Equations

Fall 2025

As we have dived into the contents of first order linear differential equations, you should have noticed many techniques, such as method of separation, integrating factors, or exactness (about to come). Meanwhile, we are about the see practice on these solving techniques and explore an explanation of such concept.

Clubs & Orgs Bulletin

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There is no bulletin this week.

Tip of the Week

There is no tip of the week this week.

1. (Another Separable ODE). Solve the following initial value problems (IVPs) on $y = y(x)$, and specify the domain for your solution:

$$\begin{cases} y' = y(y+1), \\ y(0) = 1. \end{cases}$$

2. (Integrating Factor). Solve for the general solution to the following ODEs with $y = y(t)$:
   (a) $2y' + y = 3t$.
   (b) $y' + \log(t)y = t^{-t}$.

3. (Linearity of Solutions). Let $y = y_1(t)$ be a solution to $y' + p(t)y = 0$, and let $y = y_2(t)$ be a solution to $y' + p(t)y = q(t)$. Show that $y = y_1(t) + y_2(t)$ is then also a solution to $y' + p(t)y = q(t)$.

4. (Integrating Factor for IVP). Given an initial value problem:

$$\begin{cases} \dfrac{dy}{dt} - \dfrac{3}{2}y = 3t + 2e^t, \\ y(0) = y_0. \end{cases}$$

   (a) Find the integrating factor $\mu(t)$.

   (b) Solve for the particular solution for the initial value problem.

   (c) Discuss the behavior of the solution as $t \to \infty$ for different cases of y_0.