



## Problem Set 8

### Differential Equations

Fall 2025

Up to right now, we have completed our expenditure of higher order differential equations. You should be familiar with the following concepts:

- Concepts:
  - Set of Solutions
  - Linear Independence
  - Existence & Uniqueness Theorem
- Methods to solve higher order ODEs:
  - Characteristic Equation
  - Euler's Formula
  - Undetermined Coefficients
  - Reduction of Order
  - Variation of Parameters

Now, as we step into more linear algebra, we are going to review the key contents of this part of the class, and explore how are they related to linear systems.

### Clubs & Orgs Bulletin

Promote your club! <https://forms.gle/V19BipzLyuAaWMyz8>

**Violet Patient Volunteering Program:** Violet is a fantastic opportunity for clinical volunteering, peer education, and community engagement. We spread sexual and reproductive health education across 3 clinics in Baltimore, and our applications for our Spring 2026 volunteering program are open! The written application is due Sunday, October 26th at 11:59 pm. Check our instagram @violet4teens for the link :)

**Society of Women Engineers:** SWE is hosting a Professor Panel dinner on 10/22 from 6:30pm! Meet female WSE professors, learn about their research and paths into academia, and get advice on post-college opportunities. Registration on HopkinsGroups is required and food is provided. Follow us on Instagram @jhuswe to learn more!

### Tip of the Week

*The beginning of fall also means the start of flu season—get your shot today! Hopkins holds flu clinics on all of its campuses. Students must upload verification of their immunization or a valid exemption by Friday, Nov 21st, 2025. Find out more information here: <https://wellbeing.jhu.edu/PrimaryCare/annual-flu-vaccine-requirement/>.*

1. (Finding General Solutions). Find the general solution to the following differential equation.

(a)  $y'''(x) - 6y''(x) + 11y'(x) - 6y(x) = 0.$

(b)  $y''' = -y.$

2. (Non-homogeneous Solutions). Find the general solution to the following differential equations:

(a)  $y''' - 4y' = e^{-2t}.$

(b)  $y'' + 36y = e^t \sin(6t).$

3. (Non-homogeneous Cases of Higher Order ODEs). Let a third order differential equation be as follows:

$$\ell[y(t)] = y^{(3)}(t) + 3y''(t) + 3y'(t) + y(t).$$

Let  $\ell[y(t)] = 0$  be trivial initially.

- (a) Find the set of all linearly independent solutions.

Then, assume that  $\ell[y(t)]$  is non-trivial.

- (b) Find the particular solution to  $\ell[y(t)] = \sin t$ .

- (c) Find the particular solution to  $\ell[y(t)] = e^{-t}$ .

- (d) Suppose that  $\ell[y_1(t)] = f(t)$  and  $\ell[y_2(t)] = g(t)$  where  $f(t)$  and  $g(t)$  are “good” functions. Find an expression to  $y_3(t)$  such that  $\ell[y_3(t)] = f(t) + g(t)$ .

4. (Solving Linear Systems). Let  $\mathbf{x} \in \mathbb{R}^2$ , find the general solution of  $\mathbf{x}$  for:

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \cdot \mathbf{x}.$$