PILOT

Problem Set 7

Differential Equations

Fall 2025

1. (A Symmetric Solution). Given the following second order initial value problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + \sin^2(1-x)y = \cosh(x-1), \\ y(1) = e,\ \dfrac{dy}{dx}(1) = 0. \end{cases}$$

Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.

Hint: Consider the interval in which the solution is unique. Note that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

2. (Non-homogeneous Differential Equations). Solve the following differential equations.

   (a) $y'' + 4y = t^2 + 3e^t$.

   (b) $y'' + 2y' + y = \dfrac{e^{-x}}{x}$.

3. (Repeated Roots and Wronskian). Let a differential equation of $y := y(x)$ be:

$$y''' + 3y'' + 3y' + y = 0.$$

Find the general solution the differential equation and give the Wronskian of your set of solutions.

4. (Higher order IVP with Dirichlet Condition). Consider the following initial value problem:

$$\begin{cases} 2y''' - 11y'' + 17y' - 6y = 0, \\ y(0) = 3, y\big(\log(4)\big) = 82, y\big(\log(9)\big) = 813. \end{cases}$$

Find the specific solution to the IVP.