



Problem Set 3

Differential Equations

Fall 2025

As we have dived into the contents of first order linear differential equations, you should have noticed many techniques, such as method of separation, integrating factors, or exactness (about to come). Meanwhile, we are about to see practice on these solving techniques and explore an explanation of such concept.

Clubs & Orgs Bulletin

Promote your club! <https://forms.gle/V19BipzLyuAaWMyz8>

- APO: Want to earn community service hours? JOIN APO! We are a Co-Ed Service Frat that engages in leadership development, service in the Baltimore area, and brotherwide fellowships. Also, we are the largest volunteering org (5000+ hours in SP25)! Interest form: <https://forms.gle/gPt9LwKM9z93QmqHA>
- Hopkins Political Union: Interested in politics or debate? Join the Hopkins Political Union JHU's largest forum for robust, cross-ideological dialogue. In collaboration with the SNF Agora Institute, we will host two debates on 9/19 & 11/14. Welcoming all views! Sign up here: https://jhu.campusgroups.com/HPU/club_signup/

Tip of the Week

The Hopkins Food Pantry is a free resource for JHU affiliates facing food insecurity. Located at the LaB (right next to Homewood Apartments), it's open weekly on Mondays and Tuesdays to registered shoppers.

Learn more at <https://studentaffairs.jhu.edu/student-life/student-outreach-support/hopkins-food-pantry/>.

1. (Stability of Autonomous ODEs). Draw the phase line and determine the stability of each equilibrium for the following autonomous differential equations:

(a) $y' = y^4 - 3y^3 + 2y^2.$

(b) $y' = y^{2025} - 1.$

(c) $y' = y^2 + 2y + C,$ where $C \in \mathbb{R}$ is a constant.

For part (c), determine the bifurcation values for the parameter C and sketch a bifurcation diagram.

2. (More IVPs). Given following IVPs:

(a)
$$\begin{cases} y' + \frac{1}{2}y = \sin t, \\ y(0) = 1. \end{cases}$$

(b)
$$\begin{cases} y' = \frac{1}{x^4 - 1}, \\ y(0) = 0. \end{cases}$$

Find the specific solution of the equation, note the domain of the solution, and describe the end behavior of the solution if it appears in the valid domain.

3. (Existence and Uniqueness for IVP). Suppose $f(x)$ is non-zero, let an initial value problem be:

$$\begin{cases} \frac{1-y}{x} \cdot \frac{dy}{dx} = \frac{f(x)}{1+y}, \\ y(0) = 0. \end{cases}$$

- (a) Show that the differential equation is **not** linear.

For the next two parts, suppose $f(x) = \tan x$.

- (b) State, without justification, the open interval(s) in which $f(x)$ is continuous.
(c) Show that there exists some $\delta > 0$ such that there exists a unique solution $y(x)$ for $x \in (-\delta, \delta)$.

Now, suppose that $f(x)$ is some function, **not** necessarily continuous.

- (d) Suppose that the condition in (c) does **not** hold, give three examples in which $f(x)$ could be.

4. (Existence of Largest Interval). For the following IVPs, determine the largest interval in which a solution is guaranteed to exist.

(a)
$$\begin{cases} (t-3)y' + (\log t)y = 2t, \\ y(1) = 2. \end{cases}$$

(b)
$$\begin{cases} (4-t^2)y' + 2ty = 3t^2, \\ y(1) = -3. \end{cases}$$

(c)
$$\begin{cases} y' + (\tan t)y = \sin t, \\ y(\pi) = 0. \end{cases}$$