PILOT Activity Slides

Differential Equations

Johns Hopkins University

Fall 2025

Welcome to ODE PILOT Session!

PILOT Sessions varies for each PILOT leader, please check in with your leader.

Session Information

- Term: Fall 2025
- Dates: Between September 1st and December 5th (inclusive).
  - The sessions on Labor's Day (September 1st) and during the Fall break (October 16th - October 17th) are optional. Please refer to your assigned PILOT leader for details.
  - There will be no sessions during the Fall recess (November 24th - November 28th).

PILOT Webpage for ODEs

`https://jhu-ode-pilot.github.io/FA25/`

Resources

Review problem sets (and possibly review sessions) will be planned prior to assessments (quizzes and exams).

- The review problem sets and review session information will be updated on the webpage. Please often check for new updates.

If you are looking for more conceptual challenge, there is an **Additional Problem Set** on the webpage that you can try to consolidate your understanding on the course.

Ground Expectations

- Present to the weekly meeting. If you have any time conflicts or reasons, please notify your PILOT leader in advance. Note that attendance will be taken, and multiple absences might result in removal from this session.
- Discuss with other students and/or the PILOT leader during meetings, while you may propose any questions and/or concerns if you have any.

Please be respectful and polite to other students!

If you found any of the contents a mental challenge or uncomfortable, please contact your leader or contact the Director of PILOT at Jenna Hoffman (`jhoffm71@jhu.edu`).

Introducing yourselves

Let's get to know each other.

Introduction Questions

This section aims to help you introduce yourselves to the other students, please use a few minutes to think about the problems and introduce yourselves to your peers.

Think about yourself. Get ready to introduce yourself by addressing the following information:

- Your name,
- Your expected graduation year,
- Your major(s) and minor(s),
- Your interested area(s) in mathematics.

Outreach Problems

ODEs are useful tools at many places.

Here are some open questions

If you do not prefer tedious introductions, choose one of the following questions and give a creative answer.

ODEs Outreach

What is one thing in your life, that you imagine ODEs can model. Explain why?

Examples

- How will the population of _____ change over time.
- How will a _____ dynamical system look like.
- Can I model _____.
- ...

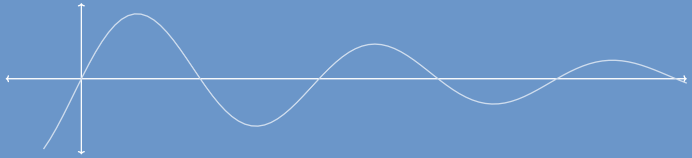
ODEs Outreach

Use the function:

$$f(x) = 10\sin\left(\frac{x}{5}\right)\exp\left(-\frac{x}{50}\right)$$

to describe something.

The graph of the above function.



ODEs Outreach

If you can define a mathematical constant, what would you define?

Example mathematical constants.

- $\pi, \tau, \cdots$.
- $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.
- $\ldots$

ODEs Outreach

Do you have a favorite theorem/formula/kernel?

Kernel

A good kernel $K_\delta(x)$ should be integrable (on $\mathbb{R}$) and satisfies the following for all $\delta > 0$:

1. $\int_{\mathbb{R}^d} K_\delta(x)dx = 1$,
2. $\int_{\mathbb{R}^d} |K_\delta(x)|dx \leq A$, and
3. for every $\eta > 0$, $\int_{|x|\geq\eta} |K_\delta(x)|dx \to 0$ as $\delta \to 0$,

where A is a constant depending on δ.

ODEs Outreach

Weierstrass Approximate Theorem guarantees uniform convergence for continuous functions, whereas *Fourier Convergence Theorem* only guarantees convergence for square integrable functions. Can you think of some places where you find trade-off situations?

ODEs Outreach

In mathematics, we call a question *well-posed* if it aligns with the following properties:

1. Existence: There exists at least one solution;
2. Uniqueness: There exists at most one solution;
3. Continuity: The solution depends continuously on the data, *i.e.*, a small error on initial/boundary data entails a small error on the solution.

Can you think of any "well-posed" questions?

Ordering Game

Ordered Sets

The field of real numbers is ordered. Thus, each person can select a number, and thus determining an order for the group.

Of course, a partial order $\prec$ must satisfy a few rules, and there are many other orders.

Dictionary Order

Of course, there are different ordering methods. For examples, you can look up *dictionary order* for complex numbers.

Ordering Game

Below are subsets of real numbers, select a number from a set:

- $\left\{0, 1, 2, -1, \frac{1}{2}, \sqrt{2}, \pi, e\right\}$,
- $\left\{\sin\frac{2k\pi}{15} : k \in \mathbb{Z} \wedge 0 \leq k \leq 14\right\}$,
- $\mathbb{R}\backslash\mathbb{Q}$ (irrational numbers),
- $\left\{\det\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}, \det\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \det\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}, \det\begin{pmatrix}-1 & 0\\-1 & 2\end{pmatrix},\right.$
  $\left.\det\begin{pmatrix}1 & 3\\4 & 7\end{pmatrix}, \det\begin{pmatrix}2 & 0\\0 & 7\end{pmatrix}, \det\begin{pmatrix}1 & 0\\4 & 3\end{pmatrix}, \det\begin{pmatrix}1 & -2\\12 & 13\end{pmatrix}\right\}$,

Ordering Game (Cont.)

- $\mathbb{A} \cap \mathbb{R}$ (real, algebraic number),
- $\{f(-10), f(-2), f(0), f(3), f(5), f(20)\}$,
  where $f(x) = \int_0^\infty e^{-xt} \sin t dt$,
- $\mathbb{Q}(\sqrt{2}, \sqrt{3}) := \left\{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} : a, b, c, d \in \mathbb{Q}\right\}$,
- $\{n :$ regular n-gon is constructible$\}$,
  Hint: Regular n-gon is constructible $\iff$ $\phi(n)$ is an integral power of 2,
- $\{F_n\}_n$ (Fibonacci sequence).