PILOT

Problem Set 12

Differential Equations

Spring 2025

Up to now, we are about to finish the first order linear system. As we are about to get into the chapter, let us review what we have learned about linear systems.

- Concepts:
  - Vector space
  - Eigenspace
  - Existence & Uniqueness Theorem
- Methods to solve linear systems of ODEs:
  - Eigenvectors & Eigenvalues
  - Repeated Roots
  - Phase Portraits and Stability

1. (Directional Field for Linear System). For the following systems with $\mathbf{x} = (x_1, x_2)$, draw a direction field and plot some trajectories to characterize the solutions.
$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \cdot \mathbf{x}.$$

2. (Complex Eigenvalue and Phase Portraits). Find the general solution and sketch a few phase portraits for:
$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \cdot \mathbf{x}.$$

3. (Phase Portraits for Repeated Roots). Find the solutions to the following linear system differential equation, sketch a few phase portraits, and classify its type and stability.

   (a) $$\mathbf{x}' = \begin{pmatrix} 4 & 2 \\ -2 & 0 \end{pmatrix} \cdot \mathbf{x}.$$

   (b) $$\mathbf{x}' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \cdot \mathbf{x}.$$

4. (Critical Point). Find all the critical point in the following first order system:
$$\begin{cases} x' = 2x^3 - x^2 - 4x + 3 - y^2, \\ y' = 2x - y. \end{cases}$$

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Tip of the Week

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