PILOT Activity Slides

Differential Equations

Johns Hopkins University

Spring 2025

Welcome to ODE PILOT Session!

PILOT Sessions varies for each PILOT leader, please check in with your leader.

Session Information

- Term: Spring 2025
- Meeting Schedule:
  - Dates: Between January 21st and April 24th (inclusive).
  - Day of the Week: _____.
  - Except for Spring break (March 17th - March 21st).
  - Time: _____, Eastern Time.
  - Location: _____.
- Leader: _____ (email: _____).

Resources for PILOT

PILOT Webpage for ODEs

`jhu-ode-pilot.github.io/SP25/`

This web page will record the weekly problem sets, general resources, review sets, and announcements. Please save the page as an resource.

Review sessions will be planned prior to the final. Review sets will be published prior to quizzes and the final.

Exam Number	Exam Date	Review Session Time
Midterm 1		
Midterm 2		
Final		

Ground Expectations

In participating the PILOT program, you are expected to:

- Present to the weekly meeting. If you have any time conflicts or reasons, please notify your PILOT leader. Note that attendance will be taken, and multiple absences might result in removal from this session.
- Discuss with other students and/or the PILOT leader during meetings, while you may propose any questions and/or concerns if you have any.

Please be respectful and polite to other students!

If you found any of the contents a mental challenge or uncomfortable, please contact your leader or contact the Director of PILOT at Jenna Hoffman (`jhoffm71@jhu.edu`).

Introducing yourselves

Let's get to know each other.

Introduction Questions

This section aims to help you introduce yourselves to the other students, please use a few minutes to think about the problems and introduce yourselves to your peers.

Think about yourself. Get ready to introduce yourself by addressing the following information:

- Your name,
- Your expected graduation year,
- Your major(s) and minor(s),
- Your interested area(s) in mathematics.

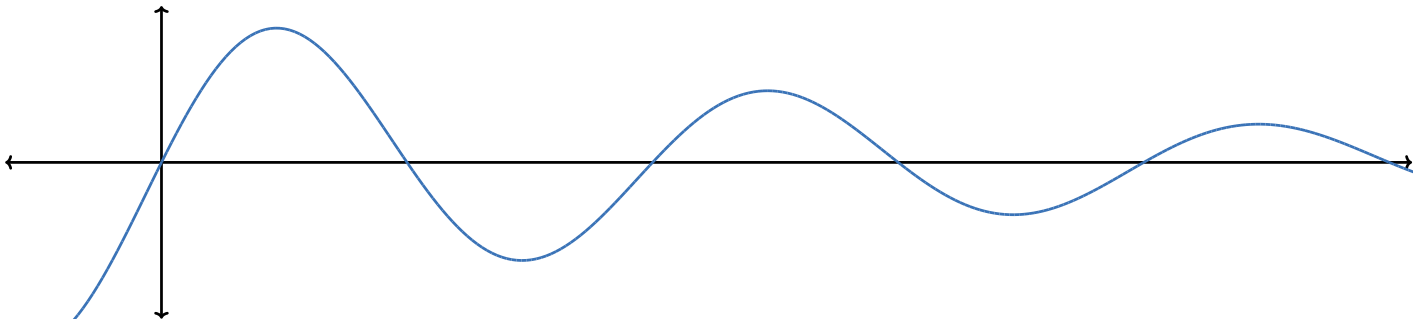
ODEs Outreach

ODEs are useful tools at many places.

Outreach Problems

If you do not prefer tedious introductions, choose one of the following questions and give a creative answer.

- What is one thing in your life, that you imagine ODEs can model. Explain why?
- Use the function $f(x) = 10 \sin\left(\frac{x}{5}\right) \exp\left(-\frac{x}{50}\right)$ to describe something. (Graph as follows.)



ODEs Outreach (Cont.)

- If you can define a mathematical constant, what would you define?
- Do you have a favorite formula/kernel? Name it.

Kernel

A good kernel $K_\delta(x)$ should be integrable (on $\mathbb{R}$) and satisfies the following for all $\delta > 0$:

1. $\int_{\mathbb{R}^d} K_\delta(x)dx = 1$,
2. $\int_{\mathbb{R}^d} |K_\delta(x)|dx \leq A$, and
3. for every $\eta > 0$, $\int_{|x|\geq\eta} |K_\delta(x)|dx \to 0$ as $\delta \to 0$,

where A is a constant depending on δ.

ODEs Outreach (Cont.)

- *Weierstrass Approximate Theorem* guarantees uniform convergence for continuous functions, whereas *Fourier Convergence Theorem* only guarantees convergence for square integrable functions. Can you think of some places where you find trade-off situations?
- In mathematics, we call a question *well-posed* if it aligns with the following properties:
  1. Existence: There exists at least one solution;
  2. Uniqueness: There exists at most one solution;
  3. Continuity: The solution depends continuously on the data, *i.e.*, a small error on initial/boundary data entails a small error on the solution.

  Can you think of any "well-posed" questions?

Ordering Game

Ordered Sets

The field of real numbers is ordered. Thus, each person can select a number, and thus determining an order for the group.

Below are subsets of real numbers, select a number from a set:

- $\left\{0, 1, 2, -1, \frac{1}{2}, \sqrt{2}, \pi, e\right\}$,
- $\left\{\sin \frac{2k\pi}{15} : k \in \mathbb{Z} \wedge 0 \leq k \leq 14\right\}$,
- $\mathbb{R} \backslash \mathbb{Q}$ (irrational numbers),
- $\left\{\det \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \det \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix}, \det \begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix}, \det \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}, \det \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}, \det \begin{pmatrix} 1 & -2 \\ 12 & 13 \end{pmatrix}\right\}$,

Ordering Game (Cont.)

- $\mathbb{A} \cap \mathbb{R}$ (real, algebraic number),
- $\{f(-10), f(-2), f(0), f(3), f(5), f(20)\}$, where $f(x) = \int_0^\infty e^{-xt} \sin t dt$,
- $\mathbb{Q}(\sqrt{2}, \sqrt{3}) := \left\{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} : a, b, c, d \in \mathbb{Q}\right\}$,
- $\{n$: regular n-gon is constructible$\}$, *Hint:* Regular n-gon is constructible $\iff$ $\phi(n)$ is an integral power of 2,
- $\{F_n\}_n$ (Fibonacci sequence).

Other Orders?

Of course, there are different ordering methods. For examples, you can look up *dictionary order* for complex numbers.