Welcome to ODEs PILOT!

webpage: jhu-ode-pilot.github.io/SU24/

Midterm 1 Review Session on Jun. 21st 8pm ET

$$\frac{dy}{dt} = 0.5\, y\left(1 - \frac{y}{K}\right) \qquad y_0 = \frac{K}{5}$$

$$\begin{cases} \dfrac{dy}{dt} = r\left(1 - \dfrac{y}{k}\right) y \\ y(0) = y_0 \end{cases} \Rightarrow y(t) = \frac{k y_0}{(k - y_0)e^{-rt} + y_0}$$

Carrying capacity

$$\frac{dy}{\left(1-\frac{y}{k}\right)\cdot y} = r\,dt$$

$$\frac{Ay + B - \frac{By}{k}}{\left(1-\frac{y}{k}\right)\cdot y} \rightarrow 1 + 0y$$

$$B = 1$$

$$A - \frac{B}{k} = 0$$

$$A = \frac{1}{k}$$

$$\left(\frac{A}{1-\frac{y}{k}} + \frac{B}{y}\right) dy = r\,dt$$

$$\int\left(\frac{1}{k-y} + \frac{1}{y}\right) dy = \int r\,dt$$

$$-\log|k-y| + \log|y| = rt + C$$

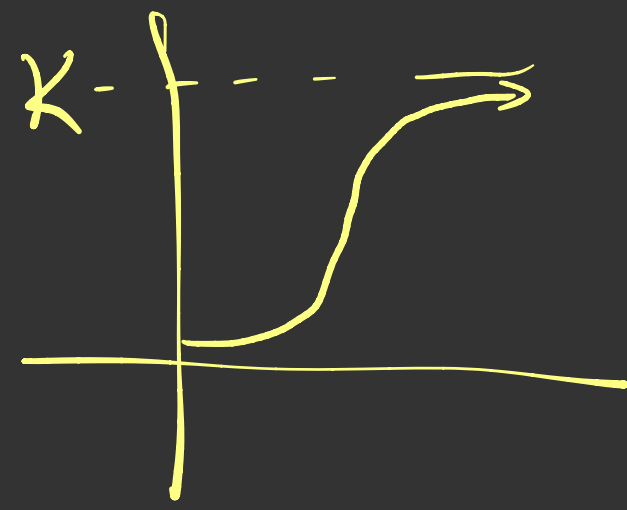
$$\log\left|\frac{y}{k-y}\right| = rt + C$$

$$\left|\frac{y}{k-y}\right| = \tilde{C}e^{rt}$$

$$y(0) = y_0.$$

$$\frac{y_0}{k-y_0} = C$$

($y_0 \to +$, $k-y_0 \to +$, $C \sim$)



$$\frac{y}{k-y} = \frac{y_0}{k-y_0} e^{rt}$$

$$y = (k-y)\frac{y_0}{k-y_0} e^{rt}$$

$$\left(1 + \frac{y_0}{k-y_0} e^{rt}\right) y = \frac{k y_0}{k-y_0} e^{rt}$$

$$y = \frac{k y_0 e^{rt}}{(k - y_0)\left(1 + \frac{y_0}{k - y_0} e^{rt}\right)}$$

$$= \frac{k y_0 e^{rt}}{k - y_0 + y_0 e^{rt}}$$

$$= \boxed{\frac{k y_0}{(k - y_0) e^{-rt} + y_0}}$$

$$y_0 = \frac{k}{5}$$

$$y(t_1) = \frac{2k}{5}$$

$$= \frac{k^2/5}{\frac{4}{5}k e^{-rt} + \frac{k}{5}} = \frac{2k}{5}$$

t?

$$\frac{k y_0}{(k - y_0) e^{-rt} + y_0} = y$$

$$k y_0 = y(k - y_0) e^{-rt} + y y_0$$

$$\frac{ky_0 - yy_0}{y(k - y_0)} = e^{-rt}$$

$$\log\left(\frac{ky_0 - yy_0}{y(k - y_0)}\right) = -rt$$

$$t = -\frac{1}{r}\log\left(\frac{y_0(k-y)}{y(k-y_0)}\right)$$

this is k

k/s