PILOT Activity Slides

Differential Equations

Summer 2024

Welcome to ODE PILOT Session!

Session Information

- Term: Summer 2024
- Dates: Thursdays between June 6th and July 25th (inc.).
- Time: 8:00pm–10:00pm, Eastern Time.
- Location: Zoom `https://jhubluejays.zoom.us/j/99589950352?pwd=b0JXY3c5ZFpnb2JtcHU4LzBIeGx5Zz09`
  - Meeting ID: 995 8995 0352
  - Passcode: 219091
- Leader: James Guo (`sguo45@jhu.edu`)

PILOT Webpage for ODE

`https://livejohnshopkins.sharepoint.com/sites/mathland/ODE_PILOT`

Ground Expectations

In participating the PILOT program, you are expected to:

- Discuss with other students and/or the PILOT leader during meetings, while you may propose any questions and/or concerns if you have any.
- Be respectful and polite to other students during the meetings. If you found any of the contents a mental challenge or uncomfortable, feel free to contact me via email or contact the Director of PILOT at Jenna Hoffman.

Summer PILOT

Summer PILOT works more like Office Hours, please join the zoom for extra help and review sessions.

Introducing yourselves

Let's get to know each other.

Introduction Questions

This section aims to help you introduce yourselves to the other students, please use a few minutes to think about the problems and introduce yourselves to your peers.

Think about yourself. Get ready to introduce yourself by addressing the following information:

- Your name,
- Your expected graduation year,
- Your major(s) and minor(s),
- Your interested area(s) in mathematics.

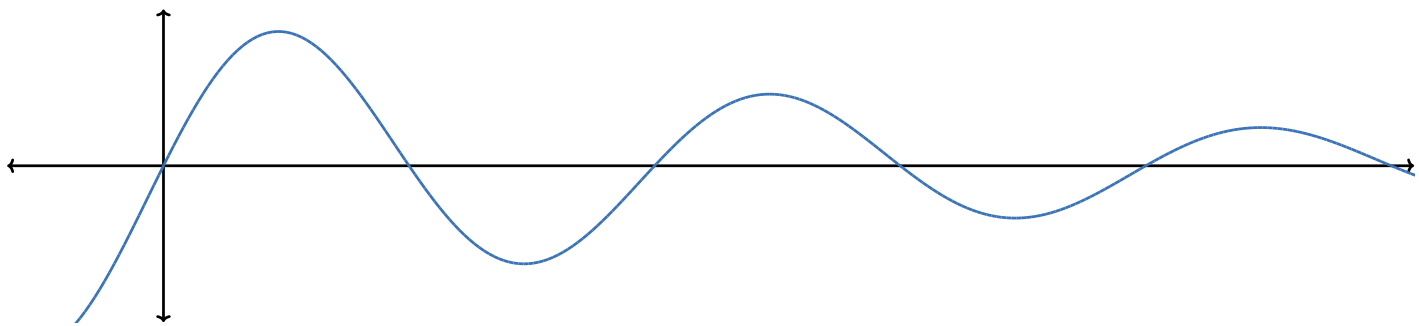
ODEs Outreach

ODEs are useful tools at many places.

Outreach Problems

If you do not prefer tedious introductions, choose one of the following questions and give a creative answer.

- What is one thing in your life, that you imagine ODEs can model. Explain why?
- Use the function $f(x) = 10 \sin\left(\frac{x}{5}\right) e^{-\frac{x}{50}}$ to describe something.



ODEs Outreach (Cont.)

- If you can define a mathematical constant, what would you define?
- Do you have a favorite formula/kernel? Name it.
- *Weierstrass Approximate Theorem* guarantees *uniform convergence* for *continuous functions*, whereas *Fourier Convergence Theorem* only guarantees *convergence* for *square integrable functions*. Can you think of some places where you find trade-off situations?
- In mathematics, we call a question *well-posed* if it aligns with the following properties:
  1. Existence: There exists at least one solution;
  2. Uniqueness: There exists at most one solution;
  3. Continuity: The solution depends continuously on the data, *i.e.*, a small error on initial/boundary data entails a small error on the solution.

  Can you think of any "well-posed" questions?

Ordering Game

Ordered Sets

The field of real numbers is ordered. Thus, each person can select a number, and thus determining an order for the group.

Below are subsets of real numbers, select a number from a set:

- $\left\{0, 1, 2, -1, \frac{1}{2}, \sqrt{2}, \pi, e\right\}$,
- $\left\{\sin \frac{2k\pi}{15} : k \in \mathbb{Z} \wedge 0 \leq k \leq 14\right\}$,
- $\mathbb{R} \backslash \mathbb{Q}$ (irrational numbers),
- $\left\{\det \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \det \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix}, \det \begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix}, \det \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}, \det \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}, \det \begin{pmatrix} 1 & -2 \\ 12 & 13 \end{pmatrix}\right\}$,

Ordering Game (Cont.)

- $\overline{\mathbb{Q}} \cap \mathbb{R}$ (real, algebraic number),
- $\{f(-10), f(-2), f(0), f(3), f(5), f(20)\}$, where $f(x) = \int_0^\infty e^{-xt} \sin t dt$,
- $\mathbb{Q}(\sqrt{2}, \sqrt{3}) := \left\{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} : a, b, c, d \in \mathbb{Q}\right\}$,
- $\{n$: regular n-gon is constructible$\}$, *Hint:* Regular n-gon is constructible $\iff$ $\phi(n)$ is an integral power of 2,
- $\{F_n\}_n$ (Fibonacci sequence).

Other Orders?

Of course, there are different ordering methods. For examples, you can look up *dictionary order* for complex numbers.