Assignment 2
MEEN 357

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Task 7

Problem i. Taylor series of $tan^{-1}(x)$

$$
\begin{gathered}
f(x) = tan^{-1}(x) \\
f(tan\theta) = \theta \\
f'(tan\theta)sec^2\theta = 1 \\
sec^2\theta = tan^2\theta + 1 \\
x = tan\theta \;\rightarrow\; sec^2\theta = 1 + x^2 \\
f'(x) = 1/(1+x^2) \\
f^{(1)}(c) = \frac{1}{1+x^2} \\
f^{(2)}(c) = \frac{-2x}{1+x^2)^2} \\
f^{(3)}(c) = \frac{8x^2}{(1+x^2)^3}\frac{-2}{(1+x^2)^2} \\
f^{(4)}(c) = \frac{-48x^3}{(1+x^2)^4} + \frac{24x}{(1+x^2)^3} \\
f^{(5)}(c) = \frac{384x^4}{1+x^2)^5} - \frac{288x^2}{1+x^2)^4} + \frac{24}{(1+x^2)^3} \\
f^{(6)}(c) = \frac{-3840x^5}{1+x^2)^6} + \frac{3840x^3}{(1+x^2)^5} - \frac{720x}{(1+x^2)^4}
\end{gathered}
$$

Continuing, one finds that

$$
\begin{gathered}
tan^{-1}(x) = f^{(0)}(c) + \frac{f^{(1)}(c)}{1!}x^1 + \frac{f^{(2)}(c)}{2!}x^2 + \ldots + \frac{f^{(n)}(c)}{n!}x^n \\
= x - \frac{x^3}{3} + \frac{x^5}{3} - \frac{x^7}{7'} + \frac{x^9}{9} - \ldots \\
= \sum_{n=1}^{\inf} -1^{n-1}\frac{x^{2n-1}}{2n-1}
\end{gathered}
$$

Problem ii. Find R_5

$$
\begin{gathered}
R_5 \leq \frac{Max|f^{5+`}(c)|}{(5+1)!}x^{5-1} = Max\left[\frac{-16c^5}{3(1+x^2)^6} + \frac{16c^3}{3(1+x^2)^5} - \frac{c}{1+x^2)^4}\right]\left(\frac{\pi}{6}\right)^4 \\
c = 0.2274 \\
R_5 = 0.0105
\end{gathered}
$$

Problem iii. Find the number of terms requried to limit the truncation error.

```
clc; clear all; close all;

syms x;
fun(x) = atan(x);

n = 1;
while true
    fun_diff = diff(fun,n+1);

    remainder = max(abs(double(fun_diff(linspace(0,pi/4,500)))))/...
        factorial(n+1)*power((pi/6),n-1);

    if remainder < power(10,-5) || n >= 50
        break
    end

    n = n + 1 ;
end
disp(n); disp(remainder);
```

$n = 15$
$remainder = 6.7654e - 06$