Vibration Analysis of Formula SAE Race Cars
Phase 1: Data Structures and Quarter-Car Models

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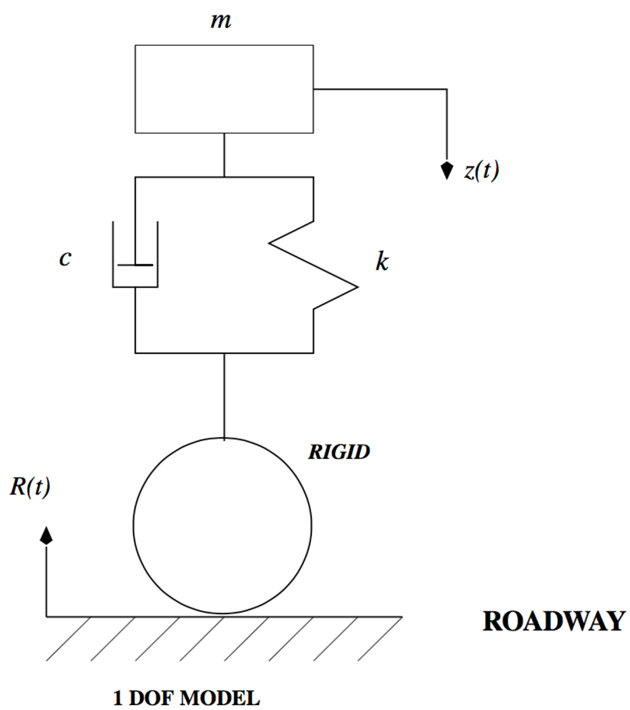
Quarter-Car Model

A 1/4-car model considers a wheel, a suspension unit (a single spring and shock absorber), and a mass. The mass is 1/4 of the total mass of the car, the damping is the average damping from all four shock absorbers, and the stiffness is the average stiffness from all four spring stiffnesses. Figure 1 shows the Free Body of this type of model. This image shows that the only one degree of freedom (DOF) present is the z-axis on the body of the vehicle.

Figure 1: Free Body Diagram for Quarter-Car Model

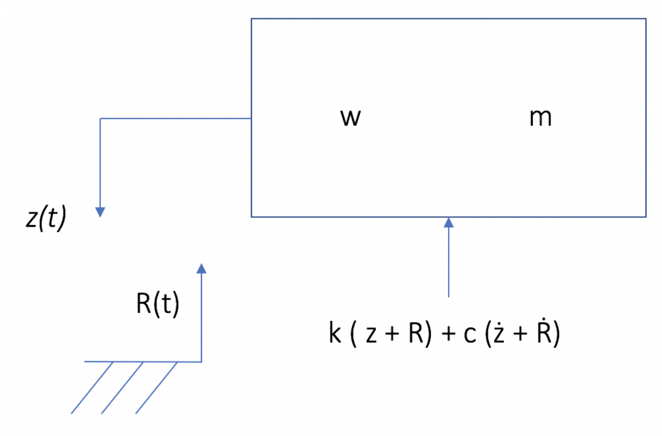


A mathematical relationship for this model is shown in Equation (1). In this equation, M is a mass matrix, C is a damping matrix, and K is a stiffness matrix. The dimensions of theses matrices are dependent on the number of DOF's in the model, and because there is only one DOF present, all of theses matrices are 1x1 scalers. The vector f(t) is a forcing function that depends on time t. The solution is described in terms of vectors for the generalized position x(t), generalized velocity $\dot{x}$(t), and generalized acceleration $\ddot{x}$(t), all of which vary with time.

$$f(t) = M\ddot{x} + C\dot{x} + Kx \tag{1}$$

In order to further analyze this function, a Force Balance Diagram must be created and is shown in Figure 2. Because the wheel is assumed to be rigid and massless, it can be effectively omitted from the Force Balance Diagram. The movement along the roadway is also taken into account with the specific function R(t). This along with its rate $\dot{R}$(t), appear as constituents within the forcing function f(t).

Figure 2: Force Balance Diagram



Applying Newton's second law of motion,(1), to the Force Balance Diagram, the solution to f(t) can be derived and is shown in Equation (4). This equation describes how the forcing function is dependent on the weight of the car, the car's movement along the roadway, and the rate at which it is driving.

$$\sum F = m\ddot{z} \tag{2}$$

$$m\ddot{z} = w - k*(z+R) - c*(\dot{z}+\dot{R}) \tag{3}$$

$$m\ddot{z} + c\dot{z} + kz = w - c\dot{R} - kR \tag{4}$$