# Production and outbound distribution scheduling with inventory cost: manufacturer dominates


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***Abstract*—In this paper we consider a two-level supply chain problem composed of a manufacturer and a third part logistic provider (3PL provider) at the operational level. The manufacturer workshop is a flow-shop and inventory cost are considered. The 3PL provider solves a routing problem minimizing routing costs. Both face tardiness penalty costs, paid for late delivery by the manufacturer to the customer and by the 3PL provider to the manufacturer. This problem is presented in [5], where the authors propose a model and several collaboration scenarios. In line with this preliminary work, we propose a new scenario where the manufacturer dominates the negotiation with the 3PL provider. He imposes the composition of batches for vehicles, the vehicle departure dates and estimates a delivery time. If the 3PL provider respects the estimated delivery times, there is no penalty, otherwise, some tardiness penalty costs apply. Mixed Integer Linear Programming Model and heuristic algorithms are proposed for this specific scenario. Some random data sets are generated and the quality and the efficiency of the different methods are evaluated.**



***Index Terms*—supply chain, manufacturer, 3PL provider, scheduling, vehicle routing, cooperation**


## I. Introduction and literature review

In todays business environment, defined by fierce competition and increasing customer expectations, companies must find solutions for providing competitive services in terms of cost and quality. Production and distribution represent the main activity of a lot of companies and good scheduling of these activities may represent a great monetary savings potential. In this environment, these services are usually performed by different agents, which work together but serve their own interests. The traditional approach is to consider the production and the distribution problems separately, in order to optimize the gain of each agent, without taking into consideration possible negotiations between them. The goal of this paper is to treat these two problems of production and distribution planning in an integrated framework and investigate one collaboration scenario between the two agents.

While there is a large amount of literature on each of the individual production and distribution problems since decades, the interest of the community for problems integrating both aspects is more recent. This interest is motivated by the possibility of making substantial savings in industries by considering the two-level problem directly rather than both problems separately.

We refer in the following to papers in the literature presenting important similarities with our problem.

In [6], the authors propose a model integrating a single machine production stage with an outbound distribution to one customer, respecting strict due dates and including inventory costs. The goal of this work is to find a coordinated production and delivery schedule, minimizing the sum of setup, delivery and inventory costs. The authors propose a non linear model, heuristic algorithms and a branch-and-bound algorithm.

A problem including inbound and outbound transportation from both sides of the production stage is treated in [4]. Inventory costs are considered, and the objective is to schedule the jobs on the machines and the vehicles so that the sum of transportation and inventory holding costs are minimized. The authors propose a MILP model, a lower bound and heuristic algorithms for solving large size instances.

In [3] the author proposes different scheduling problems with different routing hypotheses. The problems are based on the coordination and domination scenarios between agents and the author develops different models and resolution methods according to the collaboration scenario. No inventory cost is considered.

In [5], the authors describe a production problem with outbound distribution, including inventory costs. This paper proposes models for a scenario with dominance of the 3PL provider (which imposes the number of vehicles and departure dates to the manufacturer) and proposes research directions by suggesting different coordination scenarios for the two agents.

The research presented in this paper builds on this work. We propose a new scenario with a strong dominance of the manufacturer, who imposes the composition of batches for each vehicle, departure dates, and evaluates delivery times. To evaluate the delivery dates, the MILP model of the manufacturer incorporates a routing phase. Even if the sequencing of jobs in each batch follows a given rule (EDD order), this makes the problem more difficult to solve.

Section II gives a formal definition of the problem and introduces the notations. In Section III resolution methods are presented: a Mixed Integer Linear Programming (MILP) model and a heuristic to solve the first part of the problem, which is the manufacturer problem. A MILP for the 3-PL provider is also proposed. In section IV, we present the results of the resolution methods on randomly generated instances. Section V concludes the paper and gives some directions for future research.

## II. A MULTI AGENT MODEL

This research aims to propose an integrated model for a supply chain problem, where a manufacturer and a 3PL-provider are involved. For the manufacturer, we consider several costs (inventory costs, transportation costs, penalty costs, ...) that are related to the production phase and the transportation phase, in order to obtain a more realistic model. Despite inventory costs play a major role in production planning, they are often neglected in production scheduling models. Two kinds of inventory costs are considered: work in progress (WIP) inventory and finished products inventory (FIN).

The problem considered here is an integrated production and distribution scheduling problem. This multi-agent problem consists of two sub-problems: a permutation flowshop scheduling problem for the production problem of the manufacturer and a vehicle routing problem for the 3PL provider. According to the five-field notation $\alpha|\beta|\pi|\delta|\gamma$ proposed in [1], the notation of the problem that we consider is $Fm||V(\infty,\infty), fdep, routing|n|\gamma$, to indicate an $m$-machine flow shop, $V(\infty,\infty)$ meaning that a sufficient number of vehicles are available and the capacity of each vehicle is unbounded, $fdep$ to indicate *shipping with fixed delivery departure dates*, and $routing$ meaning that orders going to different customers can be transported in the same shipment, and a vehicle routing problem has to be solved. $n$ indicates that each job belongs to one customer. Finally, $\gamma$ is the objective function, detailed later in this section.

### A. Production Problem

The production problem considered in this paper is an $m$-machine permutation flow shop with work in progress (WIP) and finished product inventory considerations. We suppose that we have a set $\{J_1, ..., J_n\}$ of $n$ jobs to schedule. A manufacturer is looking to design a production schedule for all jobs on the machines. Each job $J_j$ has a given processing time $p_{i,j}$ on machine $M_i$. After the completion of a job on a machine, a work-in-process inventory is generated, before the processing of the job on the next machine. The holding cost for this inventory depends on the job and is denoted by $h_j^{WIP}$ for job $J_j$. After the completion of the job on the last machine, the finished product inventory is kept until the departure of the job for delivery. The holding cost for the finished product inventory depends on the product and is denoted by $h_j^{FIN}$ for job $J_j$. Job $J_j$ has to be delivered to customer $j$ for a given due date denoted by $d_j$. A delay in the delivery of the product generally results in a loss for the manufacturer, that can be financial, hence the manufacturer incurs a penalty cost $\pi_j^M$ per time unit of late delivery of $J_j$, that is paid to the customer.

We denote by $C_{i,j}$ the completion time of job $J_j$ on machine $M_i$, $1 \leq i \leq m$. In our model, the manufacturer decides the composition of vehicles and their departure dates. Furthermore, the manufacturer estimates a delivery date for each job, assuming that the jobs are delivered in EDD order. Therefore, he has an estimation of the amount of the penalty costs which is called a Pseudo Penalty Cost. The objective of the manufacturer is to minimize his total cost, which is composed by the inventory cost $IC$, a vehicle cost $VC$ (related to the number of vehicles used), and the pseudo penalty costs $PPC^M$. At this step, the manufacturer does not know the real delivery dates.

We denote by $IC$ the total inventory cost. Its expression is the following:

$$IC = \sum_{j=1}^{n}(C_{m,j} - C_{1,j})q_j h_j^{WIP} + \sum_{j=1}^{n}(f_j - C_{m,j})q_j h_j^{FIN} \quad (1)$$

where $f_j$ denotes the departure time of $J_j$ and $q_j$ is the quantity of items of job $J_j$.

We denote by $PC^M$ the penalty cost for tardiness. The expression of $PC^M$ is:

$$PC^M = \sum_{j=1}^{n} \pi_j^M T_j^M \quad (2)$$

Remember that $T_j^M$ depends on the delivery dates (denoted $D_j^{3PL}$) that will be given by the 3PL provider and that are not known at the moment. For this reason, the manufacturer considers a Pseudo Penalty Cost $PPC^M$ involving an estimation of the tardiness $PT_j^M$ of $J_j$, based on an estimation of the delivery time $D_j^M$ of $J_j$, defined by: $PT_j^M = \max(0, D_j^M - d_j)$, where $D_j$ is determined assuming that the jobs in a batch are delivered in EDD order.

$$PPC^M = \sum_{j=1}^{n} \pi_j^M PT_j^M \quad (3)$$

We denote by $VC$ the vehicle cost. This cost is defined as follows:

$$VC = c^V V \quad (4)$$

where $c^V$ is the cost of one vehicle and $V$ is the number of vehicles required by the manufacturer.

The pseudo total cost for the manufacturer is given by:

$$PTC^M = IC + PPC^M + VC \quad (5)$$

### B. Distribution Problem

The distribution is done by a third-party-logistics (3PL) provider. For each batch of jobs to deliver, he wants to find an optimal route for the delivery of products from the manufacturer site to the multiple customer locations. Two hypotheses are considered:

1) Vehicles stay at the manufacturing site and their departure times are imposed by the manufacturer, with at most one departure time per job.
2) A vehicle takes all the jobs that are available (completed).

Assuming this condition, we can have at most $n$ potential departure times, with the departure time of vehicle $k$ denoted by $F_k$, $k \in \{1, ..., n\}$, given by the manufacturer.

We denote by $t_{i,j}$ the travel time between site $i$ and site $j$ ($i, j \in \{0, ..., n+1\}$), where site 0 corresponds to the manufacturer site, site $n+1$ corresponds to the depot of the 3PL provider and site $j$ corresponds to the site of the customer waiting for job $J_j$ ($j \in \{1, ..., n\}$).

For each trip, the 3PL provider bears the costs for the routing, denoted by $RC_k$ for the route of vehicle $k$, which depends on the total travel time and a penalty cost per time unit to the manufacturer if the final delivery date ($D_j^{3PL}$) is greater than $D_j^M$. The total routing cost is equal to:

$$RC = \sum_{k=1}^{V} RC_k \tag{6}$$

The routing cost is the sum of the costs for all the routes, leaving from the manufacturer location and returning to the depot of the 3PL provider. The penalty cost is denoted by $PC^{3PL}$. We assume that this function is related to the total tardiness, i.e. we denote by $T_j^{3PL}$ the tardiness of delivery of $J_j$ from the point of view of the 3PL provider (related to the due date $D_j^M$ estimated by the manufacturer): $T_j^{3PL} = \max(0, D_j^{3PL} - D_j^M)$. $PC^{3PL}$ is defined by:

$$PC^{3PL} = \sum_{j=1}^{n} \pi_j^{3PL} T_j^{3PL} \tag{7}$$

The total cost for the 3PL provider is given by:

$$TC^{3PL} = RC + PC^{3PL} - VC \tag{8}$$

### C. Integrated Problem

The two problems outlined above are interconnected and dependent on each other. We assume that:

- the number of vehicles $V$,
- the dates of departure of the vehicles ($F_k$, $1 \leq k \leq V$),
- the first estimation of the completion delivery date $D_j^M$,

are determined by the manufacturer during the construction of his schedule, minimizing $PTC^M$. The vehicles of the 3PL provider take the jobs and distribute them to the customers at minimum cost. The 3PL provider minimizes $TC^{3PL}$ costs. Then, the 3PL provider gives the delivery dates to the manufacturer with the implications on tardiness, so that the real penalty cost $PC^M$ for the manufacturer can be computed.

The real cost for the manufacturer and for the 3PL provider are:

$$TC^M = IC + PC^M + VC - PC^{3PL} \tag{9}$$

$$TC^{3PL} = RC + PC^{3PL} - VC \tag{10}$$

The whole process is described more formally in Alg. 1.

## III. Resolution approaches

This section presents two different approaches for solving the integrated production outbound distribution problems described in this paper. The first is a two-step MILP model that solves the problem for the two agents to optimality and the second applies two heuristics algorithms.

**Algorithm 1** General framework

The manufacturer optimizes his production schedule: MIN $PTC^M$
*// From the schedule we deduce the jobs completion times, the number of vehicles used, the batch compositions, their departure times and an estimation $D_j^M$ of the delivery completion times*
**for** $k$ **in** 1 **to** $V$ **do**
  The 3PL provider delivers the jobs optimally to minimize his costs $RC_k + PC_k^{3PL}$.
  *// From the routing of vehicle $k$ we deduce the delivery dates of jobs $D_j^{3PL}$*
**end for**
Compute the total cost of the 3PL provider: $TC^{3PL} = \sum_{k=1}^{V} RC_k + PC_k^{3PL} - VC$
Compute the total cost of the manufacturer: $TC^M = IC + PC^M - PC^{3PL}$+VC

### A. Mixed Integer Linear Programming models

The data of the problem are the following.

| | |
|---|---|
| $n$ | number of jobs |
| $m$ | number of machines |
| $p_{i,j}$ | processing time of job $J_j$ on machine $M_i$ |
| $q_j$ | quantity of items of job $J_j$ |
| $d_j$ | delivery due date of job $J_j$ |
| $t_{s_1,s_2}$ | travel time between site $s_1$ and site $s_2$ |
| $HV$ | an arbitrary high value |

The costs that have to be taken into account are the following.

| | |
|---|---|
| $h_j^{WIP}$ | holding cost for WIP inventory of job $J_j$ |
| $h_j^{FIN}$ | holding cost for finished product inventory of job $J_j$ |
| $\pi_j^M$ | penalty cost of the manufacturer for late delivery of job $J_j$ |
| $c^V$ | cost per vehicle |

The variables to determine are the following:

| | |
|---|---|
| $y_{j1,j2}$ | $= 1$ if job $J_{j1}$ is scheduled before job $J_{j2}$, 0 otherwise |
| $C_{i,j}$ | Completion time of job $J_j$ on machine $M_i$ |
| $Z_k$ | $= 1$ if vehicle $k$ is used |
| $z_{j,k}$ | $= 1$ if job $J_j$ departs on vehicle $k$, 0 otherwise |
| $F_k$ | departure time of vehicle $k$ |
| $x_{j1,j2,k}$ | $= 1$ if job $J_{j1}$ and job $J_{j2}$ are transported in the same vehicle and $J_{j1}$ is delivered before $J_{j2}$ |
| $D_j^M$ | estimation of the delivery due date of job $J_j$ |
| $PT_j^M$ | estimation of the tardiness of job $J_j$ for the manufacturer |
| $IC$ | total inventory costs |
| $PPC^M$ | pseudo penalty cost of the manufacturer |

The following variables will be known after the 3PL provider gives the delivery dates to the manufacturer.

| | |
|---|---|
| $D_j^{3PL}$ | delivery completion time of job $J_j$ |
| $T_j^M$ | tardiness of job $J_j$ |
| $VC$ | vehicle cost |

The scheduling problem of the manufacturer is solved by the following Mixed Integer Linear Programming model.

$$\text{Minimize } PTC^M = IC + PPC^M + VC \quad (11)$$

The relative order between two jobs $J_{j1}$ and $J_{j2}$ ($\forall j1, j2 \in \{1, ..., n\}, j1 \leq j2$) is given by the following constraints:

$$y_{j1,j2} + y_{j2,j1} = 1 \quad (12)$$

The resource constraints allow to define the completion time of a job on any machine $M_i$ ($\forall i \in \{1, ..., m\}$, $\forall j1, j2 \in \{1, ..., n\}$, $j1 \neq j2$):

$$C_{i,j2} \geq C_{i,j1} + p_{i,j2} - y_{j1,j2}HV \quad (13)$$

The routing constraints are given on any machine $M_i$ ($i \in \{2, ..., m\}$) and for any job $J_j$ ($j \in \{1, ..., n\}$):

$$C_{i,j} \geq C_{i-1,j} + p_{i,j} \quad (14)$$

Each job $J_j$ is transported in a vehicle ($\forall j \in \{1, ..., n\}$), therefore:

$$\sum_{k=1}^{n} z_{j,k} = 1 \quad (15)$$

Each vehicle $k$ ($\forall k \in \{1, ..., n\}$) leaves after the completion time of all jobs transported by this vehicle ($\forall j \in \{1, ..., n\}$):

$$F_k \geq C_{m,j} - HV(1 - z_{j,k}) \quad (16)$$

A job cannot leave before its vehicle leaves ($\forall j \in \{1, ..., n\}$):

$$f_j \geq F_k - HV(1 - z_{j,k}) \quad (17)$$

A vehicle $k$ ($\forall k \in \{1, ..., n\}$) is used once it transports a job:

$$Z_k HV \geq \sum_{j=1}^{n} z_{j,r} \quad (18)$$

In the same batch $k$ ($\forall k \in \{1, ..., n\}$), job $J_{j1}$ ($\forall j1 \in \{1, ..., n\}$) has a predecessor with a smaller due-date, or has the manufacturer site as predecessor:

$$\sum_{i \in \{0\} \cup \{j/d_j \leq d_{j1}\}} x_{i,j1,k} = z_{j1,k} \quad (19)$$

In the same batch $k$ ($\forall k \in \{1, ..., n\}$), job $J_{j1}$ ($\forall j1 \in \{1, ..., n\}$) has a successor with a greater due-date, or has the depot site as successor:

$$\sum_{i \in \{n+1\} \cup \{j/d_j \geq d_{j1}\})} x_{j1,i,k} = z_{j1,k} \quad (20)$$

The manufacturer site has a successor during the route of vehicle $k$ ($\forall k \in \{1, ..., n\}$) only if vehicle $k$ is used:

$$\sum_{j=1}^{n} x_{0,j,k} \leq Z_k \quad (21)$$

The estimation of the delivery date is given by the following constraints ($\forall j1, j2 \in \{1, ..., n\}$ and $\forall k \in \{1, ..., n\}$):

$$D_{j2}^M \geq D_{j1}^M + t_{j1,j2} - HV(1 - x_{j1,j2,k}) \quad (22)$$

The following constraints give a lower bound of the delivery date of job $J_j$ ($\forall j \in \{1, ..., n\}$) transported in vehicle $k$ ($\forall k \in \{1, ..., n\}$):

$$D_j^M \geq F_k + t_{0,j} - M(1 - z_{j,k}) \quad (23)$$

To ensure the non negativity of the estimation of the tardiness for any job $J_j$ ($\forall j \in \{1, ..., n\}$):

$$PT_j^M \geq 0 \quad (24)$$

The estimation of the tardiness for job $J_j$ ($\forall j \in \{1, ..., n\}$) is given by the following constraint:

$$PT_j^M \geq D_j^M - d_j \quad (25)$$

The costs are given in the following expressions:

$$PPC^M = \sum_{j=1}^{n} \pi_j^M PT_j^M \quad (26)$$

$$IC^{WIP} = \sum_{j=1}^{n} (C_{m,j} - C_{1,j} - \sum_{i=2}^{m} p_{i,j}) q_j h_j^{WIP} \quad (27)$$

$$IC^{FIN} = \sum_{j=1}^{n} (\sum_{k=1}^{V} f_j - C_{m,j}) q_j) h_j^{FIN} \quad (28)$$

$$IC = IC^{WIP} + IC^{FIN} \quad (29)$$

$$VC = c^V \sum_{k=1}^{n} Z_k \quad (30)$$

The objective function is:

$$\text{Minimize } IC + PPC^M + VC \quad (31)$$

### B. 3PL provider

We assume that the optimization of the routing of the vehicles are independent and that they can be optimized separately. Therefore, for each trip, the set of delivery dates $D_j^M$ is given by the manufacturer and we assume that vehicle $k$ has to leave the manufacturer site at time $F_k$, which is also given.

The data required by the 3PL provider are the following:

- $D_j^M$ estimated delivery date for site j
- $F_k$ departure time from the manufacturer site
- $s$ number of sites to visit (sites $1..s$ are the customer sites, site 0 is the manufacturer site and site $s+1$ is the 3PL provider site)
- $t_{j1,j2}$ travel time between site $j1$ and site $j2$ ($0 \leq j1, j2 \leq s+1$)
- $HV$ an arbitrary high value (can be set to $2 \times \sum_{j1} \sum_{j2} t_{j1,j2}$)

The costs to be taken into account are the following.

- $c_{j1,j2}$ cost of travel time between site $j1$ and site $j2$ ($0 \leq j1, j2 \leq s+1$)
- $\pi_j^{3PL}$ penalty cost of 3PL provider for an excessive tardiness of job $J_j$ ($1 \leq j \leq s$)

The variables that have to be determined are the following.

| | |
|---|---|
| $x_{j1,j2}$ | = 1 if site $j1$ is visited just before site $j2$, 0 otherwise $(0 \leq j1, j2 \leq s+1)$ |
| $D_j^{3PL}$ | date at which site $j$ is visited, i.e. delivery date of $J_j$ $(1 \leq j \leq s)$ |
| $T_j^{3PL} \geq 0$ | tardiness of delivery of $J_j$ $(1 \leq j \leq s)$ |
| $RC$ | routing cost |
| $PC^{3PL}$ | total penalty cost of 3PL provider |

The routing of the 3PL provider is determined by solving the following Mixed Integer Linear Programming model.

$$\text{Minimize } TC^{3PL} = RC + PC^{3PL} - VC \tag{32}$$

Any site $s_1$ $(\forall s_1 \in \{0, ..., s\})$ (excluding 3PL depot), has one successor site in $\{1, ..., s+1\}$.

$$\sum_{s2=1}^{s+1} x_{s1,s2} = 1 \tag{33}$$

Any site $s_1$ $(\forall s_1 \in \{1, ..., s+1\})$ (excluding manufacturer site), has one predecessor site in $\{0, ..., s\}$.

$$\sum_{s2=0}^{s} x_{s1,s2} = 1 \tag{34}$$

A lower bound of the delivery date of job $J_j$ $(\forall j \in \{1, ..., n\})$ is the following (remember that $F_k$ is a data here):

$$D_j^M \geq F_k + t_{0,j} \tag{35}$$

The arrival date of each job at the customer site is given by the following constraints $(\forall j1 \in \{1, ..., s\}, \forall j2 \in \{1, ..., s\})$:

$$D_{j2}^{3PL} \geq D_{j1}^{3PL} + t_{j1,j2} - HV(1 - x_{j1,j2}) \tag{36}$$

The following constraints ensure the non negativity of the tardiness $(\forall j \in \{1, ..., s\})$.

$$T_j \geq 0 \tag{37}$$

The tardiness expression and the costs are given in the following expressions (remember that $D_j^M$ is a data here):

$$T_j^{3PL} \geq D_j^{3PL} - D_j^M, \quad \forall j \in 1, .., s \tag{38}$$

$$PC^{3PL} = \sum_{j=1}^{s} \pi_j^{3PL} T_j^{3PL} \tag{39}$$

$$RC = \sum_{j1=0}^{s} \sum_{j2=1}^{s+1} c_{j1,j2} x_{j1,j2} \tag{40}$$

### C. *Heuristic algorithms*

A heuristic algorithm is proposed in order to find a solution to the production scheduling problem. It is an ad-hoc GRASP (Greedy Randomized Adaptive Search Procedure) algorithm described below. The main idea is to generate quickly a large set of solutions and to keep the best one.

The process to generate one solution is the following: in a first step, we determine the scheduling of jobs and in a second step we choose the composition of vehicles (or batching).

For building the schedule, we choose the first job randomly among a restricted list based on the job's due dates. The restricted list is composed by the jobs with a due date in an interval. The size of this interval is related to a chosen parameter $\lambda$. The details of the algorithm are given in Alg. 2.

**Algorithm 2** GRASP heuristic

```
Parameters λ
Initialization
σ = ∅
𝒥 = {J_j, j ∈ {1, ..., n}}
Start
while 𝒥 ≠ ∅ do
    d^min = min_{J_j ∈ 𝒥} d_j
    d^max = max_{J_j ∈ 𝒥} d_j
    List = {J_j / d^min ≤ d_j ≤ d^min + λ × (d^max − d^min)}
    Select randomly J_j in List
    Add J_j to σ
    𝒥 = 𝒥 \ {J_j}
end while
return σ
```

This heuristic computes the inventory cost $IC$ for the manufacturer and the completion time variables $C_{k,j}$ $(\forall k \in \{1, ..., m\}$ and $\forall j \in \{1, ..., n\})$. The departure times $F_k$ are determined in a second heuristic.

Notice that we can generate a solution by the end of the schedule. Heuristic GRASP is used several times ($nb_{iter}$ times) to generate several solutions and the best is kept. Half of these solutions is built starting by the beginning of the schedule and the half is built starting by the end.

From the schedule, we build batches by evaluating the additional cost of a new job in the current batch or in a new batch created especially. The additional cost represents the increase of $PPC^M$ by adding a new job to the current batch. If create a new batch (and hire a new vehicle) is cheaper than add the job to the current batch, this option is chosen. Otherwise, we refer to the parameter *deterministic* of the algorithm. If *deterministic* is true, the new job is assigned to the current batch, else the assignment is done randomly in correlation with the ratio between the both assignment cost.

This heuristic determines variables $Z_k$, $F_k$ $(\forall k \in \{1, .., n\})$ and $z_{j,k}$ $(\forall j \in \{1, .., n\}$ and $\forall k \in \{1, .., n\})$. The number of batches is known and we can compute $VC$.

From the result of Alg. 3, we use the earliest due-date heuristic (EDD) to determine the routing of jobs in each batch, according to the previous hypotheses. It finally gives $D_j$ and $T_j$ $(\forall j \in \{1, .., n\})$ and the *pseudo* penalty cost for manufacturer $PPC^M$.

## IV. Computational results

### A. *Instance specification*

Results presented in this section have been realized with instances generated according to the following specifications.

We consider a 2-machine flow shop problem.

We set a parameter $SSQ = 100$. For each job $J_j$ and each machine $M_i$, the processing time $p_{i,j}$ is randomly chosen between 1 and $SSQ$. The different sites are placed randomly

**Algorithm 3** Batching heuristic

Parameters $deterministic$
Data
$\sigma$ *(from GRASP heuristic)*
Initialization
$\mathcal{B} = \emptyset$ // set of batches
Start
$B = \{\sigma[1]\}$ // current batch
$k = 2$
**while** $k \leq n$ **do**
  $J_j = \sigma[k]$
  Cost_1 = cost of assignment of $J_j$ to batch $B$
  Cost_2 = cost of assignment of $J_j$ to a new batch
  **if** $deterministic$ **then**
    in_same_batch = $Cost\_1 \leq Cost\_2$
  **else**
    in_same_batch = $\frac{Cost\_1}{Cost\_2} \leq rand(0, 1)$
  **end if**
  **if** in_same_batch **then**
    $B = B \cup \{J_j\}$ // $J_j$ is put in current batch $B$
  **else**
    $\mathcal{B} = \mathcal{B} \cup \{B\}$ // batch $B$ is added to $\mathcal{B}$
    $B = \{J_j\}$ // a new batch is created
  **end if**
  $k = k + 1$
**end while**
$\mathcal{B} = \mathcal{B} \cup \{B\}$
**return** $\mathcal{B}$

on a square of size $3SSQ \times 3SSQ$ and the distance $t_{j1,j2}$ is the classical euclidian distance.

Costs are fixed in order to obtain an optimal solution where the number of vehicles is not equal to 1 and not equal to $n$. For each job $J_J$ the quantity $q_j$ is equal to 100, the work-in-process inventory cost $h_j^{WIP}$ is equal to 1, the finished product inventory cost $h_j^{FIN}$ is equal to 2, the manufacturer penalty cost $\pi^M$ is equal to 2 and the 3PL penalty cost $\pi^{3PL}$ is equal to 9. The cost of vehicle $c^V$ is 200000.

Sets of 10 instances are generated for each value of $n \in \{6, 7, 8, 9\}$.

### *B. Results of the MILP*

The solver used is CPLEX 2012, the machine has an Intel Core i7-7820HQ and 16,00 Go RAM. The size of an instance is correlated to the number of scheduled jobs. The maximum computation time is limited to 30 min. Table I gives the results of the MILP and compare with the GRASP performance.

We can notice that the computation time quickly increases with $n$. No instance with 9 jobs is solved.

The MILP for the 3PL provider is very efficient and can solve all the instances in less than one second (each batch contains very few jobs).

### *C. Result of the heuristic algorithms*

The proposed heuristic is based only on the first part of the model where the manufacturer chooses the schedule of

TABLE I
MODEL EFFICIENCY

| n | Percentage of instance solved | Average time CPLEX | Average time GRASP | Average $\Delta$ % |
|---|---|---|---|---|
| 6 | 100% | < 1s | < 1s | 4.5 % |
| 7 | 100% | 14s | < 1s | 3.8% |
| 8 | 100% | 169s | < 1s | 4.1% |
| 9 | 0% | – | < 1s | 2.1% |

$\Delta = \frac{GRASP - CPLEX}{GRASP}$

his jobs and the vehicle composition. As a reminder, the objective function of this model is the sum of the inventory cost ($IC$) plus the vehicle cost ($VC$) plus the pseudo penalty cost ($PPC^M$). The parameter $\lambda$ (related to the acceptance capacity of the restricted list) is set to 0.3 and $nb_{iter}$ (number of solutions generated by GRASP) is set to 20000. The results are presented in Table III for $n \in \{10, 20, 30, 40, 50, 60, 100\}$. $\Delta$ indicates the average relative deviation between the GRASP algorithm with $\lambda = 0.3$ and with $\lambda = 0$, which corresponds to EDD sequence. $\lambda = 0.3$ have been chosen after exhaustive tests on instance with $n \in \{6, 7, 8, 9, 10, 20, 30\}$.

TABLE II
MODEL EFFICIENCY *with deterministic = false*

| n | Average time | Average $\Delta$ % |
|---|---|---|
| 10 | 1 s | 12.2% |
| 20 | 3 s | 8.8% |
| 30 | 5 s | 5.6% |
| 40 | 10 s | 1% |
| 50 | 15 s | 0% |
| 60 | 19 s | 1% |
| 100 | 57 s | 1% |

$\Delta = \frac{GRASP\_EDD - GRASP}{GRASP\_EDD}$

TABLE III
MODEL EFFICIENCY *with deterministic = true*

| n | Average time | Average $\Delta$ % |
|---|---|---|
| 10 | 1 s | 13.1% |
| 20 | 3 s | 13.6% |
| 30 | 5 s | 13.8% |
| 40 | 10 s | 11.0% |
| 50 | 15 s | 9.56% |
| 60 | 19 s | 10.9% |
| 100 | 57 s | 8.08% |

$\Delta = \frac{GRASP\_EDD - GRASP}{GRASP\_EDD}$

The GRASP algorithm based on non deterministic schedule gives results better than algorithm with EDD schedule up to 40 jobs. After, the both methods become equivalent. We suppose that the random parameter always lead to bad decision when the size of instance increase.

Otherwise, deterministic schedule achieves to improve EDD up to 100 jobs with a $\Delta$ around 8%. In general, results of algorithm with *deterministic = true* are better than algorithm with *deterministic = false* as soon as instances get 20 jobs.

## V. Conclusion and future research directions

In this paper, we consider a two-level supply-chain problem where a manufacturer and a 3PL provider cooperate to satisfy the customers demands. The manufacturer faces inventory costs, vehicle costs and penalty costs, paid to the customer for late delivery. The 3PL provider faces routing cost and penalty cost, paid to the manufacturer for extra-late delivery. In this paper, we consider a scenario where the manufacturer dominates the 3PL provider, by imposing the number of vehicles used, the batch composition, and the departure times of vehicles.

Mixed Integer Linear Programming models are proposed for the optimization of the manufacturer problem and the 3PL provider problem. These models are integrated in a global algorithm (the customer penalty costs are only known after the delivery) and its efficiency is evaluated on different sets of instances. At the end, a heuristic is proposed for solving both problems. Computational results are provided. The results show that the MILP model for the manufacturer is not efficient and may certainly be improved. With respect to the heuristic methods, other methods have to be developed in order to make comparisons for instances with more jobs.

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