International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: https://www.tandfonline.com/loi/tprs20

Integrated model of preventive maintenance, quality control and buffer sizing for unreliable and imperfect production systems

M. Radhoui , N. Rezg & A. Chelbi

To cite this article: M. Radhoui , N. Rezg & A. Chelbi (2009) Integrated model of preventive maintenance, quality control and buffer sizing for unreliable and imperfect production systems, International Journal of Production Research, 47:2, 389-402, DOI: 10.1080/00207540802426201

To link to this article: https://doi.org/10.1080/00207540802426201

Published online: 20 Nov 2008.

International Journal of Production Research,
Vol. 47, No. 2, 15 January 2009, 389–402

Taylor & Francis
Taylor & Francis Group

Integrated model of preventive maintenance, quality control and buffer sizing for unreliable and imperfect production systems

M. RADHOUI*†‡, N. REZG† and A. CHELBI‡

†LGIPM, Ile du Saulcy 57045 Metz Cedex, France
‡CEREP, ESSTT, 5, avenue Taha Hussein, Tunis, Tunisia

(*Revision received September 2008*)

In this paper, we develop a joint quality control and preventive maintenance policy for a randomly failing production system producing conforming and non-conforming units. The considered system consists of one machine designed to fulfil a constant demand. According to the proportion l of non-conforming units observed on each lot and compared to a threshold value lm, one decides to undertake or not maintenance actions on the system. In order to palliate perturbations caused by the stopping of the machine to undergo preventive maintenance or an overhaul, a buffer stock h is built up to ensure the continuous supply of the subsequent production line. A mathematical model is developed and combined with simulation in order to determine simultaneously the optimal rate, l_m^* and the optimal size h^* which minimize the expected total cost per time unit including the average costs related to maintenance, quality and inventory.

Keywords: Quality control; Preventive maintenance; Buffer stock

1. Introduction

In many production systems (typically those operating under a just-in-time configuration), buffer stocks are built between successive machines in order to guarantee the continuous supply during disruptions due to breakdowns or to the execution of planned preventive maintenance actions.

This problem of buffer stock deployment in production lines with unreliable machines has been tackled by many authors (Groenevelt *et al*. 1992b, Van Der Duyn Schouten and Vanneste 1995, Meller and Kim 1996). Different operating strategies of these production lines have been proposed and discussed by many other authors such as Balasubramanian (1987), Groenevelt *et al*. (1992a) and Salameh and Ghattas (2001).

Other works deal with the control of production systems subjected to preventive maintenance (Boukas and Haurie 1990, Gharbi and Kenne 2000, Kenne and Gharbi 2001). They developed analytical approaches and simulation models to tackle this problem.

Chelbi and Aït-Kadi (2004) analyse, using an analytical approach, a randomly failing system made up of a single machine and having to feed a subsequent

*Corresponding author. Email: radhoui@univ-metz.fr

International Journal of Production Research
ISSN 0020–7543 print/ISSN 1366–588X online © 2009 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/00207540802426201

production unit at a constant rate. They present a mathematical model to determine simultaneously the optimal size of the buffer stock and the period according to which preventive maintenance actions should be undertaken, minimizing the total unitary cost.

More recently, Chelbi and Rezg (2006) consider, instead of a periodic maintenance strategy, a policy based on the age of the equipment. They determine the optimal age of the system at which it should be submitted to preventive maintenance, and the optimal size of the buffer stock, so as to minimize the total cost per time unit, taking into account a minimum required system availability level.

It is clearly shown in all these works and in many others that the equipment condition plays a crucial role in controlling the buffer or lot size. In other respects, it is also well known that the equipment state plays an important role in controlling quality of produced items. Despite the strong link between production, quality and maintenance, only a few research works have attempted to catch their underlying relationship through a single integrated model.

For example, Ben Daya (1999) developed an integrated model for the joint optimization of the economic production quantity, the economic design of x-control chart, and the optimal maintenance level, for a deteriorating process where the in-control period follows a general probability distribution with increasing hazard rate.

Wang *et al.* (1999) treated the case of a production system in which some of the products made are defective. They assumed that the distribution of such defective products depends on the total number of products made since the last maintenance action.

Ben Daya (2002) proposed an integrated model for the joint determination of economic production quantity and preventive maintenance level for imperfect production process. He proved that performing preventive maintenance gives way to a reduction of quality control related costs.

The main contribution of this work compared with the existing literature consists of relating directly the type of maintenance actions to be performed, to the observed rate of non-conforming items, in a context of unknown production system lifetime or in-control periods probability distributions. In fact, we propose a joint quality control, preventive maintenance and production policy for randomly failing systems producing conforming and non-conforming items. We consider a production system consisting of a single machine having an increasing failure rate. The machine lifetime cumulative distribution function is not known. Each lot produced by the machine is subject to a quality control and according to the observed percentage of non conforming units found, one decides to perform or not maintenance actions and which type of maintenance to carry out.

Production has to be stopped while the machine is submitted to preventive or corrective maintenance. In order to palliate these perturbations, a buffer stock is built up. A mathematical model is developed in order to determine the optimal values of both decision variables which are: the threshold level of the rate of non-conforming units on the basis of which maintenance actions are to be performed, and the size of the buffer stock. The optimal values are those which minimize the average total cost per time unit including inventory cost, maintenance cost and quality cost.

In the next section, the proposed policy is defined; the working assumptions and the necessary notation are stated. In section 3, we develop the mathematical model.

section 4 shows an illustrative numerical example. Finally, the main conclusions of this work are presented in section 5.

2. Strategy definition, assumptions and notation

Consider a production unit subject to a deterioration process (with an increasing failure rate) and producing conforming and non-conforming items. The rate of non-conforming items is perfectly correlated with the degradation process of the production unit. The latter is considered as a single machine which must satisfy a constant demand. Each produced lot is entirely subjected to an automated quality control of negligible duration and cost in order to determine the number of non conforming items. According to the observed rejection rate l, one decides to undertake or not maintenance actions. As shown in figure 1, if the rate of non-conforming units l is found higher than a certain threshold lm and lower than a maximum value $l_{\max}$ ($l_m < l < l_{\max}$), the machine is stopped to undergo a preventive maintenance action. If l is found higher than $l_{\max}$, the machine is considered to be in a quasi-failed state and consequently a major corrective maintenance (overhaul) is performed. Finally, if l is found between 0 and lm, no action is undertaken. After a maintenance action (preventive and overhaul), the system's state is considered as good as new and rejection rate starts again at 0. These two actions have the same effect to restore the system to as good as new but the difference between them is in the cost. Indeed, if the rejection rate is between l_m and $l_{\max}$ only light maintenance operations are needed to restore the system to as good as new. In the other situation where a rejection rate is found greater than $l_{\max}$ more expensive operations (overhaul) are needed.

We suppose in this paper that maintenance actions guarantee that the probability of breakdown of the production unit before the next maintenance action is approximately zero.

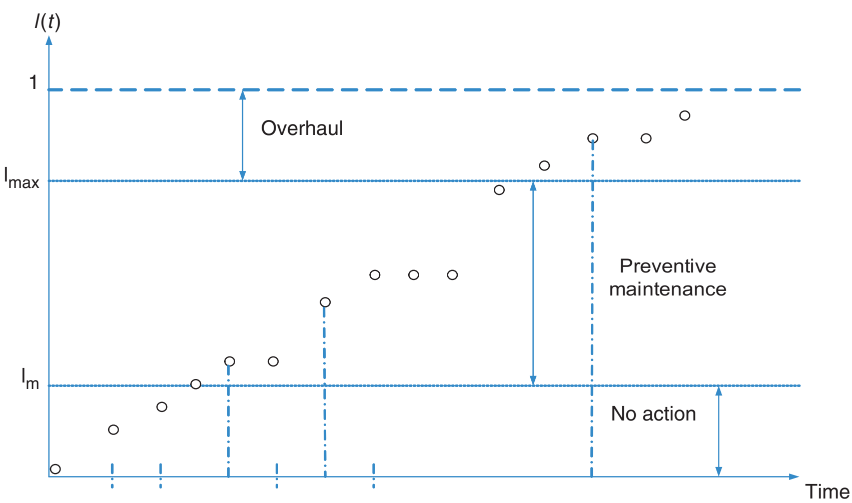

Figure 1. Actions to be performed according to the evolution of the rate of non-conforming units.

A buffer stock of h units is built to ensure the continuous satisfaction of the demand while the production unit is stopped to undergo maintenance actions. The buffer stock is refilled at the end of the maintenance actions.

Other assumptions are made as follows:

- The production unit lifetime cumulative distribution function is not known.
- The cumulative distribution functions associated to preventive maintenance actions and overhaul durations are known.
- The demands which cannot be satisfied are lost.
- The produced items are imperishable with time.
- The non-conforming items are not reinserted in the production process.
- All costs related to maintenance, quality and inventory are known and constant.
- The resources necessary for the achievement of maintenance actions are always available.
- The size of the lot subject to quality control is known and constant.

The following notation will be used:

C_s	Holding cost per item per unit time.
C_p	Shortage cost per item.
C_{nc}	Cost incurred by producing a non-conforming item per unit time.
h	Size of the buffer stock.
δs	Total holding cost.
δq	Total quality cost.
M_p	Preventive maintenance action cost.
M_c	Overhaul cost ($M_c \gg M_p$).
δm	Total Maintenance cost.
d	The demand (units/unit time).
l_m	Threshold level of non-conforming units rate.
$g_p(t)$	Probability density function associated to preventive maintenance actions duration.
$g_c(t)$	Probability density function associated to overhaul duration.
$P(l)$	Probability distribution function associated to the rejection rate l. It is assumed to be a continuous distribution function bounded by 0 as a minimum and 1 as a maximum.
μ_c	Average duration of an overhaul.
μ_p	Average duration of a preventive maintenance action.
$U_{\max}$	Maximum production rate.
$\Pi(h, l_m)$	Operating total average cost per time unit.

Other notation will be introduced through the mathematical model presented in the next section.

3. Mathematical model development

Our objective is to determine simultaneously the optimal rejection rate threshold level 'l_m' and the size of the buffer stock 'h' which minimize the total average cost per time unit. The following analysis will lead to the expression of this unitary cost.

The total cost corresponds to the sum of the maintenance cost, the inventory holding cost, and the quality cost. This total cost will then be divided by the average duration of a production cycle which is the average time between consecutive overhauls.

For a production cycle period T_K, two scenarios are possible depending on whether shortage occurs or not. The following figure describes the evolution of the buffer stock taking into account both scenarios. The stock behaviour is based on the average rejection rate per time unit which will be defined below. The analytical model developed in the rest of this paper will be based on average rejection rates of non-confirming units per time unit.

Notation

- W_K Period during which the production unit is operating within a production cycle period T_K.
- D_K Time of service interruption of the production unit for preventive or corrective maintenance.
- $\overline{\alpha_{R1}}$ Average rejection rate of non-confirming units by time unit over period T_1, which is equivalent to the production rate of non-conforming units over T_1. It is given by the following expression:

$$\overline{\alpha_{R1}} = U_{\max} \cdot \int_0^1 [1 - P(l)]\mathrm{d}l. \tag{1}$$

- $\overline{\alpha_{R2}}$ Average rejection rate of non-confirming units by time unit over period T_2, which is equivalent to the production rate of non-conforming units over T_2. It is given by the following expression:

$$\overline{\alpha_{R2}} = \frac{d \cdot \int_0^1 [1 - P(l)]\mathrm{d}l}{\left(1 - \int_0^1 [1 - P(l)]\right)\mathrm{d}l}. \tag{2}$$

- T_1 Time necessary to build a buffer stock of h units during which the machine produces at the maximum rate $U_{\max}$.
- T_2 Time during which the machine produces only to satisfy the demand. The production rate during this period is equal to $d + \overline{\alpha_{R2}}$, $(d + \overline{\alpha_{R2}}) \leq U_{\max}$.

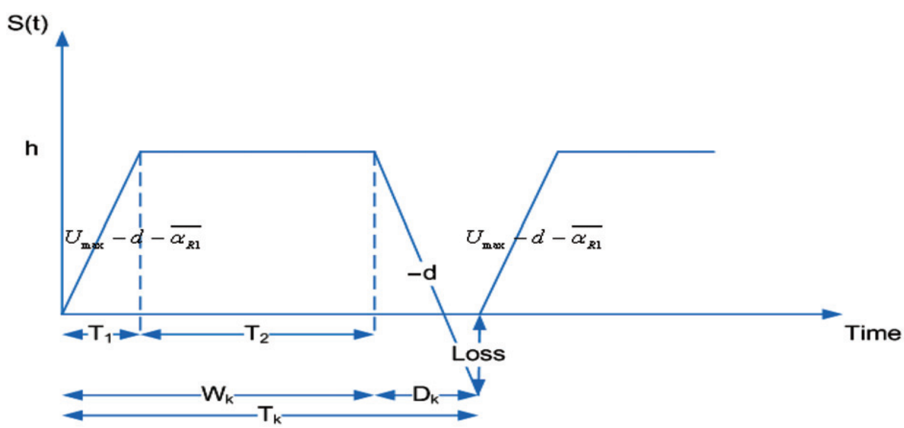

Figure 2. Evolution of the buffer stock level.

Formally, the production rate is written as follows:

$$\text{Production rate } = \begin{cases} U_{\max} & \text{for } S(t) < h \\ \dfrac{d}{1 - \int_0^{l\max}[1 - P(l)]\mathrm{d}l} & \text{for } S(t) = h \end{cases}.$$

Next, we develop the expressions needed to evaluate the total expected cost per time unit which includes inventory, maintenance and quality average costs.

3.1 *The average inventory cost*

To calculate the inventory cost, we must take into account the two possible cases. The first case (described in figure 3) characterizes a cycle without shortage, in which time D_K of interruption of service does not exceed the time of consumption of the buffer stock. The second case (figure 4) describes a production cycle with loss due to excessive maintenance time.

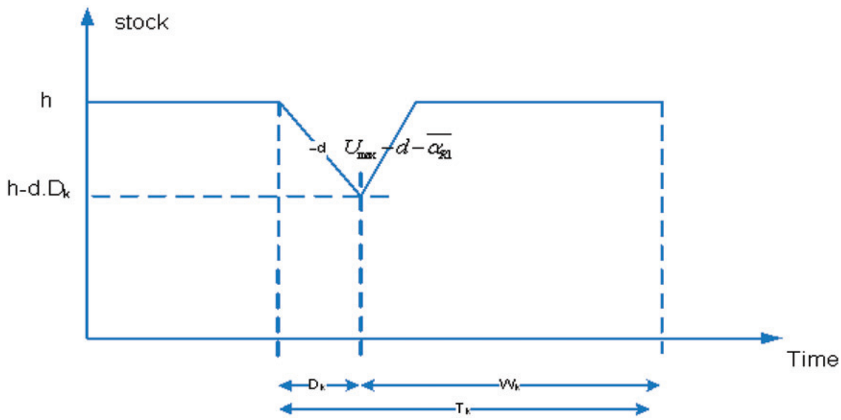

Figure 3. Production cycle without loss.

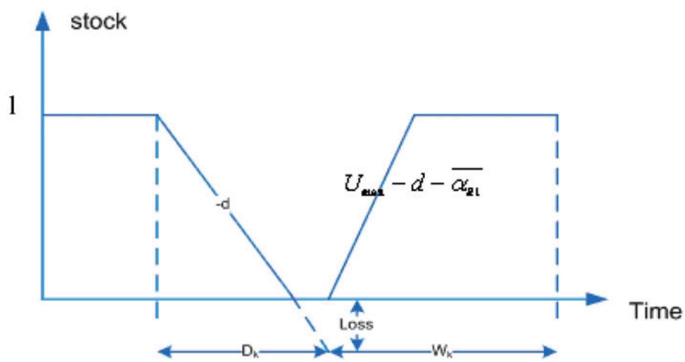

Figure 4. Production cycle with loss.

The first case is that where the period D_K of interruption of service does not exceed the period of consumption of the buffer stock. In this situation, the average inventory cost Γ_{NL} is given by:

$$\Gamma_{NL} = C_s\left[h \cdot (W_K + D_K) - \left[\frac{D_k^2 \cdot d}{2} + \frac{(D_K \cdot d)^2}{2(U_{\max} - d - \overline{\alpha_{R1}})}\right]\right] \tag{3}$$

$$\Gamma_{NL} = C_s\left[h \cdot (W_K + D_K) - \left[\frac{D_K^2 \cdot d \cdot (U_{\max} - d - \overline{\alpha_{R1}}) + D_K^2 \cdot d^2}{2 \cdot (U_{\max} - d - \overline{\alpha_{R1}})}\right]\right] \tag{4}$$

$$\Gamma_{NL} = C_s\left[h \cdot (W_K + D_K) - \left[\frac{D_K^2 \cdot d \cdot U_{\max} - d^2 \cdot D_K^2 - D_K^2 \cdot d \cdot \overline{\alpha_{R1}} + D_K^2 \cdot d^2}{2 \cdot (U_{\max} - d - \overline{\alpha_{R1}})}\right]\right] \tag{5}$$

$$\Gamma_{NL} = C_s\left[h \cdot (W_K + D_K) - \left[\frac{D_K^2 \cdot d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2 \cdot (U_{\max} - d - \overline{\alpha_{R1}})}\right]\right] \tag{6}$$

$$\Gamma_{NL} = C_s h \cdot (W_K + D_K) - C_s \cdot \left[\frac{D_K^2 \cdot d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2 \cdot (U_{\max} - d - \overline{\alpha_{R1}})}\right]. \tag{7}$$

For a production cycle with loss, the average inventory cost Γ_{WL} is expressed as follows summing up the inventory holding cost and the cost of loss due to shortage:

$$\begin{aligned}\Gamma_{wL} = C_s&\left[\frac{h^2}{2d} + \frac{h^2}{2(U_{\max} - d - \overline{\alpha_{R1}})} + h \cdot \left(W_K - \frac{h}{U_{\max} - d - \overline{\alpha_{R1}}}\right)\right] \\ &+ C_p \cdot d \cdot \left(D_K - \frac{h}{d}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}\Gamma_{wL} = C_s&\left[\frac{h^2}{2d} + \frac{h^2}{2(U_{\max} - d - \overline{\alpha_{R1}})} + h \cdot W_K - \frac{h^2}{U_{\max} - d - \overline{\alpha_{R1}}}\right] \\ &+ C_p \cdot d \cdot \left(D_K - \frac{h}{d}\right)\end{aligned} \tag{9}$$

$$\begin{aligned}\Gamma_{wL} = C_s \cdot h^2&\left[\frac{1}{2d} + \frac{1}{2(U_{\max} - d - \overline{\alpha_{R1}})} - \frac{1}{U_{\max} - d - \overline{\alpha_{R1}}}\right] \\ &+ (C_s W_K - C_p) \cdot h + C_p \cdot d \cdot D_K\end{aligned} \tag{10}$$

$$\Gamma_{wL} = C_s \cdot h^2\left[\frac{U_{\max} - 2d - \overline{\alpha_{R1}}}{2(U_{\max} - d - \overline{\alpha_{R1}})}\right] + (C_s W_K - C_p) \cdot h + C_p \cdot d \cdot D_K. \tag{11}$$

The total average inventory cost considering the two scenarios is expressed by:

$$\Gamma = \Gamma_{NL} \cdot \left(1 - R_D\left(\frac{h}{d}\right)\right) + \Gamma_{WL} \cdot R_D\left(\frac{h}{d}\right) \tag{12}$$

with

$$R_D\left(\frac{h}{d}\right) = \int_{h/d}^{+\infty} g_D(u) \cdot du \tag{13}$$

and

$$g_D(x) = g_c(x) \cdot (1 - P(l_{\max})) + g_p(x) \cdot (P(l_{\max}) - P(l_m)). \tag{14}$$

Thus,

$$\begin{aligned}\Gamma = &\left(1 - R_D\left(\frac{h}{d}\right)\right) \cdot \left[C_s h \cdot (W_K + D_K) - C_s \cdot \left[\frac{D_k^2 \cdot d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2 \cdot (U_{\max} - d - \overline{\alpha_{R1}})}\right]\right] \\ &+ R_D\left(\frac{h}{d}\right) \cdot \left[C_s \cdot h^2\left[\frac{U_{\max} - 2d - \overline{\alpha_{R1}}}{2(U_{\max} - d - \overline{\alpha_{R1}})}\right] + (C_s W_K - C_p) \cdot h + C_p \cdot d \cdot D_K\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\Gamma = &\; C_s \cdot h \cdot (W_K + D_K) - \frac{C_s \cdot D_K^2 \cdot d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2(U_{\max} - d - \overline{\alpha_{R1}})} - R_D\left(\frac{h}{d}\right) \cdot C_s \cdot h \cdot W_K \\ &- R_D\left(\frac{h}{d}\right) \cdot C_s \cdot h \cdot D_K + R_D\left(\frac{h}{d}\right)\frac{C_s \cdot D_K^2 \cdot d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2(U_{\max} - d - \overline{\alpha_{R1}})} \\ &+ R_D\left(\frac{h}{d}\right) \cdot C_s \cdot h^2 \frac{(U_{\max} - 2 \cdot d - \overline{\alpha_{R1}})}{2(U_{\max} - d - \overline{\alpha_{R1}})} + R_D\left(\frac{h}{d}\right) \cdot h \cdot C_s \cdot W_K \\ &- R_D\left(\frac{h}{d}\right) \cdot h \cdot C_p + C_p \cdot d \cdot D_K \cdot R_D\left(\frac{h}{d}\right).\end{aligned} \tag{16}$$

The average inventory cost per time unit, δ, is given by:

$$\begin{aligned}\delta = &\frac{1}{E[T_K]}\left[C_s \cdot E[W_K] + C_s \cdot h^2 \cdot R_D\left(\frac{h}{d}\right)\frac{U_{\max} - 2 \cdot d - \overline{\alpha_{R1}}}{2 \cdot d \cdot (U_{\max} - d - \overline{\alpha_{R1}})}\right. \\ &+ C_s \int_0^{h/d} xh - x\frac{d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2(U_{\max} - d - \overline{\alpha_{R1}})} g_D(x)\mathrm{d}x \\ &\left. + R_D\left(\frac{h}{d}\right) \cdot C_p \int_{h/d}^{+\infty} (x \cdot d - h) g_D(x)\mathrm{d}x\right]\end{aligned} \tag{17}$$

where $E[T_K]$ stands for the average duration of a production cycle:

$$E[T_K] = E[W_K] + E[D_K] \tag{18}$$

$$E[D_K] = \mu_p \cdot [P(l_{\max}) - P(l_m)] + \mu_c \cdot [1 - P(l_{\max})] \tag{19}$$

and

$P(l_{\max}) - P(l_m)$	expresses the probability to have $l_m \leq l < l_{\max}$ (preventive maintenance).
$1 - P(l_{\max})$	stands for the probability to have $l_{\max} \leq l$ (overhaul).
$E[W_K]$	will be expressed further in the paper.

3.2 *The average maintenance cost*

The total expected cost of maintenance is given by:

$$\delta m = \frac{1}{E[T_K]}\left(M_p \cdot [P(l_{\max}) - P(l_m)] + M_c \cdot [1 - P(l_{\max})]\right). \tag{20}$$

3.3 *The average quality cost*

The average quality cost corresponds to the cost of the average number of rejected items during a production cycle.

According to figure 2, the operating time (W_K) is divided in two parts T_1 and T_2:

$$T_2 = W_K - T_1. \tag{21}$$

The average value of T_2 is:

$$E[T_2] = E[W_K] - E[T_1]. \tag{22}$$

With

$$E[T_1] = \frac{h}{U_{\max} - d - \overline{\alpha_{R1}}}. \tag{23}$$

The average number of rejected non-conforming items during T_1 is:

$$Qp_1 = \overline{\alpha_{R1}} \cdot E[T_1]. \tag{24}$$

During T_2, the average number of rejected items is given by:

$$Qp_2 = \overline{\alpha_{R2}} \cdot E[T_2] = \overline{\alpha_{R2}} \cdot (E[W_k] - E[T_1]). \tag{25}$$

Hence, using equations (22), (24), (25) and summing up Qp_1 and Qp_2, we obtain the following expression of the total average quality cost:

$$\begin{aligned}\delta q &= \frac{\mathrm{C_{nc}}}{E[T_K]}[Qp_1 + Qp_2] \\ &= \frac{\mathrm{C_{nc}}}{E[T_K]}[\overline{\alpha_{R1}} \cdot E[T_1] + \overline{\alpha_{R2}} \cdot (E[W_K] - E[T_1])].\end{aligned} \tag{26}$$

Using equation (24), we can write:

$$\delta q = \frac{\mathrm{C_{nc}}}{E[T_K]}\left[\overline{\alpha_{R1}} \cdot \frac{h}{U_{\max} - d - \overline{\alpha_{R1}}} + \overline{\alpha_{R2}} \cdot (E[W_K] - \frac{h}{U_{\max} - d - \overline{\alpha_{R1}}})\right]. \tag{27}$$

Combining equations (17), (20) and (27), the expression of the total average cost per time unit is obtained as follows:

$$\begin{aligned}\prod(h,l_m) = \frac{1}{E[T_k]}\Bigg[& C_s \cdot E[W_K] + C_s \cdot h^2 \cdot R_D\left(\frac{h}{d}\right)\frac{U_{\max} - 2 \cdot d - \overline{\alpha_{R1}}}{2 \cdot d \cdot (U_{\max} - d - \overline{\alpha_{R1}})} \\ &+ C_s \cdot R_D\left(\frac{h}{d}\right) \cdot \int_{h/d}^{+\infty} xh - x^2 \frac{d \cdot (U_{\max} - \overline{\alpha_{R1}})}{2(U_{\max} - d - \overline{\alpha_{R1}})} g_D(x)\mathrm{d}x \\ &+ R_D\left(\frac{h}{d}\right) \cdot C_p \int_{h/d}^{+\infty} (x \cdot d - h) g_D(x)\mathrm{d}x + M_p \cdot [P(l_{\max}) - P(l_m)] \\ &+ M_c \cdot [1 - P(l_{\max})] \\ &+ C_{nc}\left[\overline{\alpha_{R1}} \cdot \frac{h}{U_{\max} - d - \overline{\alpha_{R1}}} + \overline{\alpha_{R2}} \cdot \left(E[W_k] - \frac{h}{U_{\max} - d - \overline{\alpha_{R1}}}\right)\right]\Bigg].\end{aligned} \tag{28}$$

In this equation, the expression of the average operating period $E[W_K]$ needs to be established in order to be able to compute the total expected cost per time unit for a given set of input parameters.

According to the proposed strategy, one can notice that the end of period W_K coincides with the moment at which the rejection rate is found having exceeded the threshold level l_m. Let's designate this instant by t_m. Let $l(t)$ be a continuous and increasing function which expresses the evolution of the rejection rate as a function of time. $l(t)$ verifies:

$$l(t_m) = l_m \tag{29}$$

Hence, $E[W_K]$ corresponds to tm which can be obtained as follows:

$$t_m = l^{-1}(l_m) \tag{30}$$

Due to the difficulty of obtaining an exact analytical expression of $E[W_K]$, we propose to derive it using simulation. To do so, we will develop, for a given set of input parameters in the next section, a simulation model allowing finding an estimate of the function $l(t)$. Once such an estimate is obtained, $E[W_K]$ can easily be determined using equation (30).

4. Numerical example and simulation model

To illustrate our approach, we consider a situation with the following input data which have been arbitrarily chosen considering nevertheless realistic settings:

- Overhaul duration probability distribution: normal law: mean 2 hours and standard deviation 0.5 hour.
- Preventive maintenance duration probability distribution: normal law: mean 0.5 hour and standard deviation 0.1 hour.
- P is a beta law with minimum 0, maximum 1, $p = 3$, $q = 3$. The choice of the Beta distribution for the rejection rate l is justified by the fact that it has both upper and lower finite bounds (0 and 1 in this case). The Beta density function can approach zero or infinity at either of its bounds, with p controlling the lower bound and q controlling the upper bound. Uniform or triangular distributions can also be associated to the rejection rate l. However, they are known to be less accurate than the Beta distribution.
- $C_s = 5$ \$/unit/hour, $C_p = 450$ \$/unit short, $M_p = \$500$, $M_c = \$2000$, $l_{\max} = 0.8$.
- $C_{nc} = 150$ \$/unit lost, $U_{\max} = 100$ units/hour, $d = 20$ units/hour.

The expression of the total average cost per time unit given by equation (28) must be used to determine the optimal values of the decision variables h^* and l_m^*. As mentioned in the end of the previous section, to be able to use equation (28), we need to estimate the average operating period $E[W_K]$ using equation (30) which is based on the function $l(t)$ expressing the evolution of the rejection rate as a function of time.

In order to estimate this function $l(t)$, for any given set of input data, we developed a simulation model for the production process. The simulator considers the production system producing, over a large horizon, lots of a given size and generating non-conforming items according to the following relationship:

$$L(t + \Delta t) = L(t) \cdot (1 + \xi) \tag{31}$$

where $L(t)$ is the cumulative quantity of non-conforming items at instant t; ξ is a random variable between 0 and 1 following a Beta distribution; Δt is a time increment equal to the period needed to produce a lot.

The rejection rate is calculated as follows:

$$l(t) = \frac{L(t)}{m \cdot x} \tag{32}$$

where m stands for the number of produced lots. The size of a lot being equal to x.

The detailed simulation procedure is described in figure 5 for which the following notations were adopted:

L° The quantity of non-conforming units at $t = 0$.
n_{rep} Number of replications.

A lot of size x is produced during a period T, a quantity of non-conforming items is then generated randomly according to equation (31) and the corresponding rejection rate $l(t)$ is calculated using equation (32). This procedure is repeated for m periods T as long as the rejection rate does not exceed 1. A total of n_{rep} replications are made. Finally, the average values of $l(iT)$ for $(i = 1, \ldots, n_{\text{rep}})$ are considered to find an approximate expression of the function $l(t)$ using the basic least squares method.

Hence, once $l(t)$ is obtained, an estimate of the average operating period $E[W_K]$ can be found using equation (30) for any given threshold value l_m. Let us consider the following setting as an example:

$L^\circ = 0$
$x = 50$ units
$n_{\text{rep}} = 10$
$m = 7$

The following results are obtained from the simulator:

Table 1. Results of simulation.

	l(t) rep 1	l(t) rep 2	l(t) rep 3	l(t) rep 4	l(t) rep 5	l(t) rep 6	l(t) rep 7	l(t) rep 8	l(t) rep 9	l(t) rep 10	Average l(t)
T	0.074	0.066	0.057	0.055	0.067	0.067	0.068	0.065	0.069	0.071	0.066
2^*T	0.123	0.116	0.089	0.084	0.116	0.105	0.120	0.123	0.123	0.101	0.111
3^*T	0.172	0.207	0.148	0.152	0.224	0.172	0.204	0.223	0.191	0.205	0.188
4^*T	0.305	0.385	0.252	0.217	0.360	0.314	0.347	0.375	0.337	0.335	0.321
5^*T	0.514	0.595	0.475	0.323	0.601	0.514	0.635	0.678	0.515	0.602	0.539
6^*T	0.804	0.887	0.807	0.490	0.959	0.955			0.827	0.906	0.818
7^*T	0.834	0.900	0.843	0.884							0.865

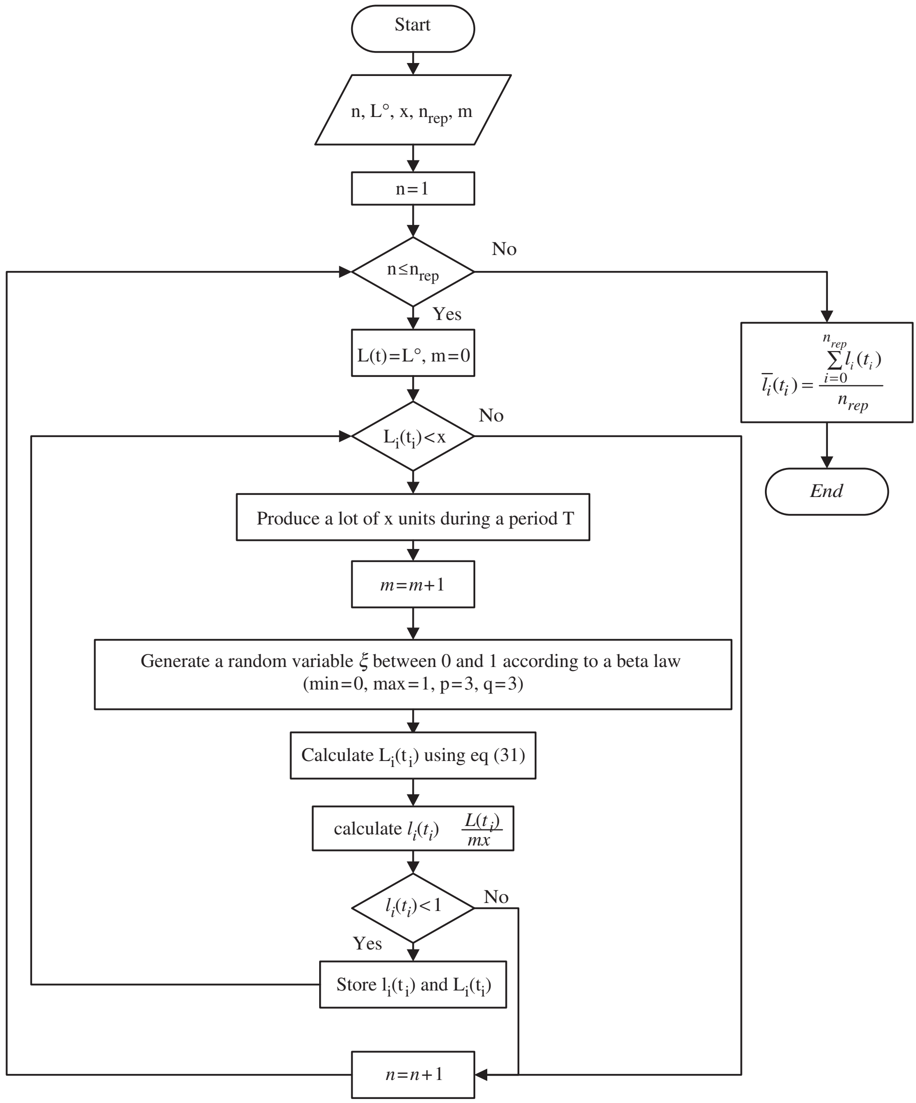

Figure 5. Simulation model.

The next figure shows the shape of the obtained function $l(t)$ as well as the shape of its estimate using a least square curve fitting tool. The obtained estimated expression is given by:

$$l(t) = 0.4328\mathrm{l}_n(t) - 0.1089. \tag{33}$$

Hence, using equation (30), for a given lm, $E[W_K]$ can be found as follows:

$$E[W_K] = \exp\left(\frac{l_m + 0.1089}{0.4328}\right). \tag{34}$$

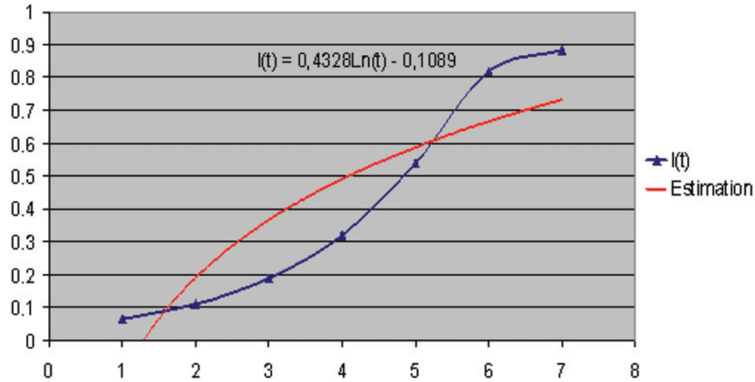

Figure 6. Approximate evolution of the rejection rate as a function of time.

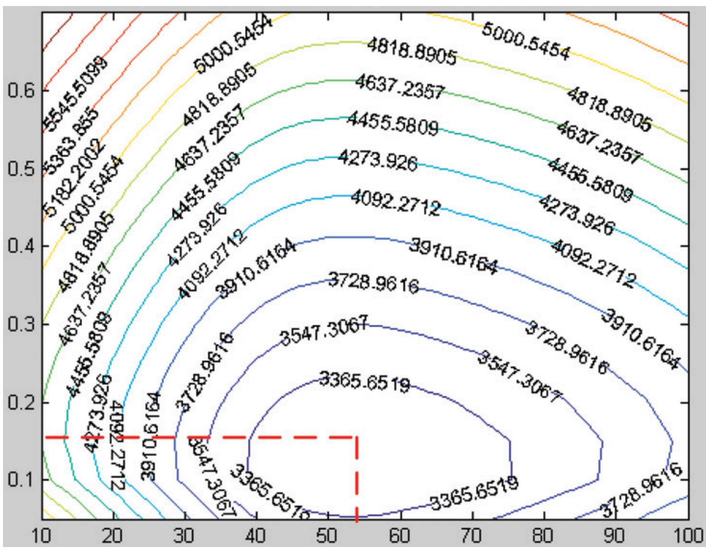

Figure 7. Contour plot of the response surface.

Now that all the terms of the total average cost per time unit (equation (28)) are defined for any given values of the decision variables h and lm, considering the numerical input data presented at the beginning of this section, we used a simple enumeration procedure to find the optimal couple (h^*, l_m^*) which minimizes the total average cost per time unit. Figure 7 shows the contour plot of the response surface of this cost rate $\Pi(h, l_m)$.

The optimal strategy thus consists of starting an action of preventive maintenance only when the rejection rate is found equal to or higher than $l_m^* = 15\%$. It will also be necessary to build a buffer stock of $h^* = 55$ units to continue to satisfy the demand when the production is stopped for maintenance. By doing so, it would cost on average 3365.65 ($/hour).

5. Conclusion

This paper presented a joint strategy of quality control and preventive maintenance for an imperfect production process producing conforming and non-conforming items. Each produced lot is submitted to a quality control during which the rate of non-conforming units is determined.

Two decision variables characterise the proposed strategy: the rate, l_m, of non-conforming units on the basis of which preventive maintenance actions should be performed, and the size, h, of the buffer stock to be built in order to palliate perturbations caused by stopping production and performing maintenance actions of random durations.

A mathematical model and a simulation program have been developed to generate the optimal strategy (l_m^*, h^*) which minimizes the total average unitary cost which includes quality, maintenance and inventory costs.

This work constitutes an attempt to integrate in a single model quality control issues, buffer stock sizing and preventive maintenance. The proposed methodology can be extended to the case of stochastic demand. Another possible extension is to consider situations where the produced items are of a perishable nature.

References

Balasubramanian, R., Preventive maintenance scheduling in presence of a production plan. *Prod. Invent. Man.*, 1987, **1**, 80–87.

Ben Daya, M., Integrated production maintenance and quality model for imperfect processes. *IIE Trans.*, 1999, **31**, 491–501.

Ben Daya, M., The economic production lot-sizing problem with imperfect production processes and imperfect maintenance. *Int. J. Prod. Econ.*, 2002, **76**, 257–264.

Boukas, E.K. and Haurie, A., Manufacturing flow control and preventive maintenance: a stochastic control approach. *IEEE Trans. Auto. Cont.*, 1990, **35**, 1024–1031.

Chelbi, A. and Aït-Kadi, D., Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance. *Euro. J. Oper. Res.*, 2004, **156**(3), 712–718.

Chelbi, A. and Rezg, N., Analysis of a production/inventory system with randomly failing production unit subjected to a minimum required availability level. *Int. J. Prod. Econ.*, 2006, **99**, 131–143.

Gharbi, A. and Kenne, J.P., Production and corrective maintenance planning of FMS through simulation optimisation, in *4th International Conference on Engineering Design and Automation*, Orlando, Florida, USA, 2000.

Groenevelt, H., Pintelon, L. and Seidmann, A., Production lot sizing with machine breakdowns. *Man. Sci.*, 1992a, **38**(1), 104–123.

Groenevelt, H., Pintelon, L. and Seidmann, A., Production batching with machine breakdowns and safety stocks. *Oper. Res.*, 1992b, **40**(5), 959–971.

Kenne, J.P. and Gharbi, A., A simulation optimization approach in production planning of failure prone manufacturing systems. *J. Intel. Manuf.*, 2001, **12**(5), 421–431.

Meller, R.D. and Kim, D.S., The impact of preventive maintenance on system cost and buffer size. *Euro. J. Oper. Res.*, 1996, **95**, 577–591.

Salameh, M.K. and Ghattas, R.E., Optimal just-in-time buffer inventory for regular preventive maintenance. *Int. J. Prod. Econ.*, 2001, **74**, 157–161.

Van Der Dayn Schouen, F.A. and Vanneste, S.G., Maintenance optimization of a production system with buffer capacity. *Euro. J. Oper. Res.*, 1995, **82**(2), 323–338.

Wang, P., William, J., Gray, W.J. and Meckinley, S., Quality-related measures of unreliable machines with preventive maintenance. *Comput. Oper. Res*, 1999, **10**(23), 981–996.