# Variance Tolerancing and Decomposition in Short-Staple Spinning Processes

## Part I: Modeling Spun Yarn Strength Through Intrinsic Components

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ABSTRACT

A variance tolerancing method is developed as a means for separating and estimating random errors associated with raw materials and yarn structures from process-induced errors based on structural relationships governing the strength of a spun yarn. The method is successfully used to estimate the mean and variance of spun yarn strength by propagating the statistical parameters of fiber properties onto those of the resulting spun yarns. In developing the new estimation procedure, probabilistic models based on the distributions of fiber length and tensile properties are combined with geometric and structural models of fiber arrangements in spun yarns. For the first time, the concept of "effective gauge length" is used to model and simulate the breakage process of spun yarns. The new concept and specific methodology are aimed at better controlling process and product characteristics by quantifying the variances according to their sources.

In textile manufacturing, large variations in process and product characteristics are often not well understood due to lack of an effective way to separate random variations ("noise") from structural or known scientific relations ("signal") that are often unknown or hidden. As pointed out by Suh [8], traditional quality control techniques based on variance analysis alone have failed due to the large number of unaccounted variations in raw materials and processes in addition to the complex structural relationships that are largely unknown or difficult to verify in the presence of huge process variations. In this situation, an improvement in signals cannot be easily verified without reducing and estimating the random noise. The focus of this paper is to re-examine the process of quantifying random and process-induced errors under a set of simplified structural relations rather than trying to improve the relationships themselves.

Much textile research in the past has been devoted to "forward prediction and characterization" of physical properties through various modeling approaches. In predicting such yarn and fabric properties as strength and evenness, the forward prediction methods were also the most prevalent. To model spun yarn strengths, various forward prediction equations have long been used to estimate the "average" pattern of output characteristics from available averages of input characteristics. For those, however, the precision as well as the accuracy of the characteristic to be predicted becomes extremely low as the complexity of the functional form and the number of predictor variables increases. This is due to the non-uniform and confounding response pattern of the output variable over the entire ranges of the predictor variables that are often highly correlated among themselves. When this reality is added to the large variance introduced at each process stage (process-induced variance), it is not surprising at all that the forward prediction equations are neither precise nor accurate enough to control or improve the qualities in textile manufacturing.

As we have indicated (Suh [8]), traditional research methodology has not been highly effective in improving textile product and process qualities due to large variations coming from the mixture of signals and noise. Therefore, the control and improvement strategies for textile process qualities will have to come from better analyses of variance on the input and output variables imbedded in simplified and verifiable structural relationships.

Statistical tolerancing techniques [3, 12] have long been widely used to determine the probability distributions of output characteristics from a set of input component variances when the structural cause-and-effect relationship is known functionally. This technique, however, is not easily applicable to textile products and processes, where reliable functional forms of the structural relationships are either not known at all or only partially understood. For this reason, we present a new concept called "variance tolerancing" based on "intrinsic components," which uses geometric and probabilistic models to depict characteristics of textile products and to estimate the mean and variance of yarn tensile strength. Variance tolerancing can facilitate decomposing of the spun yarn strength variance into responsible subcomponents.

## Variance Tolerancing Based on Intrinsic Components

Conventional "statistical tolerancing" methods [3, 12], while effective in many well-defined processes, are inadequate in many ways for textile products and processes. In this section, we explain the reasons and introduce a new variance tolerancing technique.

Let the forward prediction equation of a textile product characteristic $Y$ be

$$Y = f(X_1, \ldots, X_n) \quad , \tag{1}$$

where $f$ is a deterministic, statistical, or empirical function, and $X_1, \ldots, X_n$ is a set of input variables. If each $X_i$ is associated with independent errors or variances, the classical statistical tolerancing formula by Tukey [12] can be applied to Equation 1 to give

$$\mathrm{Var}(Y) \approx \sum \left(\frac{\partial f}{\partial X_i}\right)_0^2 \mathrm{Var}(X_i) = \sum f_i^2 \,\mathrm{Var}(X_i) \quad , \tag{2}$$

where the subscript 0 means that each element of $X_1, X_2, \ldots, X_n$ assumes its mean value, $X_i = \mu_i$, $i = 1, 2, \ldots, n$, for all $i$s after partial differentiation, and $f_i$ is the functional form of the $i$th partial derivative of $f$. Equation 2 has frequently been used to predict the variance of response $Y$ in terms of the variances of $X_i$ variables within their tolerance ranges by employing the classical one-variable-at-a-time approach. This method, however, is not highly effective in variance tolerancing because it tacitly assumes that the effects of a given variable on the variance of $Y$ are fixed and independent of the levels of the others. This problem becomes even more serious as the complexity of the functional form and the number of predictor variables increases. For this reason, we have developed a better alternative procedure called "variance tolerancing" to estimate the variance of $Y$ when only the essential components are varied simultaneously. In developing a forward prediction equation, difficulties are often due to the "trivial many" components that inhibit formulation of an exact functional relationship. In estimating the variance of a response variable with high precision, therefore, only the "vital few" must be chosen for variance tolerancing. This may be accomplished in two steps: filtering only the intrinsic components for variance tolerancing, and finding a geometric, probabilistic, or structural model that is able to estimate the variance of a product characteristic defined by its intrinsic components.

The definition of an intrinsic component is one that is measurable or countable and is structurally imbedded for explaining the variance of $Y$. The intrinsic component can be an input material characteristic or a geometric or structural factor. In modeling the tensile strength of a yarn, fiber length, strength, and fineness and yarn twist may be chosen to be the intrinsic components.

Let a textile product characteristic $Y$ be

$$Y = g(Z_1, \ldots, Z_m),\ m \leq n \quad , \tag{3}$$

where $Z_1, \ldots, Z_m$ are the intrinsic components and $g$ is a geometric, probabilistic, or structural model reflecting only the effects of $\mathrm{Var}(Z_i)$ without regard to $E(Z_i)$; $i = 1, 2, \ldots, m$. Unlike the statistical tolerancing process shown in Equation 2, the variance is toleranced through a set of structural, geometric, or probabilistic relationships instead of partial differentiation with respect to the input variables. Here, $g(Z_1, \ldots, Z_m)$ is a composite structure consisting of the intrinsic components rather than a closed functional form. The variance of a textile product characteristic is then estimated in terms of the statistical distributions of the intrinsic components. In turn, this can facilitate an optimal selection of input materials, process conditions, and prediction/optimization of textile product characteristics without having to rely heavily on the existing structural relationships that are often inadequate for variance tolerancing. This process is best illustrated by the specific structure of the response variable $Y$, discussed next.

## Variance Tolerancing in Spun Yarn Strengths

In a spun yarn or a woven fabric, the variance of each quality characteristic may be estimated in terms of the variances of subcomponents under certain structural relationships. In this section, we show a method for estimating the variance of yarn tensile strength as a function of the variance of the intrinsic fiber properties and a random variable associated with the arrangements of the constituent fibers through the new variance tolerancing technique.

A spun yarn is considered to be a continuous chain of twisted parallel fiber bundles with a known average number of fibers of which only some are "continuous" within a given segment of size $L$, as shown in Figure 1.

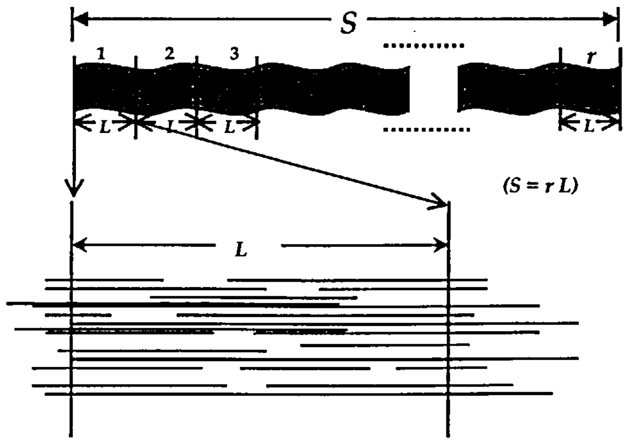


FIGURE 1. Model for spun yarn strength through effective gauge length $L$ and test gauge $S$ $(=rL)$.

$L$ is an arbitrary value to be estimated based on actual yarn strength data. For the time being, we have developed a model for spun yarn strength assuming that the value of $L$ is fixed. In addition, we assume the model and analysis are invariant with respect to the specific beginning and ending points of each $L$. The essence of the analysis to follow based on $L$ is to apply the optimum value of $L$, denoted as $L_0$, based on a set of experimental data. Earlier studies on yarn structures by Sullivan [11], Platt [7], Pitt and Phoenix [6], and Suh [9] included some discussions of fiber arrangement within a yarn. According to the weakest-link theory of Peirce [5], the strength of a yarn can be estimated by the strength of the weakest bundle within the chain, when the strength of each bundle is the sum of the strengths of continuous fibers within length $L$. By "continuous fiber," we mean that either end of the fiber is not found within the particular yarn length segment $L$. In this model, we assume the yarn strength is proportional to the strength of the bundle with length $L$, which contains the smallest number of continuous fibers among $r$ such bundles (links) making up a specimen of size equal to the test gauge length. First, we must find the distribution and moments of the number of continuous fibers within $L$. We can then obtain the expectation and variance of the specimen of size $rL$ by equating it to the strength of the weakest bundle among $r$, or the bundle with the smallest number of continuous fibers within $L$.

## NUMBER OF CONTINUOUS FIBERS WITHIN $L$

Suh [9] derived the exact distribution and moments of the number of fibers to be found within $L$ on the premise that the fiber lengths and the number of fibers found within an interval of size $L$ follow certain statistical distributions. For the purpose of expanding his results, we show the following summary: Let

$k$ = average number of fibers per yarn cross section

$L$ = size of an arbitrary interval along the yarn axis, called the "effective gauge length"

$X$ = fiber length

$F(x), f(x)$ = cumulative distribution function and probability density function, respectively, of $X$

$S(L)$ = length of a fiber segment contained within $L$

$N(L)$ = total number of fibers to be found within $L$

$S_T(L)$ = aggregate length of all fibers found within $L$

$$= \sum_{i=1}^{N(L)} S_i(L) \quad ,$$

where $S_i(L)$ is $S(L)$ for fiber $i$, $i = 1, 2, \ldots, N(L)$. Then, the expected number of fibers within the effective gauge length is

$$E[N(L)] = \frac{kL}{E[S(L)]} \quad . \tag{4}$$

Suh [9] has shown that $E[S(L)] = E_x\left[\frac{Lx}{x+L}\right]$ and $N(L)$ is asymptotically Poisson ($\lambda$), yielding

$$\mathrm{Var}[N(L)] = \lambda = \frac{k}{E[S(L)]} \quad . \tag{5}$$

By applying Suh's results [9], we can now derive the exact distribution and moments of the number of continuous fibers to be found within $L$ as follows: Let

$N_C(L)$ = number of continuous fibers to be found within $L$, *i.e.*, the number of fibers having $S(L) = L$,

$N_D(L)$ = number of discontinuous or floating fibers to be found within $L$, *i.e.*, the number of fibers having $S(L) < L$,

$N(L) = N_C(L) + N_D(L)$,

$E[N_C(L)]$, $\mathrm{Var}[N_C(L)]$ = expectation and variance of $N_C(L)$, respectively.

By writing

$P[S(L) = L] = P[\text{a fiber is continuous within } L] = p$

$$= \int_L^\infty \frac{x-L}{x+L} f(x)dx \quad , \tag{6}$$

the conditional distribution of $N_C(L)$ given $N(L)$ is *binomial* $\{N(L), p\}$. Since $N_C(L)$ has a mixture distribution combining a binomial $\{N(L), p\}$ with $N(L) \sim$ Poisson($\lambda$), the distribution of $N_C(L)$ is a Poisson with intensity parameter $\lambda p$ [1, 4]. Hence, by applying Equation 4,

$$E[N_C(L)] = \mathrm{Var}\,[N_C(L)] = \lambda p$$

$$= \frac{k}{E[S(L)]} \int_L^\infty \frac{x - L}{x + L} f(x) dx \quad . \tag{7}$$

SMALLEST NUMBER OF CONTINUOUS FIBERS WITHIN $L$

Assuming that the strength of a yarn with length $rL$ is proportional to the strength of the weakest bundle with size $L$ among $r$ such bundles, the next step is to estimate the number of continuous fibers within the weakest bundle. Let

$S$ = test gauge length of a spun yarn after removing the twists $= rL$

$r$ = number of bundles with length $L$ within $S$

$Y_1$ = the smallest number of continuous fibers within $L$ among $r$ samples, *i.e.*,

$$N_C(L)_{(1)} = \min_{1 \le i \le r} \{N_C(L)_i\} \quad ,$$

$E[Y_1]$, $\mathrm{Var}[Y_1]$ = expectation and variance of $Y_1$, respectively.

Then, we can find the distribution of $Y_1$ from the distribution of $N_C(L)$. Let $N_C(L)_1, N_C(L)_2, \ldots, N_C(L)_r$ be random samples drawn from a Poisson ($\lambda p$) population, where $N_C(L)_{(1)}$ $(=Y_1) < N_C(L)_{(2)} < \ldots < N_C(L)_{(r)}$ are the order statistics formed by the samples. Then, we can show the distribution function of $Y_1$ based on a discrete random variable $N_C(L)$ as follows (see Casella and Berger [1] for similar approaches):

$$P(Y_1 > n) = \prod_{i=1}^{r} P\{N_C(L)_i > n\} = \{1 - F_{N_C(L)}(n)\}^r \quad ,$$

$$P(Y_1 \le n) = 1 - P(Y_1 > n)$$
$$= 1 - \{1 - F_{N_C(L)}(n)\}^r \quad , \tag{8}$$

$$P(Y_1 \le n - 1) = 1 - P(Y_1 > n - 1)$$
$$= 1 - \{1 - F_{N_C(L)}(n - 1)\}^r \quad , \tag{9}$$

where $F_{N_C(L)}(n) = P\{N_C(L)_i \le n\}$ for all $i = 1, 2, \ldots, r$.

From Equations 8 and 9, the probability that $Y_1$ assumes value $n$ is

$$g_{Y_1}(n) = P(Y_1 = n)$$
$$= P(Y_1 \le n) - P(Y_1 \le n - 1)$$
$$= \{1 - F_{N_C(L)}(n - 1)\}^r - \{1 - F_{N_C(L)}(n)\}^r$$
$$= \left[1 - \sum_{i=1}^{n-1} \frac{e^{-\lambda p}(\lambda p)^i}{i!}\right]^r - \left[1 - \sum_{i=1}^{n} \frac{e^{-\lambda p}(\lambda p)^i}{i!}\right]^r \quad . \tag{10}$$

Hence, the expectation and variance of the smallest number of continuous fibers within $L$ are

$$E[Y_1] = \sum_{n=1}^{\infty} n g_{Y_1}(n) = \sum_{n=1}^{k} n g_{Y_1}(n) \tag{11}$$

and

$$E[Y_1^2] = \sum_{n=1}^{\infty} n^2 g_{Y_1}(n) = \sum_{n=1}^{k} n^2 g_{Y_1}(n)$$

$$\mathrm{Var}[Y_1] = E[Y_1^2] - [E[Y_1]]^2 \quad . \tag{12}$$

In order to simplify the computation, the upper limit for $n$ in Equations 11 and 12 may be placed to be equal to the average number of fibers per yarn cross section without reducing the precision of the estimates.

MEAN AND VARIANCE OF SPUN YARN STRENGTH

Under the structural geometry of a spun yarn shown in Figure 1, we can estimate the mean and variance of yarn tensile strength based on a conceptual gauge length $S$ rather than the actual test gauge length $S'(<S)$ resulting from the twist retraction. For simplicity, we use the conceptual gauge length in both the theory development and the computer simulations to be made in Part II. Based on $E(Y_1)$ and $\mathrm{Var}(Y_1)$ in Equations 11 and 12, we now estimate the mean and variance of the spun yarn strength as a function of the effective gauge length $L$, which we have yet to determine. The ultimate goal is to obtain $E(Y_1)$ as an estimate of the yarn strength. The schematics of the necessary steps are given in Figure 2. Knowing the minimum number of continuous fibers within $L$ among $r$ length segments, we must know what fraction of the aggregate fiber strength is translated into the yarn strength. Suh *et al.* [9] have shown the exact distribution and moments of bundle strength as a func-

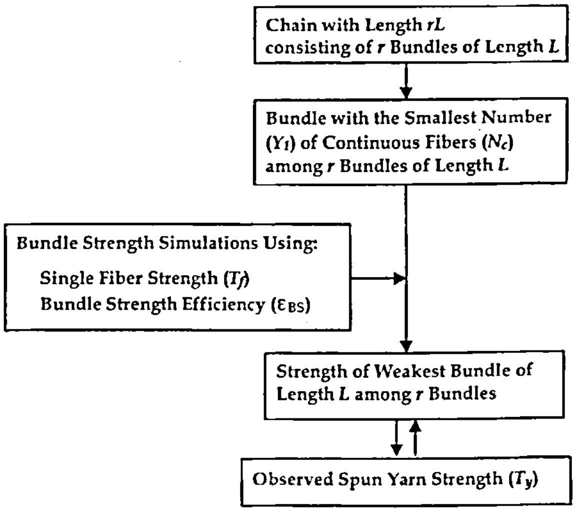


FIGURE 2. Schematics of intrinsic modeling and analysis for spun yarn strength.

tion of the distribution of single fiber strength. While they are complex for "small bundles," the asymptotic forms for "large bundles" can be obtained in a much simpler way [5, 10] when the distribution function for the single fiber strength $T_f$ is known. In a later study, Cui [2] observed that bundle strength efficiency, defined as a ratio between the realized bundle strength and the aggregate fiber strength in the bundle, decreases rapidly to 0.6 for $n = 60$ and to 0.58 for $n = 400$ for most cotton fibers. As we will show in Part II of this paper, the size of the bundle $E(Y_1)$, or the number of continuous fibers in the weakest bundle, is within this range for the majority of spun yarns coarser than 40/1 Ne. We will discuss the actual values of $\varepsilon_{BS}$ in Part II.

Therefore, by letting

$T_f$ = breaking strength of a single fiber tested with gauge length $L$

$E[T_f]$, $\mathrm{Var}[T_f]$ = expectation and variance of single fiber strength, respectively

$\varepsilon_{BS}$ = bundle strength efficiency

$T_y$ = tensile strength of a spun yarn with length $S = rL$ $= \varepsilon_{BS} \sum_{i=1}^{Y_1} T_{fi}$ .

We can estimate $E(T_y)$ and $\mathrm{Var}(T_y)$ as follows using the well known conditional expectation formula [1, 4]:

$$E[T_y] = E_{Y_1} E_{T_y}[T_y/Y_1] = \varepsilon_{BS} E_{Y_1}[\sum_{i=1}^{Y_1} T_{f_i}]$$

$$= \varepsilon_{BS}\, E[T_f]\, E[Y_1] \quad , \tag{13}$$

$$\mathrm{Var}[T_y] = E_{Y_1} \underset{T_y}{\mathrm{Var}}[T_y/Y_1] + \underset{Y_1}{\mathrm{Var}} E_{T_y}[T_y/Y_1]$$

$$= E_{Y_1} \underset{T_y}{\mathrm{Var}}[\varepsilon_{BS} \sum_{i=1}^{Y_1} T_{f_i}]$$

$$+ \underset{Y_1}{\mathrm{Var}}\, E_{T_y}[\varepsilon_{BS} \sum_{i=1}^{Y_1} T_{f_i}]$$

$$= \varepsilon_{BS}^2 \{\mathrm{Var}[T_f]\, E[Y_1] + (E[T_f])^2\, \mathrm{Var}[Y_1]\} \quad . \tag{14}$$

Note in Equation 14 that the variance of the yarn strength has been "toleranced" effectively by the inherent fiber properties, that is, the mean and variance of constituent fiber strengths through intrinsic structural components and $Y_1$, which, in turn, are determined by $L$ and the fiber length distribution.

DETERMINING OPTIMUM EFFECTIVE GAUGE LENGTH $L_0$ FROM YARN TEST DATA AND COMPUTER SIMULATIONS

As we noted previously, the strength of a spun yarn given by Equation 14 is a function of $E(Y_1)$, which, in turn, is a function of $r$, $L$, and $f(x)$, as evidenced by Equations 7, 10, and 11. Trivially, $E(Y_1)$ would decrease as $L$ increases or $r$ decreases, whereas $\mathrm{Var}(Y_1)$ is expected to decrease to zero for a very large $L$. With an observed value of spun yarn strength, therefore, it is possible to estimate the "optimal" effective gauge length $L_0$ in such a way that the theoretical yarn strength under $L_0$(and $r_0 = S/L_0$) would match the actual yarn strength. While we can make no *a priori* assumption about $L_0$, the objective of our study is to estimate it for different yarn sizes and twists with varying fiber properties so that we may establish its possible relationship with those structural parameters. With the advent of computing power, this approach facilitates understanding of spun yarn strengths.

## Variance Decomposition of Yarn Tensile Strength

As we already indicated, our theories on variance tolerancing methods are aimed at estimating the effects of random components that are due to raw materials and fiber arrangements in spun yarn strength. Once this is done, we can estimate the process-induced variances from the total observed variance. In practice, the random variances are estimated under $L_0$ to facilitate variance decomposition of yarn tensile strength. The conceptual frame and estimation procedures are as follows for the variance decomposition of spun yarn strengths:

In general, variabilities in spun yarn strengths are largely attributable to variabilities of raw material properties and processing conditions. The decomposition of experimentally observed variances is accomplished by variance tolerancing of the random components and by accounting for the process-induced variances out of the total observed variances.

In single-end testing, the total variance of yarn tensile strength ($\sigma_T^2$) decomposes into two components—the between-package variance ($\sigma_{bp}^2$) and the within-package variance ($\sigma_{wp}^2$):

$$\sigma_T^2 = \sigma_{bp}^2 + \sigma_{wp}^2 \quad . \tag{15}$$

The between-package variance ($\sigma_{bp}^2$) is entirely due to variations in processing machines accrued at different stages of spinning. We estimate this component from the usual analysis of variance on the actual yarn test data. The within-package variance ($\sigma_{wp}^2$), on the other hand, further decomposes into two subcomponents—random variance ($\sigma_r^2$) and nonrandom variance ($\sigma_{nr}^2$):

$$\sigma_{wp}^2 = \sigma_r^2 + \sigma_{nr}^2 \quad . \tag{16}$$

Here, we estimate the variance due to the random component ($\sigma_r^2$) by using the variance tolerancing method based on intrinsic components shown in the previous section. More specifically, $\mathrm{Var}(T_y)$ shown in Equation 16 can be rewritten as follows under an optimal $L(=L_0)$:

$$\begin{aligned}\sigma_r^2 &= \mathrm{Var}[T_f/L_0] \\ &= \varepsilon_{BS}^2\{\mathrm{Var}\,[T_f]\; E[Y_f/L_0] \\ &\qquad + (E[T_f])^2\,\mathrm{Var}[Y_f/L_0]\} \quad .\end{aligned} \tag{17}$$

As previously examined, the random component ($\sigma_r^2$) is due to variances in raw material properties and those resulting from random errors associated with fiber arrangement within the yarn structure. The variance from the nonrandom component ($\sigma_{nr}^2$) reflects the variations caused by systematic fluctuations of the fiber mass due to process-induced drafting waves, operator effects, environmental effects, etc. The magnitude of $\sigma_{nr}^2$ is estimated by subtracting the estimated value of $\sigma_r^2$ from the observed value of $\sigma_{wp}^2$ from an analysis of variance on the yarn test data.

Finally, by combining these results, we obtain the total amount of process-induced variance ($\sigma_P^2$) by adding the two nonrandom variance components ($\sigma_{nr}^2$ and $\sigma_{bp}^2$), whereas the total random variance remains the same as the toleranced variance ($\sigma_r^2$) from the intrinsic components:

$$\sigma_P^2 = \sigma_{nr}^2 + \sigma_{bp}^2 \quad , \tag{18}$$

$$\sigma_T^2 = \sigma_p^2 + \sigma_r^2 \quad . \tag{19}$$

As we will show in Part II, these variance tolerancing and decomposition methods are readily applicable to all spun yarn strengths in which the fiber arrangements are assumed to be more or less parallel before twisting.

## Conclusions

We have developed a procedure for quantifying variabilities in spun yarn strength by introducing a new variance tolerancing technique that employs geometric, probabilistic, and structural models depicting the physical characteristics, structures, and formation stages of spun yarns. In the course of development, we introduce a new concept of intrinsic components. In explaining the variance of spun yarn strength, we show that fiber length, strength, fineness, and a newly developed parameter called "effective gauge length" can be used effectively as intrinsic components for variance tolerancing.

We can estimate the random variance component of spun yarn strength from the toleranced variances of raw material properties and the random errors of fiber arrangements within the structural geometry of the yarn modeled by the concept of effective gauge length. We can also estimate the total process-induced variance by adding the between-package variance and the nonrandom variance estimated from the within-package variance.

The concept and computational steps we propose for the variance tolerancing and decomposition methods facilitate a new method of analyzing single-end strength data on any spun yarn and enable us to assess the magnitude of the process-induced variance for effective quality control and improvement of spun yarn strength. The same modeling concept can be easily applied to other physical properties of textile structures under the proper definition of intrinsic components. We present the experimental and analytic procedures for single-end strength in Part II of this paper.

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# Photodegradation of Lyocell Fibers Through Exposure to Simulated Sunlight

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ABSTRACT

This research investigates changes occurring in lyocell fibers resulting from exposure to simulated sunlight. FTIR microspectroscopy is used in combination with measurements of tensile properties, fluidity, moisture regain, yellowness, weight loss, and birefringence. X-ray diffraction and SEM are used to determine crystallinity and fiber surface changes, respectively. The results indicate no significant changes in physical properties up to 96 hours' exposure. At 168 hours, open pores appear on the fiber surface, probably due to the release of gaseous products, accompanied by an increase in moisture regain.

The effect on textile products, particularly cellulosic materials, of exposure to ultraviolet light has attracted considerable interest for many years. Hon [14] attributed Witz[3] as possibly the first researcher to establish that when cellulose is exposed to sunlight, chemical changes are induced. Later, Harrison [9] showed that light-induced reactions of cellulose were oxidative-degradative reactions. Milligan *et al.* [21] reported an investigation of the chemical and physical changes caused by prolonged exposure of wool to ultraviolet and visible radiation. They attributed phototendering to disulphide bond fission and main chain cleavage, and suggested that the latter is more important. Simpson and Page [27] postulated that light in the near ultraviolet region (310–400 nm) has the most marked degradative effect on wool fibers. Carlsson *et al.* [3] exposed polypropylene fibers to ultraviolet light and determined the changes in tensile strength, elongation to break, the shape of the stress-

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