Cheat Sheet for Exercises

Gaussian Elimination

https://en.wikipedia.org/wiki/Gaussian_elimination

Computing ranks and bases

System of equations	Row operations	Augmented matrix
$\begin{alignedat}{4} 2x &+ y &- z &= 8 \\ -3x &- y &+ 2z &= -11 \\ -2x &+ y &+ 2z &= -3 \end{alignedat}$		$\left[\begin{array}{rrr\|r} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array}\right]$
$\begin{alignedat}{4} 2x &+ y &- z &= 8 \\ &\tfrac{1}{2}y &+ \tfrac{1}{2}z &= 1 \\ &2y &+ z &= 5 \end{alignedat}$	$L_2 + \tfrac{3}{2}L_1 \to L_2$ $L_3 + L_1 \to L_3$	$\left[\begin{array}{rrr\|r} 2 & 1 & -1 & 8 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 2 & 1 & 5 \end{array}\right]$
$\begin{alignedat}{4} 2x &+ y &- z &= 8 \\ &\tfrac{1}{2}y &+ \tfrac{1}{2}z &= 1 \\ & &-z &= 1 \end{alignedat}$	$L_3 + -4L_2 \to L_3$	$\left[\begin{array}{rrr\|r} 2 & 1 & -1 & 8 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & -1 & 1 \end{array}\right]$
The matrix is now in echelon form (also called triangular form)		
$\begin{alignedat}{4} 2x &+ y & &= 7 \\ &\tfrac{1}{2}y & &= \tfrac{3}{2} \\ & &-z &= 1 \end{alignedat}$	$L_2 + \tfrac{1}{2}L_3 \to L_2$ $L_1 - L_3 \to L_1$	$\left[\begin{array}{rrr\|r} 2 & 1 & 0 & 7 \\ 0 & \frac{1}{2} & 0 & \frac{3}{2} \\ 0 & 0 & -1 & 1 \end{array}\right]$
$\begin{alignedat}{4} 2x &+ y & &= 7 \\ &y & &= 3 \\ & &z &= -1 \end{alignedat}$	$2L_2 \to L_2$ $-L_3 \to L_3$	$\left[\begin{array}{rrr\|r} 2 & 1 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$
$\begin{alignedat}{4} x & & &= 2 \\ &y & &= 3 \\ & &z &= -1 \end{alignedat}$	$L_1 - L_2 \to L_1$ $\tfrac{1}{2}L_1 \to L_1$	$\left[\begin{array}{rrr\|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$

The Gaussian elimination algorithm can be applied to any $m \times n$ matrix A. In this way, for example, some 6 × 9 matrices can be transformed to a matrix that has a row echelon form like

$$T = \begin{bmatrix} a & * & * & * & * & * & * & * & * \\ 0 & 0 & b & * & * & * & * & * & * \\ 0 & 0 & 0 & c & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & d & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

Derivative of Matrix Product

$D(||x||_2^2) = 2x^T$ Note: This is because $D(||\mathbf{x}||_2^{\,2}) = 2x^T D(\mathbf{x})$ and $D(\mathbf{x}) = \mathbf{1}$ (see next line)

$D(Ax) = A$

Where *D()* is derivative with respect to *x*

Properties of Matrix Transpose

$(AB)^T = B^T A^T$

$(A - B)^T = (A^T - B^T)$