1 (a) for $P_A = 0.2$ $P_B = 0.4$ $P_C = 0.7$.

Value of N	Best value of N_1	P	Is it the correct value of p.
20	6	0.2	NO
100	31	0.7	yes
1000	170	0.7	yes
5000	787	0.7	yes.

for $P_A = 0.215$ $P_B = 0.5$ $P_C = 0.58$.

Value of N	Best value of N_1	P	Is it the correct value of P.
20	6	0.5	NO
100	33	0.58	yes
1000	326	0.58	yes
5000	1196	0.58.	yes.

1 (b) Let $\boxed{\alpha = 0.1}$

(i) $P_A = 0.2$ $P_B = 0.4$ $P_C = 0.7$.

→ when $N = 20$

average number of times correct coin was used = 12.817

average number of heads = 11.519

(ii) ~~$P_A = 0.2$~~ when $N = 100$

avg number of times correct coin was used = 86.725

avg number of heads = 64.391

→ when $N = 1000$

avg no: of times correct coin was used = 981.822

avg no: of heads = 684.191

→ when $N = 5000$

avg no: of times correct coin was used = 4975.83

avg no: of heads = 3446.53.

(ii) $P_A = 0.45$ $P_B = 0.50$ $P_C = 0.58$

→ when $N = 20$

avg no. of times correct coin was used = 8.44

avg no: of heads = 10.282

→ when $N = 100$

avg no: of times correct coin was used = 52.446

avg no: of heads = 52.697.

→ when $N = 1000$

avg no: of times correct coin was used = 799.268.

avg no: of heads = 553.591

→ when $N = 5000$

avg no: of times correct coin was used = 4833.8

avg no: of heads = 2824.23

Let $\boxed{\alpha = 0.05}$

(i) $P_A = 0.2$ $P_B = 0.4$ $P_C = 0.7$.

→ $N = 20$

avg no: of times correct coin was used = 12.316

avg no: of heads = 11.019.

→ $N = 100$

avg no: of times correct coin was used = 84.642

avg no: of heads = 63.884

→ when $\underline{N = 1000}$

avg no of times correct coin was used = 979.318

avg no: of heads = 683.824

→ when $\underline{N = 5000}$

avg no: of times correct coin was used = 4977.36

avg no: of heads = 3445.16.

(ii) $P_A = 0.45$ $P_B = 0.5$ $P_C = 0.58$.

→ when $\underline{N = 20}$

avg no: of times correct coin was used = 8.361

avg no: of heads = 10.256

→ when $\underline{N = 100}$

avg no: of times correct coin was used = 52.168.

avg no: of ~~times~~ heads = 52.508.

→ when $\underline{N = 1000}$

avg no: of times correct coin was used = 787.915

avg no. of heads = 551.996.

→ when $\underline{N = 5000}$

avg no: of times correct coin was used = 4681.29

avg no. of heads = 2624.04

$\boxed{\alpha = 0.01}$

(i) $P_A = 0.2$ $P_B = 0.4$ $P_C = 0.7$

→ when $\underline{N = 20}$

avg no: of times correct coin was used = 11.804

avg no: of heads. = 10.801

(ii) when $\underline{N = 100}$

avg no: of times correct coin was used = 81.585

avg no: of heads = 62.247

→ when $N = 100$

avg no. of times correct coins were used = 48.519

avg no. of heads = 52.089

→ when $N = 1000$

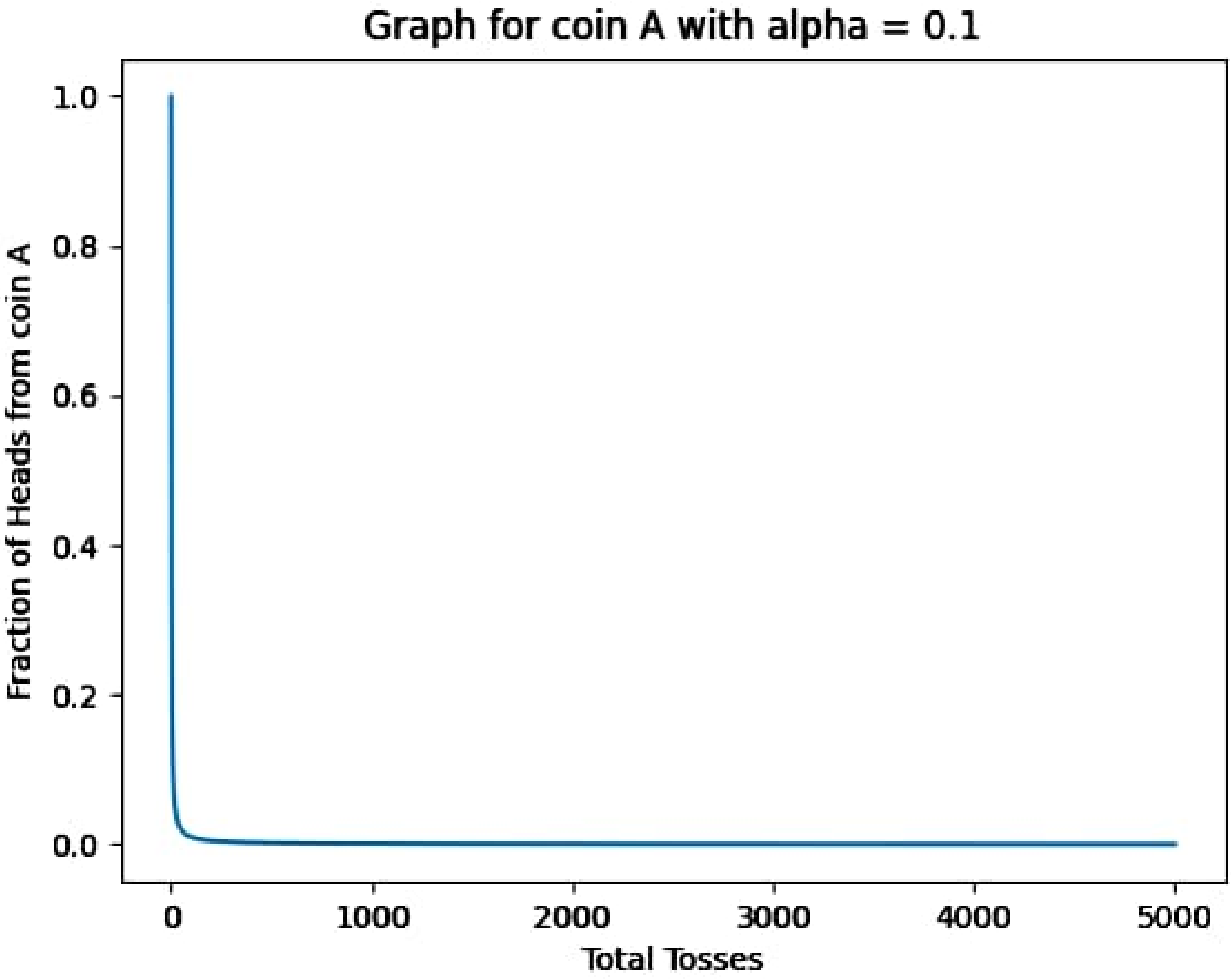
avg no. of times correct coins were used = 767.434

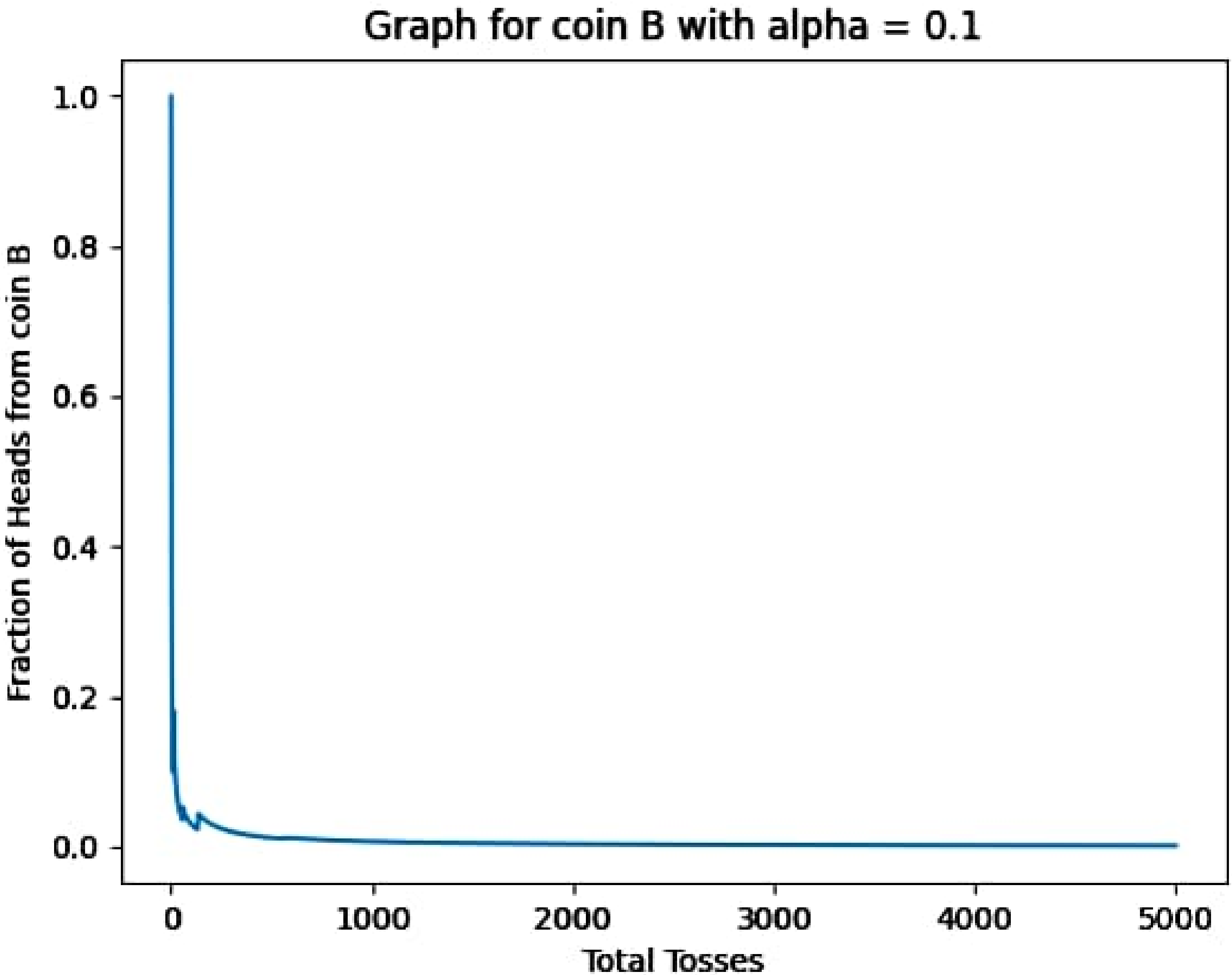
avg no. of heads = 550.282

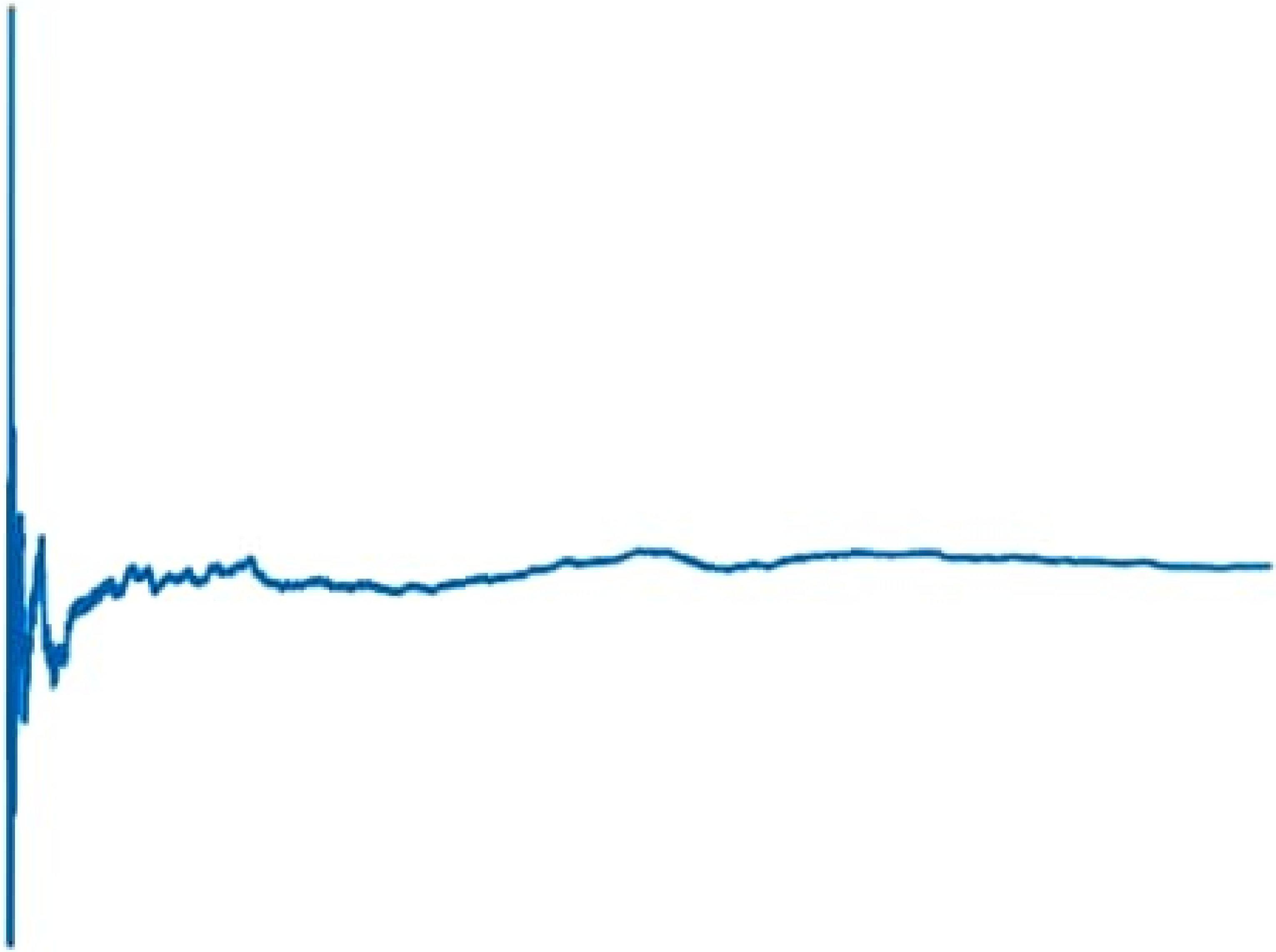
→ when $N = 8000$

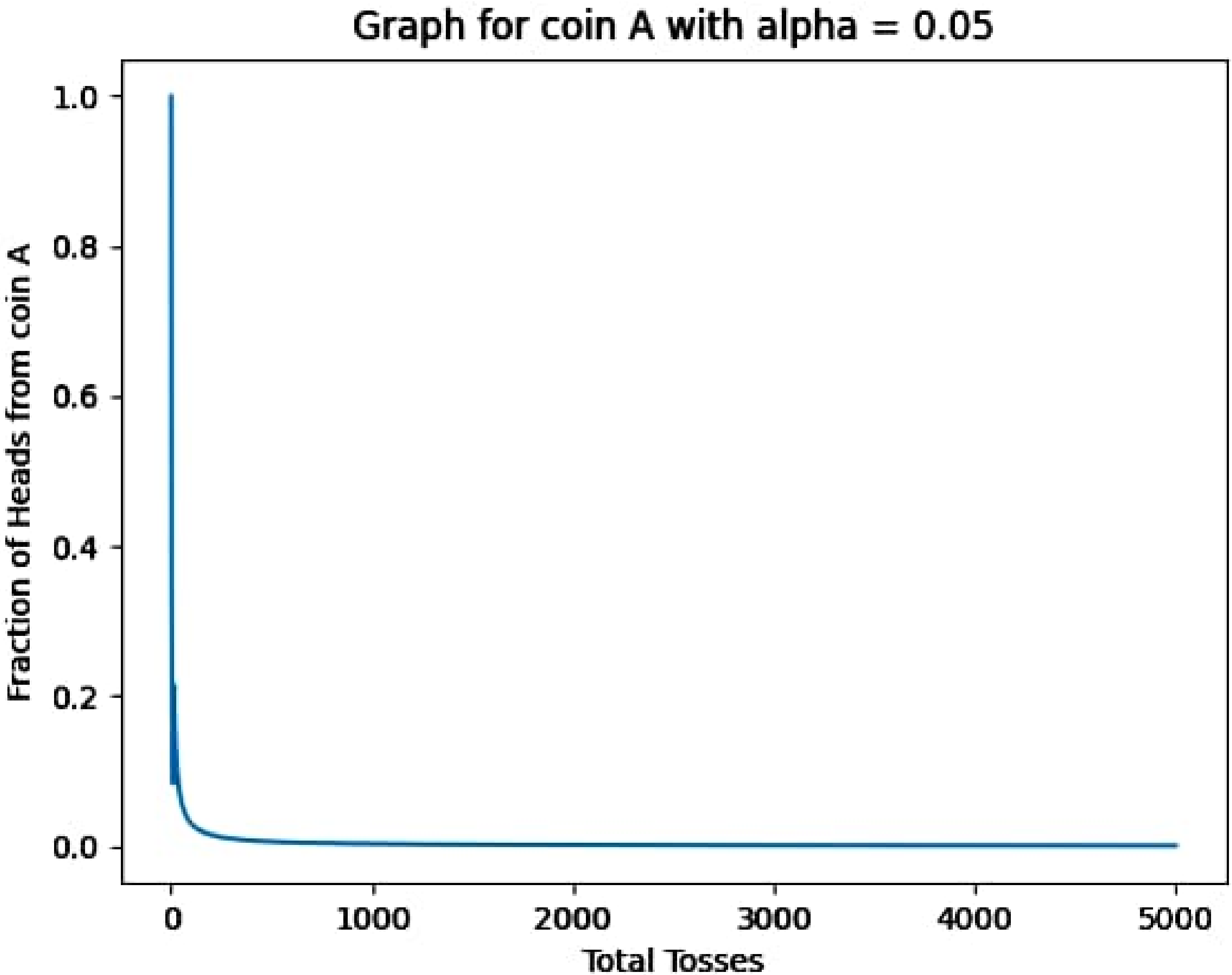
avg no. of times correct coins were used = 4621.23

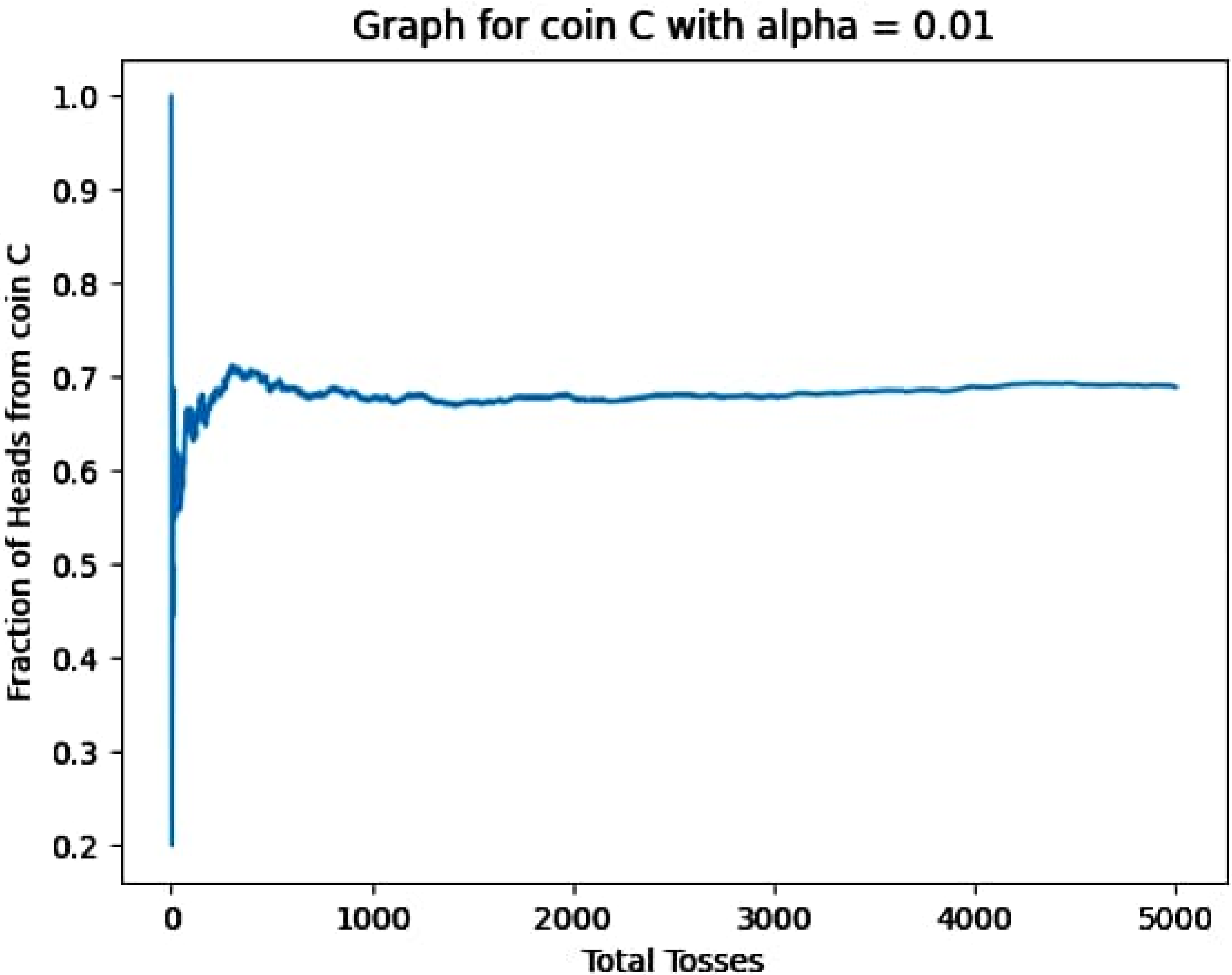
avg no. of heads = 2820.57

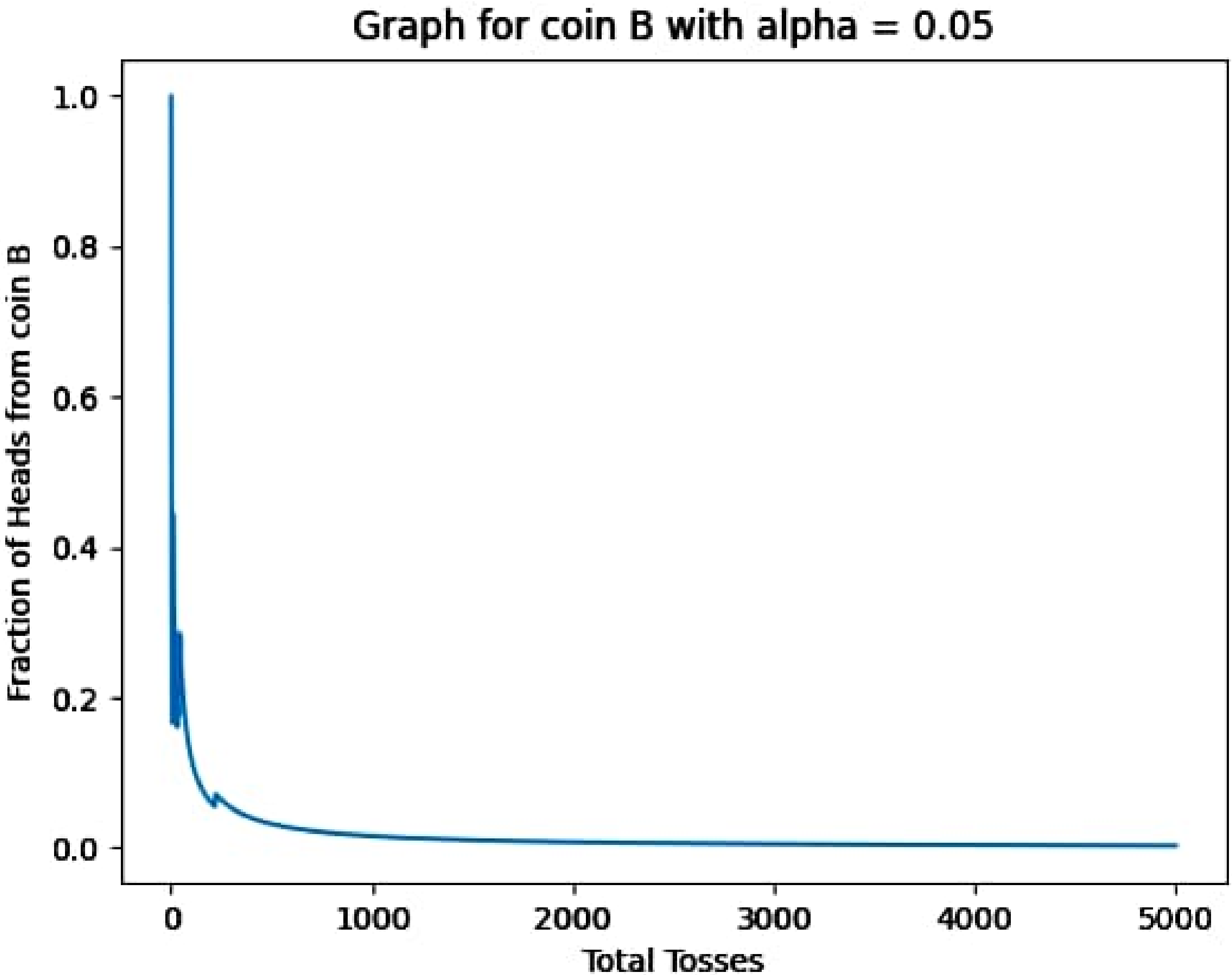

Graph for coin A with alpha = 0.1
Fraction of Heads from coin A
Total Tosses
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000

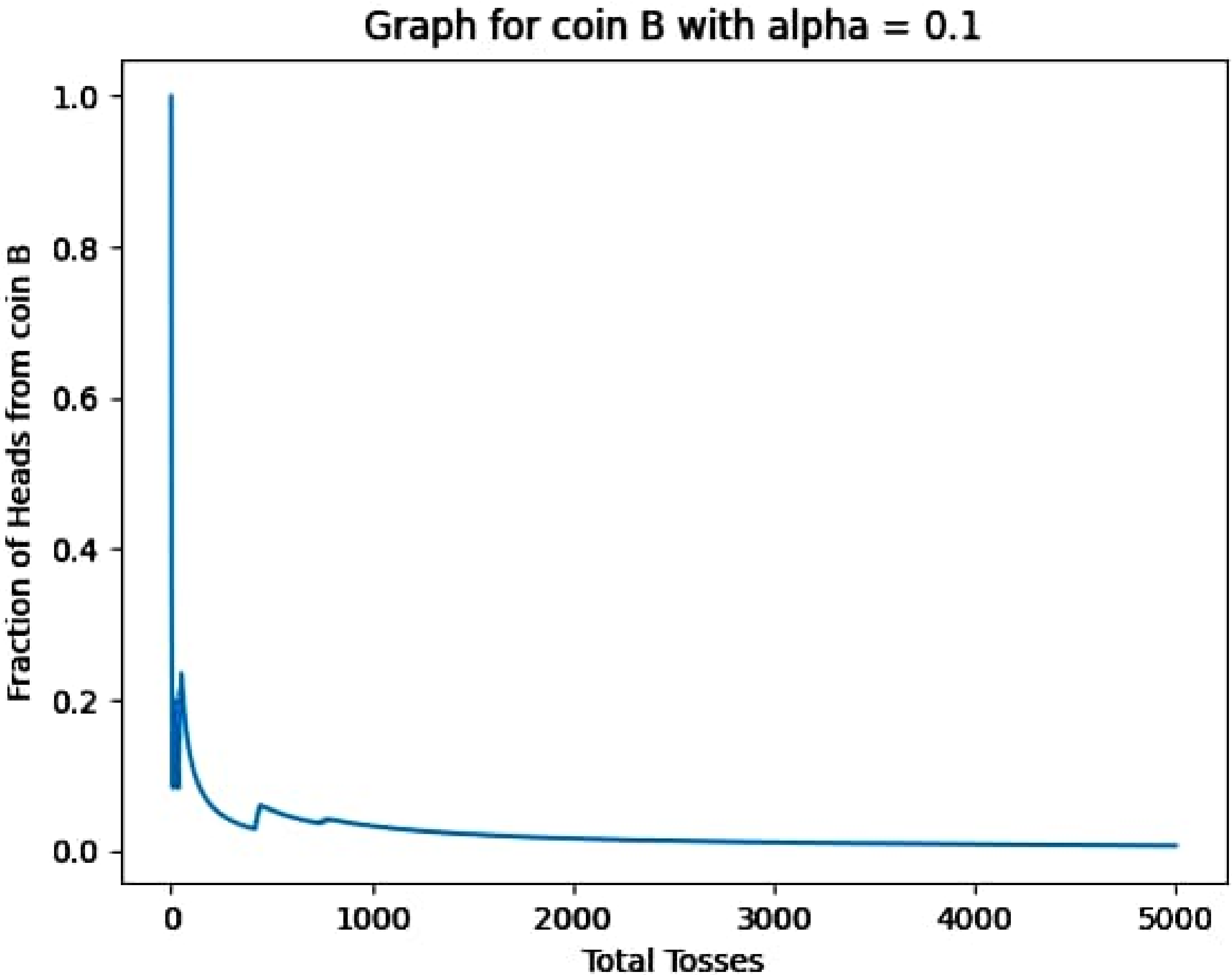

Graph for coin B with alpha = 0.1
Fraction of Heads from coin B
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000
Total Tosses

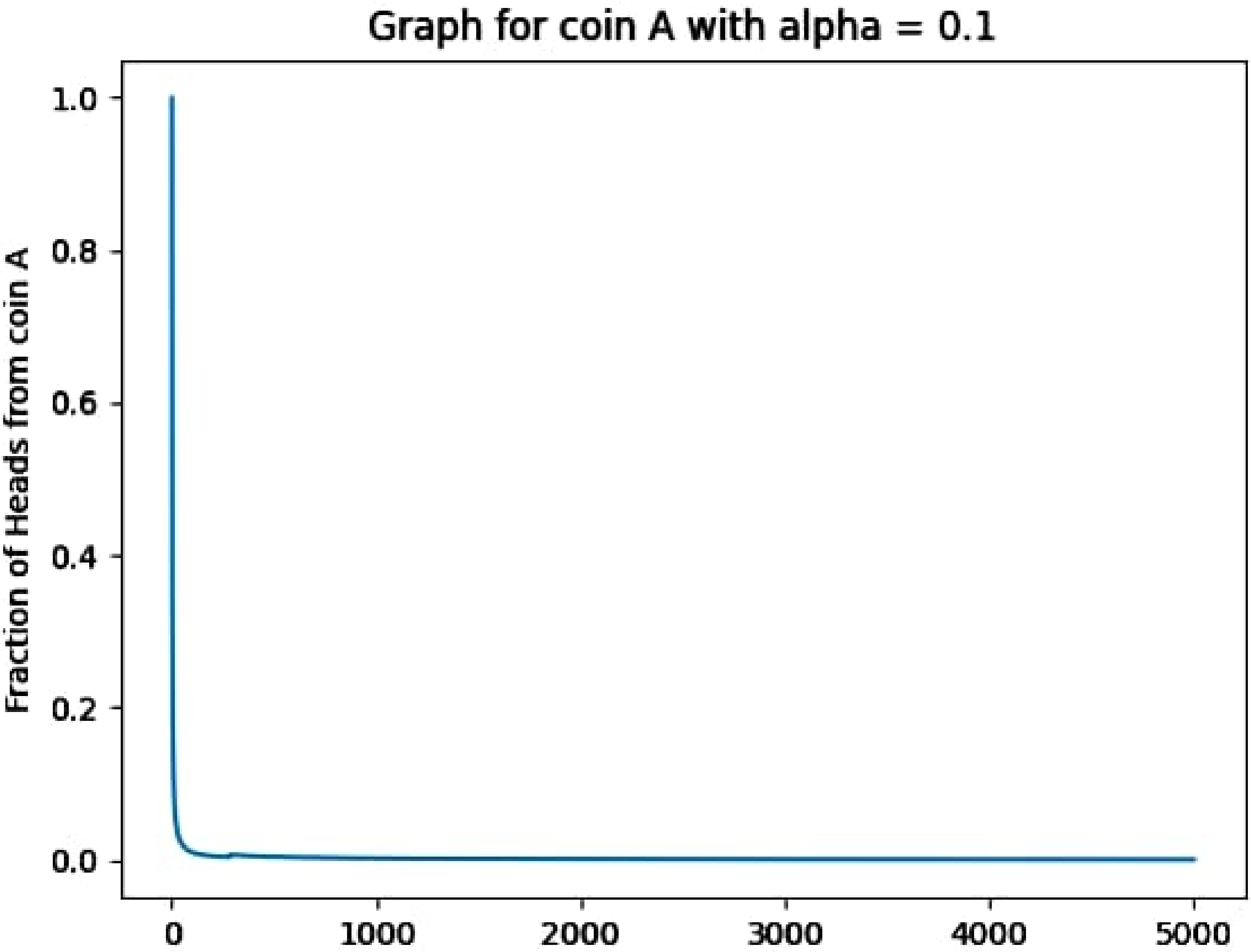


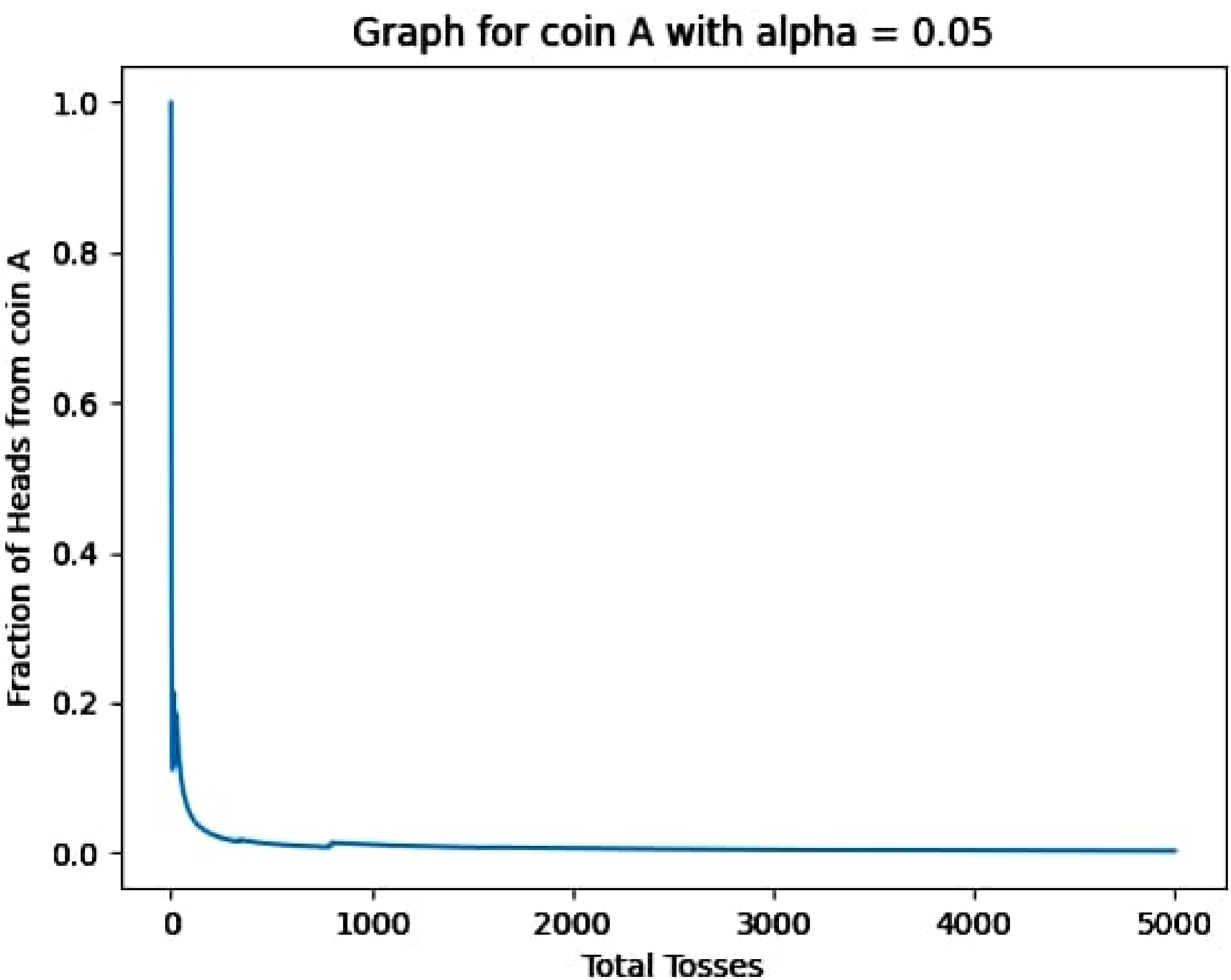

Graph for coin A with alpha = 0.05
Fraction of Heads from coin A
Total Tosses
0.0
0.2
0.4
0.6
0.8
1.0
0
1000
2000
3000
4000
5000

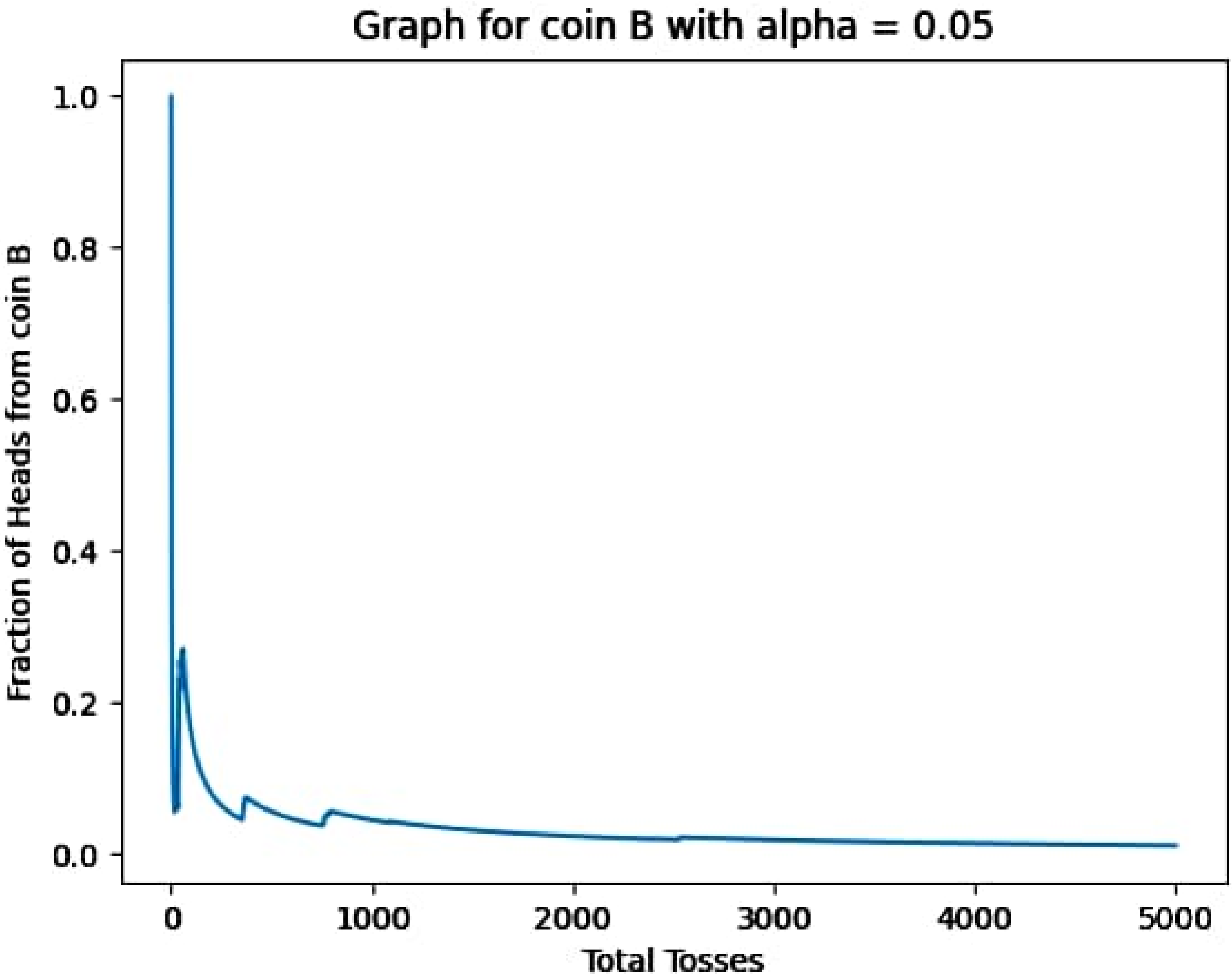

Graph for coin C with alpha = 0.01
Fraction of Heads from coin C
Total Tosses
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
1000
2000
3000
4000
5000

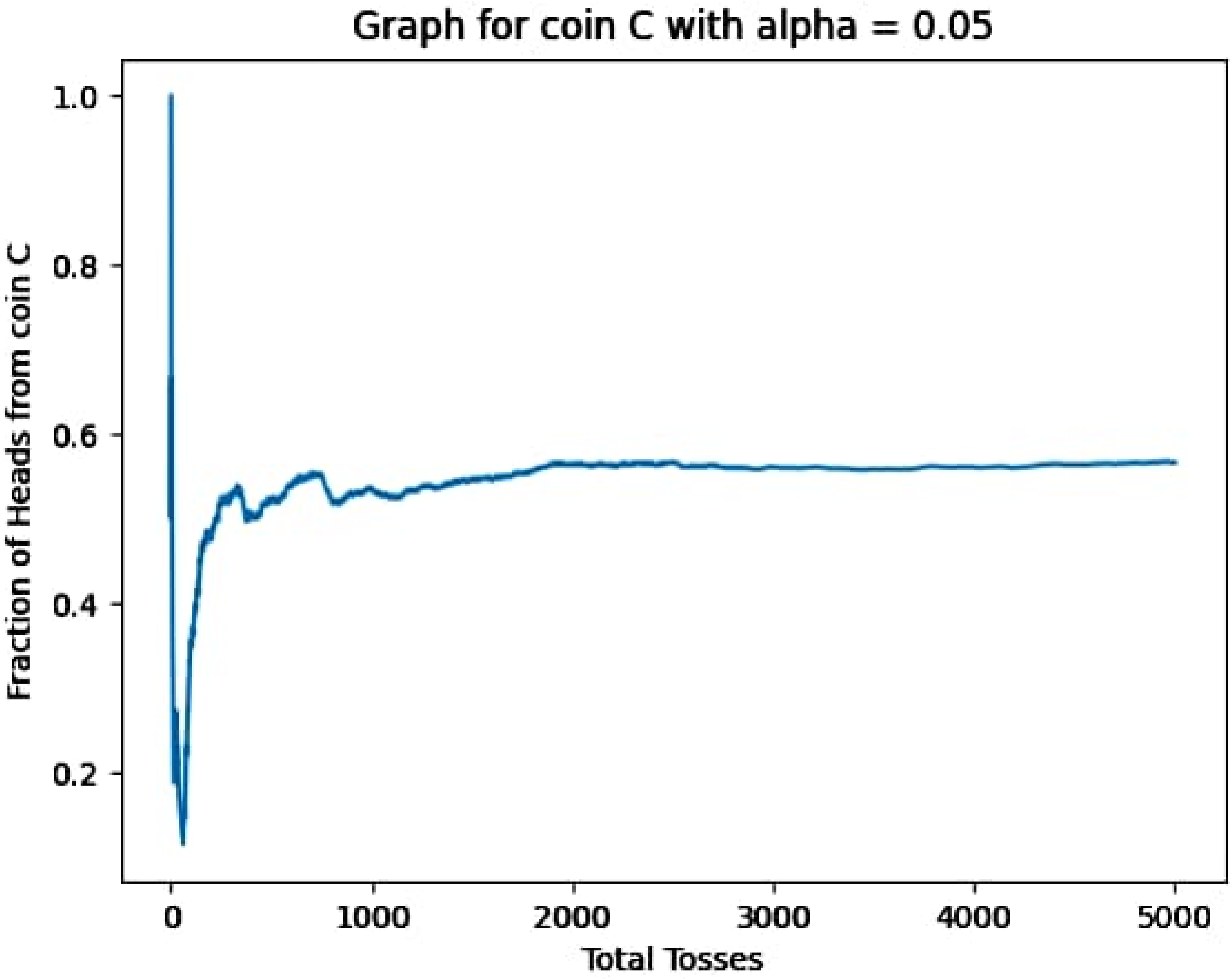

Graph for coin B with alpha = 0.05
Fraction of Heads from coin B
Total Tosses
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000


Graph for coin B with alpha = 0.1
Fraction of Heads from coin B
Total Tosses
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000

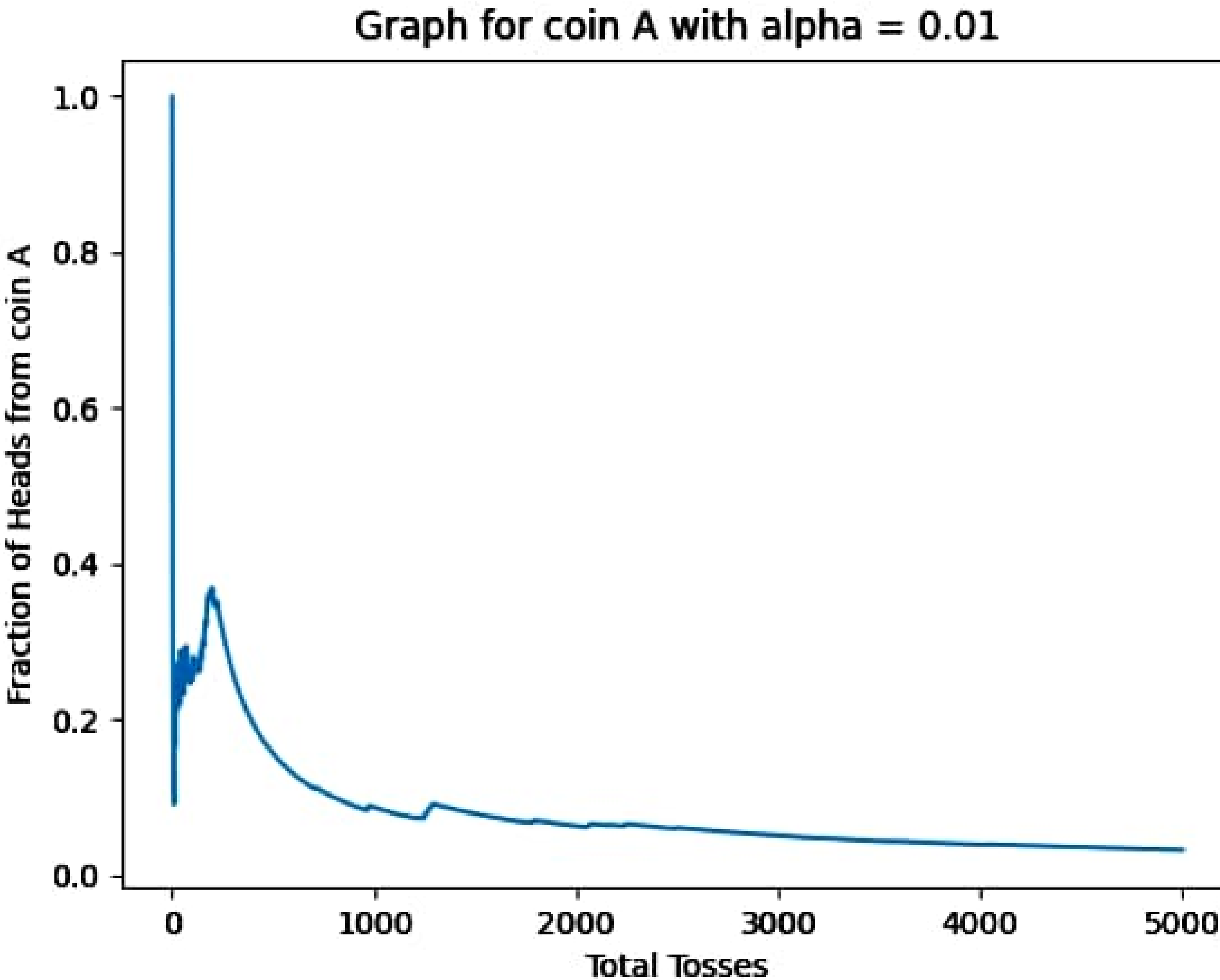

Graph for coin A with alpha = 0.1
Fraction of Heads from coin A
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000
Total Tosses

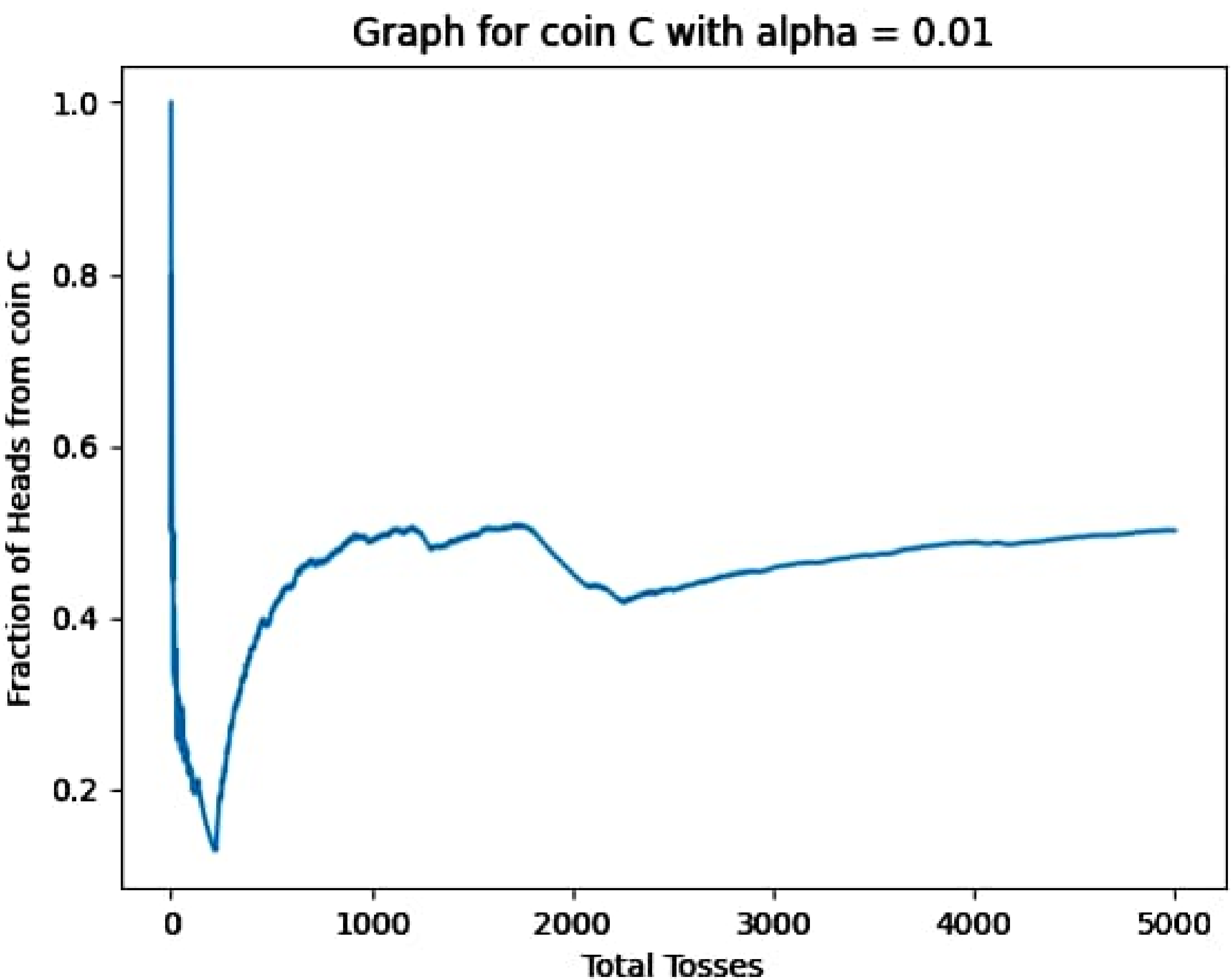

Graph for coin A with alpha = 0.05
1.0
0.8
0.6
0.4
0.2
0.0
Fraction of Heads from coin A
0
1000
2000
3000
4000
5000
Total Tosses

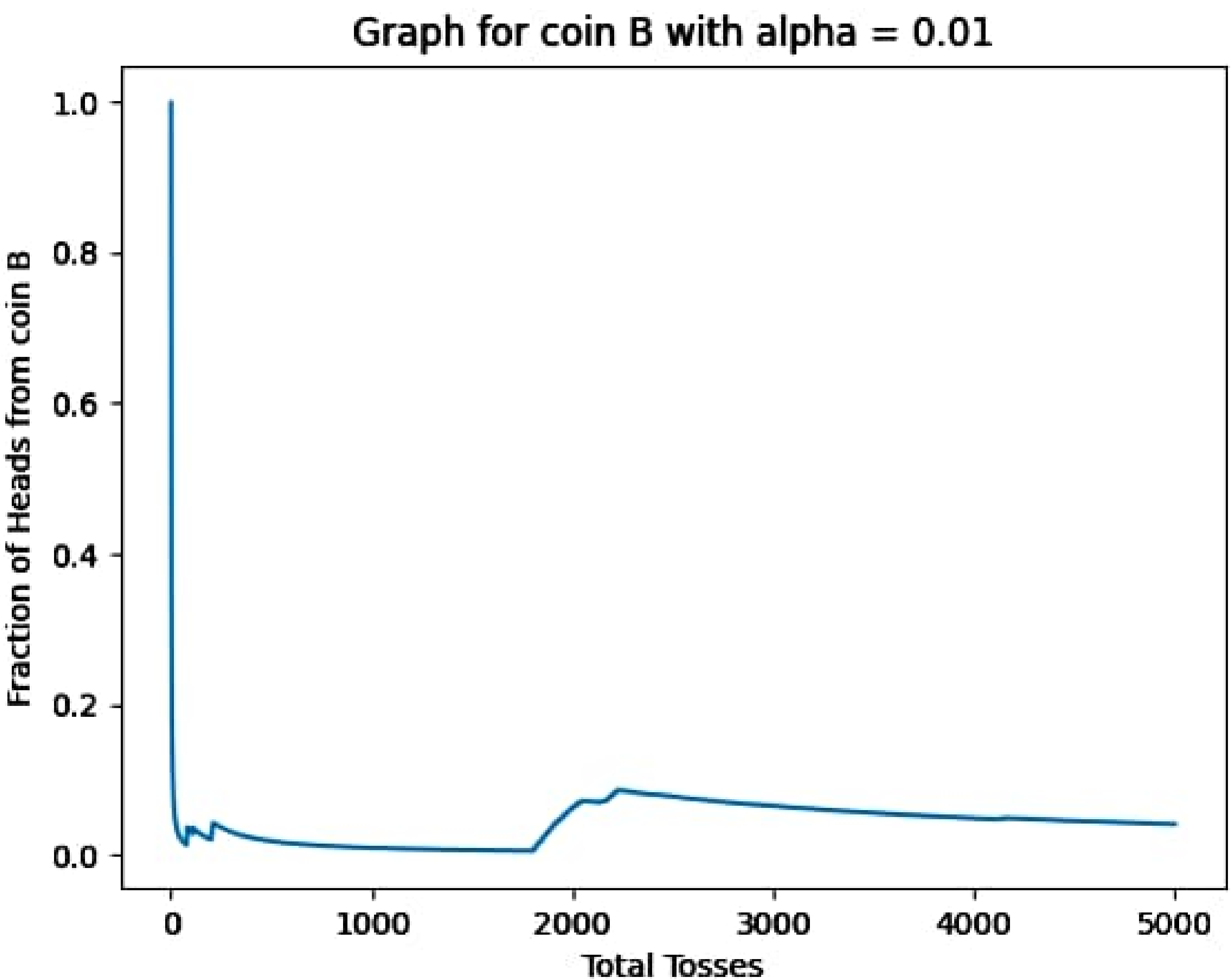

Graph for coin B with alpha = 0.05
Fraction of Heads from coin B
Total Tosses
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000



Graph for coin C with alpha = 0.05
Fraction of Heads from coin C
Total Tosses
1.0
0.8
0.6
0.4
0.2
0
1000
2000
3000
4000
5000


Graph for coin A with alpha = 0.01
Fraction of Heads from coin A
Total Tosses
0.0
0.2
0.4
0.6
0.8
1.0
0
1000
2000
3000
4000
5000


Graph for coin C with alpha = 0.01
Fraction of Heads from coin C
Total Tosses
1.0
0.8
0.6
0.4
0.2
0
1000
2000
3000
4000
5000



Graph for coin B with alpha = 0.01
1.0
0.8
0.6
0.4
0.2
0.0
Fraction of Heads from coin B
0
1000
2000
3000
4000
5000
Total Tosses

n is the random variable denoting no. of positives from 10 tubes.

Let the no. of positives in 200 tubes (i.e. Pos) be a random variable of binomial distribution

$$\therefore \quad Pr(Pos = x) = {}^{200}C_x \, p^x (1-p)^{200-x}$$

where 'p' is the individual probability of a tube being positive

Finding p

Given that $n = k$ (k can be $0, 1, 2, 3, 4$)

So, Individual prob of a tube being +ve $\Big\} = \frac{k}{10}$

$$\Rightarrow \quad p = \frac{k}{10}$$

$$\therefore \quad Pr_k(Pos = x) = {}^{200}C_x \left(\frac{k}{10}\right)^x \left(1 - \frac{k}{10}\right)^{200-x}$$

(a) Now, $E(Pos \mid n = k) = \sum_{x=0}^{200} x \, Pr_k(Pos = x)$

$$= \sum_{x=0}^{200} x \; {}^{200}C_x \left(\frac{k}{10}\right)^x \left(1 - \frac{k}{100}\right)^{200-x}$$

For $k = 2$, $E(Pos \mid n = 2) = \sum_{x=0}^{200} x \; {}^{200}C_x (0.2)^x (0.8)^{200-x}$

$$E(Pos \mid n = 2) = 40 \longrightarrow \text{①}$$

Similarly, $E(Pos \mid n = 1) = 20$

$E(Pos \mid n = 3) = 60$

$E(Pos \mid n = 4) = 80$

(b) To find: $Pr(Pos > E(Pos)+1 \mid k=2)$

For $k=2$, $E(Pos) = 40$

$$Pr(Pos > 41 \mid k=2) = \sum_{x=42}^{200} {}^{200}C_x (0.2)^x (0.8)^{200-x}$$

$$\approx 0.3891$$

(c) If total no. of tubes were 400 & tested tubes is 20, then:

$$Pr(Pos = x) = {}^{400}C_x\, p^x (1-p)^x$$

Here, p will be,

$$p = \frac{k}{20} \quad \text{(Since, no. of tested tubes is 20)}$$

$$\therefore \; Pr_k(Pos = x) = {}^{400}C_x \left(\frac{k}{20}\right)^x \left(1-\frac{k}{20}\right)^{400-x}$$

For $k=2$, $E(Pos \mid n=2) = \sum_{x=0}^{400} x \; {}^{400}C_x (0.1)^x (0.9)^{400-x}$

$$E(Pos \mid n=2) = 40 \rightarrow (2)$$

From (1) & (2), it is observed that expectation values remain unchanged on changing $200 \rightarrow 400$ &
$10 \rightarrow 20$

So, $E(Pos \mid n=1) = 20$
$E(Pos \mid n=3) = 60$
$E(Pos \mid n=4) = 80$, for this case as well

$Pr(Pos > E(Pos)+1 \mid k=2) = Pr(Pos > 41 \mid k=2)$ (Since $E(Pos \mid k=2) = 40$)

$$= \sum_{x=42}^{400} {}^{400}C_x (0.1)^x (0.9)^{400-x}$$

$$\approx 0.39329$$

There is a marginal (infact negligible) increase in the confidence of inference from the first case to second case. This change is primarily due to the increase in number of total tubes tested. As the number of tubes tested increases, the confidence of inference tend to increase, though very marginally.