Q1) $C_{xx} = R_{xx}$

$$\hat{x}(n+1) = aX(n)$$

$$E\left((x(n+1) - ax(n))^2\right)$$

$$= E[x^2(n+1) + a^2x^2(n) - 2X(n)X(n+1).a]$$

$$= E[x^2(n+1)] + a^2E[x^2(n)] - 2E[X(n)\,X(n+1)]$$

$$= C_{xx}(0) + a^2C_{xx}(0) - 2aC_{xx}(1)$$

$$= (1+a^2)C_{xx}(0) - 2aC_{xx}(1).$$

Diff. w.r.t,

$$2aC_{xx}(0) - 2C_{xx}(1) = 0$$

$$\therefore \boxed{a = \frac{C_{xx}(1)}{C_{xx}(0)}}$$

(2) $x(t)$ is real, bandpass

$$S_{xx}(\omega) = 3\left(u(\omega - 9000) - u(\omega - 11000)\right) + 400\delta(\omega - 10^4)$$

$$R_x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)e^{j\omega t}\,d\omega.$$

for $\tau = 0$ $\quad R_x(t) = E[x^2(t)]$

$$E[x^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(j\omega)\,d\omega$$

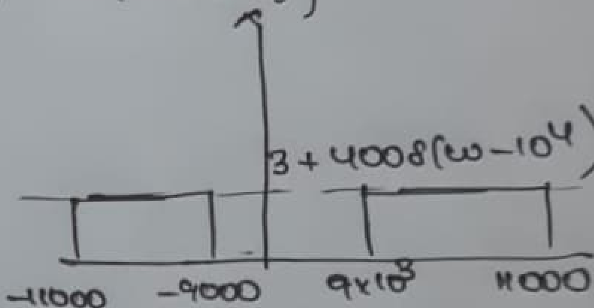
$$= \frac{1}{2\pi} \times 2 \times 2000\,[3 + 400\delta(\omega - 10^4)]$$

$$\boxed{E[x^2(t)] = \frac{2000}{\pi}[3 + 400\delta(\omega - 10^4))}$$

Now $R_x(t) \to \lim_{\tau \to \infty} R_x(\tau) = E^2[x(t)]$

$$= \frac{k}{t}[\sin(11000\pi t) - \sin(9000\pi t)]$$

$$\boxed{\lim_{\tau \to \infty} R_x(t) = 0}$$



$$\Rightarrow \boxed{E[x(t)] = 0}$$

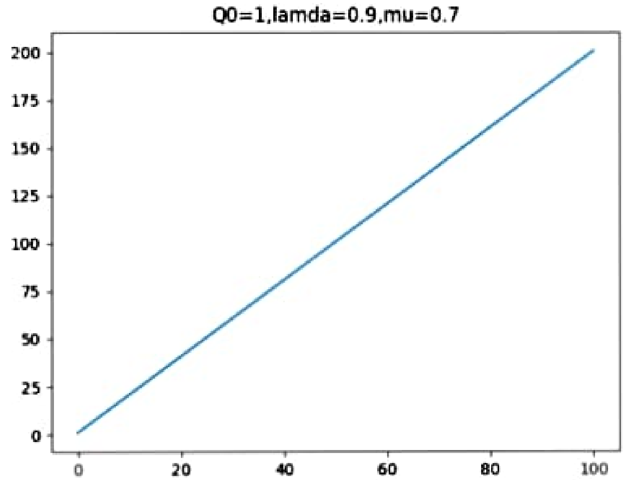
Given that,

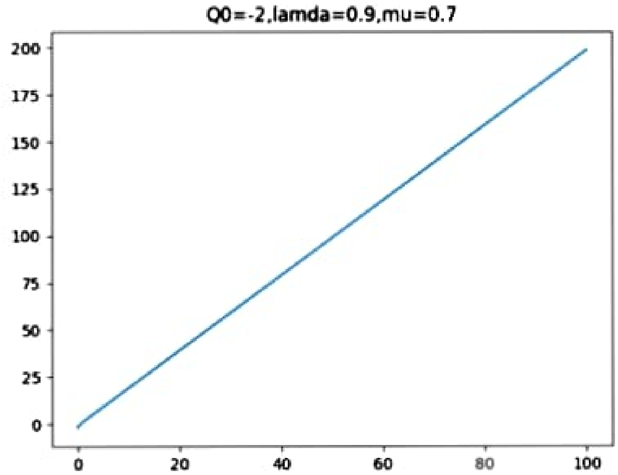
$$Q_n = \begin{cases} Q_{n-1} + 1 & \text{prob } \lambda(1-\mu) \\ \max\{Q_{n-1}-1, 0\} & \text{prob } (1-\lambda)\mu \\ Q_{n-1} & \text{prob } \lambda\mu + (1-\lambda)(1-\mu) \end{cases}$$

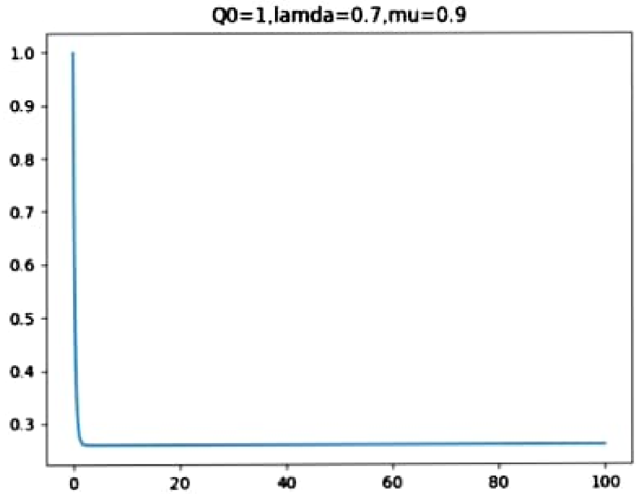
The simulations for the given values of μ & λ are given below. Here the simulation is done by plotting the expectant value for each instant of time, as a sequence. We take a time frame of say 10 seconds.

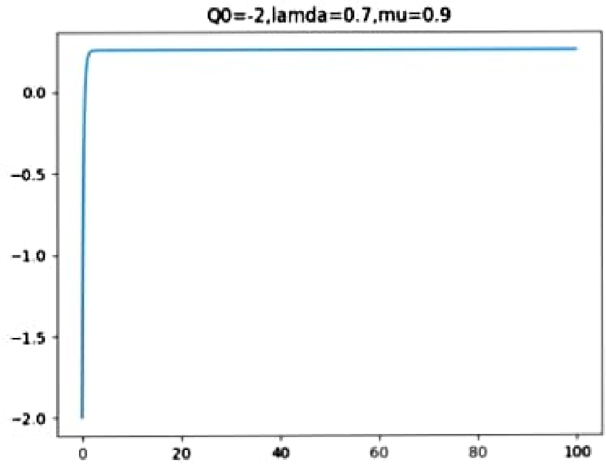
~~It~~ From the simulations it can be understood that, when $\mu = \lambda$, we get a stationary, ergodic sequence.

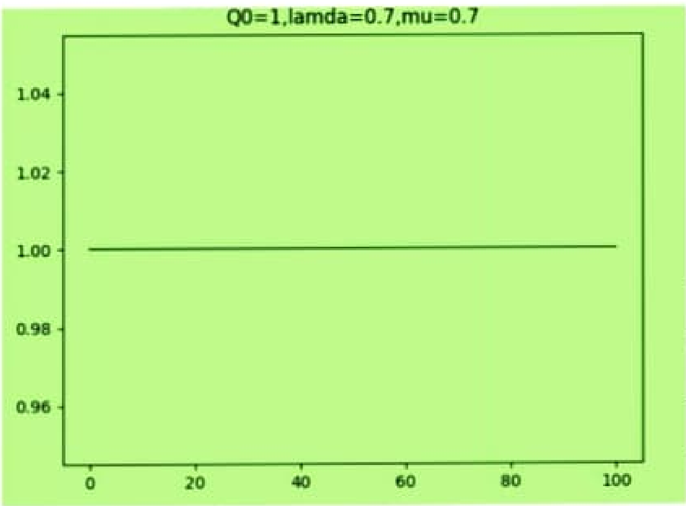
When the sequence is stationary, ergodic, the time-average and ensemble average are equal and is equal to q_0.

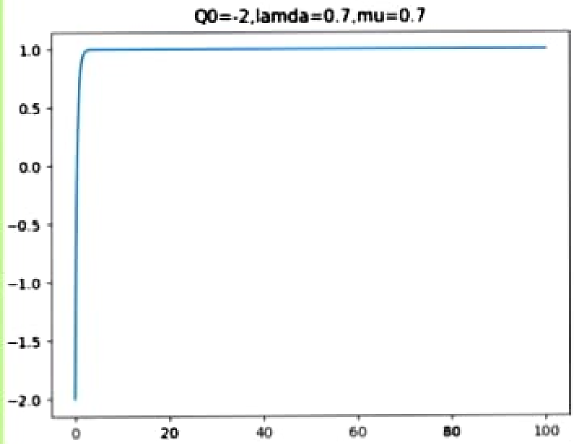












The simulation which gives a stationary and ergodic sequence is highlighted in green. Its **λ=0.7, μ=0.7**

Also, the choice of **Q_0** should be **non-negative** to simulate stationary, ergodic sequence.

4) X_n is a random sequence that takes values in the sets $\{1,2,3,4,5,6\}$. evolves as follows:

$$P_r(X_n = j \mid X_{n-1} = i) = P_{ij}$$

~~a) $P_{11} = 0.3$ for~~

i) when $X_n = 4$ for some 'n' $X_k = \{3,4\}$ $\forall\, k > n$, same for $X_n = 3$

ii) when $X_n = 5$ for some 'n' then $X_k = \{5,6\}$ $\forall\, k > n$, same fon $X_n = 6$.

ie For this set of probabilities the sequence may get stuck on 3 or 4. OR get stuck on 5 or 6

It is clear that when sequence gets stuck on 3 or 4, it means it will tend to 3.5 for higher N values and when sequence gets stuck on 5 or 6, it means it will tend to 5.5 for higher N values.

b) The probabilities are same in case a) except $p_{52} = 0.2$ and $p_{56} = 0.5$

So naturally simulation of $\frac{\sum_{n=1}^{N} X_n}{N}$

tended to 3.5 for appropriate values of N.

c). If we find for X_2 being 2, 3, 4, 5 or 6, it will come to be $1/6$

d). In this case ~~the~~ the sequence doesn't get 'stuck' and takes all values among $\{1, 2, 3, 4, 5, 6\}$.