Propositional logic

Argument: construct with one or more **premises** and one **conclusion**.

Valid argument: If the premises are true the conclusion is true.

Facts: everything there is the case (in the world).

Sentences capture (represent) facts - this is expressed by the semantics of a language. Assessment of the truth value.

Syntax: possible configurations / rules for sentence forming.

In the world: facts follow from facts

In the language: sentences entail sentences

The syntax and semantics determine the logic

Knowledge Base (KB): collection of sentences

Inference:

Given a knowledge base KB, inference is used

To generate new sentences

To check if a sentence can be entailed (obtained).

Sound inference = Truth preserving

Inference procedure that generates only entailed sentences, i.e. sentences that correspond to the facts that follow from the facts corresponding to the KB.

Proof: **steps in a sound inference procedure.**

Complete inference: **can prove ALL sentences entailed.**

LOGIC:

- **syntax and semantics**
- **a proof theory**

Propositional logic

- symbols :
    - logical constants (true and false)
    - propositions (facts): p, q, r
- operators (Boolean connectives)
    - and (^)
    - or (v)
    - implication (→)
    - not (~)
    - equivalence (← →)
- BNF grammar of sentences in propositional logic:

Sentence := AtomicSentence | ComplexSentence

AtomicSentence := True | False | p | q | r | ...

ComplexSentence := (Sentence)
| Sentence Connective Sentence
| ~Sentence

Connective := ^ | v | ← → | →

Order of precedence is needed to solve ambiguities.

Also () are used to alter this order.

Semantics of the propositional logic (truth tables):

And (^):

p ^ q		p	
		T	**F**
q	**T**	**T**	**F**
	F	**F**	**F**

Implication (p → q) is very important and its truth table is somewhat strange (no causality between its components is required):

p → q		p	
		T	**F**
q	**T**	**?**	**?**
	F	**F**	**?**

It is easy to accept F in the box above. But what should we put in the other boxes?

Another entry which is easier to accept, but not totally justified is as shown below (we would not want to put an F when both p and q are T):

p → q		p	
		T	**F**
q	**T**	**T**	?
	F	**F**	?

Why is this not totally justified? Because then we can form somewhat nonsensical implications in which both p and q are true but also totally unrelated! For example, an implication of the form, "***If water is*** H_2O ***then grass is green***" will be evaluated to true (T).

Next, when p is F and q is T: we will not want to put and F here. For example, consider the implication ***"If there is wine in a glass then there is alcohol in it"*** and assume that in fact the glass contains whiskey: antecedent is false; consequent is true. So, it does not seem right to put an F; but it does not feel very good to put a T either (think of an example).

p → q		p	
		T	**F**
q	**T**	**T**	**T**
	F	**F**	**?**

Finally, when both p and q are F what should we put? Should we put an F? Assume the glass is empty. Then p is F and q is F, however, these cannot really negate that the conditional is true! So we obtain

p → q		p	
		T	**F**
q	**T**	**T**	**T**
	F	**F**	**T**

One way to think about the implication "p → q":

"If p is true then claim that q is true otherwise we make no claim"

or

"p → q is true just in case we do not have a true antecedent and a false consequent."

Other sources for implication: ***Paradoxes of the material implication***
(http://www.earlham.edu/~peters/courses/log/mat-imp.htm)

Truth functional operators

An operator is called truth functional when it is possible to deduce the truth value of the result from the truth values of the operands.

All of the operators (connectives) above are truth functional.

Use of truth tables to check validity (semantic method)

An argument is valid (sound) if whenever all the premises are true the conclusion is also true.

Inference rules for propositional logic and why they are valid:

|=: turnstile for semantic inference (read this as "therefore")

Modus Ponens or Implication Elimination (IE)

p → q, p |= q (Read as: "p implies q and p, therefore q")

p	q	p	p→q	q
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

And-Elimination

$p \wedge q \models p$, also $p \wedge q \models q$

p	q	$p \wedge q$	q
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

$p \wedge q \models p \vee q$ it is valid because whenever the premise

$p \wedge q$ is true the conclusion $p \vee q$ is also true

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

$p \vee q \models p \wedge q$ is NOT valid because there are rows in the truth table (rows 2 and 3) where the premise is true, but the conclusion is false.

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

And-Introduction

p, q |= p ∧ q is a valid argument

Prop letters		premises		conclusion
p	q	**p**	**q**	**p ∧ q**
T	T	T	T	T
T	F	T	F	F
F	T	F	T	F
F	F	F	F	F

Or-Introduction

p |= p ∨ q

Double-negation-elimination

~~p |= p

Truthtable(p→q) is identical to the Truthtable(~p v q)

p → q		p	
		T	F
q	T	T	T
	F	F	T

~p v q		p	
		T	F
~p		F	T
~p v q		T	T
q	T	T	T
	F	F	T

~p v q is the clausal form of the implication p→q

Unit Resolution

From a disjunction in which one term is false infer the other term:

p ∨ **q, ~p |= q**

p	q	pvq	~p	q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

p ∨ **q, ~q |= p**

~p ∨ **q, p |= q** : this is in fact, the clausal form of the modus ponens (implication elimination)

p	q	~pvq	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Resolution

p ∨ **q, ~p** ∨ **r |= q** ∨ **r**

In implicative form (that is using p →q same as ~p v q) we have:

~(~p) v q: ~p → q

p v ~(~q) same as ~(~q) v p same ~q →p

~p v r same p → r

~q → r same ~(~q) v r is *the same* q v r

~q → p, p → r |= ~q → r

same as

~q → p: ~(~q) ∨ p = q ∨ p
p → r: ~p ∨ r

p ∨ q, ~p ∨ r |= q ∨ r

So, we have the table:

Propositional variables			Aux	Premises		Conclusion
p	**q**	**r**	**~p**	**p ∨ q**	**~p ∨ r**	**q ∨ r**
T	T	T	F	T	T	T
T	T	F	F	T	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Some examples/exercises

Construct truth tables for:

- **p** ∨ **~p**
- **p**
- **p → ~p**

(p →q) → (~q → ~p) material implication

p	q	~p	~q	p→ q	~q→~ p	(p→q) → (~q →~p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Modus Tollens

(p →q) |= (~q → ~p) logical implication

propositions		Auxilliary steps		premise	Conclusion
p	**q**	**~p**	**~q**	**p→q**	**~q→~p**
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

(p →q) |= (~q → ~p) is a valid argument

- **$(p \rightarrow q) \text{->} (q \rightarrow p)$**
- **$(p \rightarrow q) \text{->} (\sim p \vee q)$**
- **$p \rightarrow (p \wedge q)$**

p	q	p^q	p→p^q
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

$p \models (p \wedge q)$: invalid argument

propositions		premise	Cconclusion
p	q	p	p^q
T	T	T	T
T	F	T	F
F	T	F	F
F	F	F	F

p1 v p2 v …. v pn is true whenever there exists I such that pi is true

- **$(p \wedge (q \vee \sim q)) \rightarrow ((p \wedge q) \vee (\wedge \sim q))$**

Define: p ←→ q (where ←→ means if and only if) by

$(p \rightarrow q) \wedge (q \leftarrow p)$

Construct truth tables for:

- $p \leftrightarrow \sim p$
- $(p \wedge q) \leftrightarrow (q \wedge p)$

Evaluate the following arguments:

- $p \rightarrow q, \sim q \models p$
- $p \rightarrow \sim q \models q \rightarrow \sim p$
- $p \rightarrow \sim q \models \sim(p \rightarrow q)$
- $p \models p \rightarrow q$
- $p \models q \rightarrow p$
- $p \vee q \models \sim(\sim p \wedge \sim q)$

Formalize the following arguments and then test them for validity

- *If Diane diets, Diane will get slim. Diane diets. So Diane gets slim.*

  Solution:

  p = Diane diets;
  q= she will get slim (same as Diane will get slim)

  The argument becomes:

  $p \rightarrow q, p \models q$

Validity: VALID; it is actually Modus Ponens

- *If the Devil has no redeeming graces he is thoroughly bad. Hence if he is thoroughly bad he has no redeeming graces.*

Solution:

Set:

p = the Devil has no redeeming graces;
q= he (the Devil) is thoroughly bad

The argument becomes:

$$p \rightarrow q \models q \rightarrow p$$

Validity : NOT valid

p	q	p→q (premise)	q→p (conclusion)
T	T	T	T
T	F	F	T
F	T	**T**	**F**
F	F	T	T

In the shaded row the premise of the argument is true but the conclusion is false. Therefore the argument is NOT valid.

- *God is all good and all powerful. But if he is all powerful and good there can be no evil. But there is plenty of evil. So God is not all good or God is not all powerful.*

  Solution:
  p = God is all good;
  q= God is all powerful;
  r = there can be no evil;

The argument becomes:

$$p \wedge q \rightarrow r, \sim r \models \sim p \vee \sim q$$

Validity: valid

p	q	r	p∧q	p∧q=>r (premise)	~r (premise)	~p	~q	~p ∨ ~q (conclusion)
T	T	T	T	T	F	F	F	F
T	T	F	T	F	T	F	F	F
T	F	T	F	T	F	F	T	T
T	F	F	F	**T**	**T**	F	T	**T**
F	T	T	F	T	F	T	F	T
F	T	F	F	**T**	**T**	T	F	**T**
F	F	T	F	T	F	T	T	T
F	F	F	F	**T**	**T**	T	T	**T**

- *April is a UC student. So she either is a UC student or she is intelligent.*

- *Logic is **either** too boring **or** too difficult. For **either** it is part of mathematics **or** it is part of philosophy. And **unless** it is NOT part of mathematics it is too difficult. **Only if** it is boring will it be part of philosophy.*

p: Logic is boring
q: Logic is difficult
r: logic is part of mathematics

s: logic is part of phil

p unless q