**Homework Assignment #4**

**Assigned on October 11, 2022**

**Due on Canvas October 21, 2022**

**by 11:59PM**

**50 points**

This is a ‘paper-and-pencil’ assignment.  Please type your solutions

**Problem 1.**  (10 points) Let r, e, and c be propositional letters as follows:

r: “a person is a radical”,

e: “a person is electable”,

c: “a person is conservative”

Consider the following assertion “***a person who is a radical is electable if he/she is conservative, but otherwise the person is not electable***”.

1. Which of the following are the correct representations of this assertion?

Use &, V, ~, 🡪, 🡨🡪 to stand for conjunction, disjunction, negation, implication and bidirectional implication respectively.

1. (r & e) 🡨🡪 c
2. r --> (e 🡨🡪 c)
3. r🡪 ((c 🡪 e) V ~e)

(b) Which sentence in (a) can be expressed as a Horn clause? Explain your answer.

**Problem 2. (10 points)**Consider the following argument, where food, drinks, party denote propositions.

**[(food 🡪party) V (drinks 🡪 party)] |= [(food & drinks) 🡪 party]**

1. Use truth tables to determine whether the argument is **Valid**
2. Convert the left hand-side and the right hand-side in CNF (conjunction of clauses/disjunctions) and show how the results confirm your answer in part (a)
3. Prove your answer to (a) using resolution.

**Problem 3. (10 points)**Consider the following argument, where food, drinks, party denote propositions.

**[(food 🡪party) V (drinks 🡪 party)] |- [(food & drinks) 🡪 party]**

Investigate the validity of this argument using syntactic proofs.

**Problem 4. (5 points)**Consider now the sentence, where food, drinks, party denote propositions.

**[(food 🡪party) V (drinks 🡪 party)] 🡪 [(food & drinks) 🡪 party]**

Use truth tables to investigate whether this sentence is

1. **Satisfiable** (i.e., there exists an assignment of truth values which make it true)
2. **Unsatisfiable** (i.e., there is no assignment of truth values which make it true)

**Problem 4.  (15 points)**

Suppose you are given the following axioms:

1. 0 ≤ 4.
2. 6 ≤ 8.
3. ∀x x≤ x.
4. ∀x x≤ x + 0.
5. ∀x x+ 0 ≤ x.
6. ∀ x, y x + y ≤ y + x.
7. ∀ w, x, y, z w ≤ y ∧ x ≤ z ⇒ w + x ≤ y + z.
8. ∀ x, y, z x ≤ y ∧ y ≤ z ⇒ x ≤ z

(a) Give a **backward-chaining** proof of the sentence 6 ≤ 4 + 8. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

(b) Give a **forward-chaining** proof of the sentence 6 ≤ 4 + 8. Again, show only the steps that lead to success.