I. INTRODUCTION

Methods and applications of multiple objective decision making and multiple attribute decision making for a single decision maker have been studied and classified systematically in Hwang and Masud [H55]: Multiple Objective Decision Making - Methods and Applications, and Hwang and Yoon [H56]: Multiple Attribute Decision Making-Methods and Applications. This study - group decision making under multiple criteria - is a sequel to the above works.

Moving from a single decision maker to a multiple decision maker setting introduces a great deal of complexity into the analysis. The problem is no longer the selection of the most preferred alternative among the nondominated solutions according to one individual's (single decision maker's) preference structure. The analysis must be extended to account for the conflicts among different interest groups who have different objectives, goals, criteria, and so on.

Group decision making under multiple criteria includes such diverse and interconnected fields as preference analysis, utility theory, social choice theory, commttee decision theory, theory of voting, general game theory, expert evaluation analysis, aggregation of qualitative factors, economic equilibrium theory, etc.

The problems of group decision making under multiple criteria are widely varied. However, even the range of different problems which are considered here share some common characteristics which are outlined in the following three paragraphs.

Multiple criteria/objectives/attributes. Each problem has multiple criteria/objectives/attributes. Each decision maker must generate relevant criteria/objectives/attributes for each problem. Each of the other decision makers may share some, none or all of decision maker i's criteria. In general, decision maker i has a set of criteria indexed by $1,\ldots,\ell_i$

Conflict among criteria. Multiple criteria usually conflict with each other. For example, in designing a car, the objective of high gas mileage might reduce the

passengers' comfort due to less space. In a business context, the problem is sometimes described in terms of a single objective (criterion) only. For instance, profit may be the index chosen for maximization, yet the long range profit goal quite often conflicts with the short range profit goal, and multiple and conflicting criteria, "maximizing the quality of service" and "minimizing cost", are also the decision maker's true concerns.

Committee. A group of persons whose actions (decisions) agree with certain rules that further their interests.

"By a committee we will mean any group of people who arrive at a decision by means of voting. The voters or members of the committee may be situated in one room, as in the case of the committee meeting of a sports club, or they may be scattered over an area, as in the election of a member of parliament" [Black, B27, pp. 1]. Other names for a group of people include "team", "organization", "club", "commission", and "parliament". These names are used in various other fields.

However, the group's decision is usually understood to be the reduction of different individual preferences among objects in a given set to a single collective preference, or group preference. In recent years, the researchers have tended to concentrate on the analysis of decisions that are "correct" or "reasonable" from certain points of view, rather than on how group choices are in fact made. This approach allows one to treat the group decision problems as a completely general problem of "reasonable" transition from given "individual sets of data" to a single "group set of data". The individuals involved and their data can vary greatly from situation to situation. Members of a group use several different techniques to arrive at a final decision. Some use the social choice theory, which is voting, while others use the experts judgment/group participation analysis, which is discussing and guessing at the advantages and disadvantages of the project, while still others may use the game theory approach where each decision maker has his own strategy. There are many other techniques not listed here that may also be used.

A taxonomy of group decision making methods which we have dealt with in this monograph is shown in Figure 1.1.

Social choice theory. Voting is a group decision making method in a

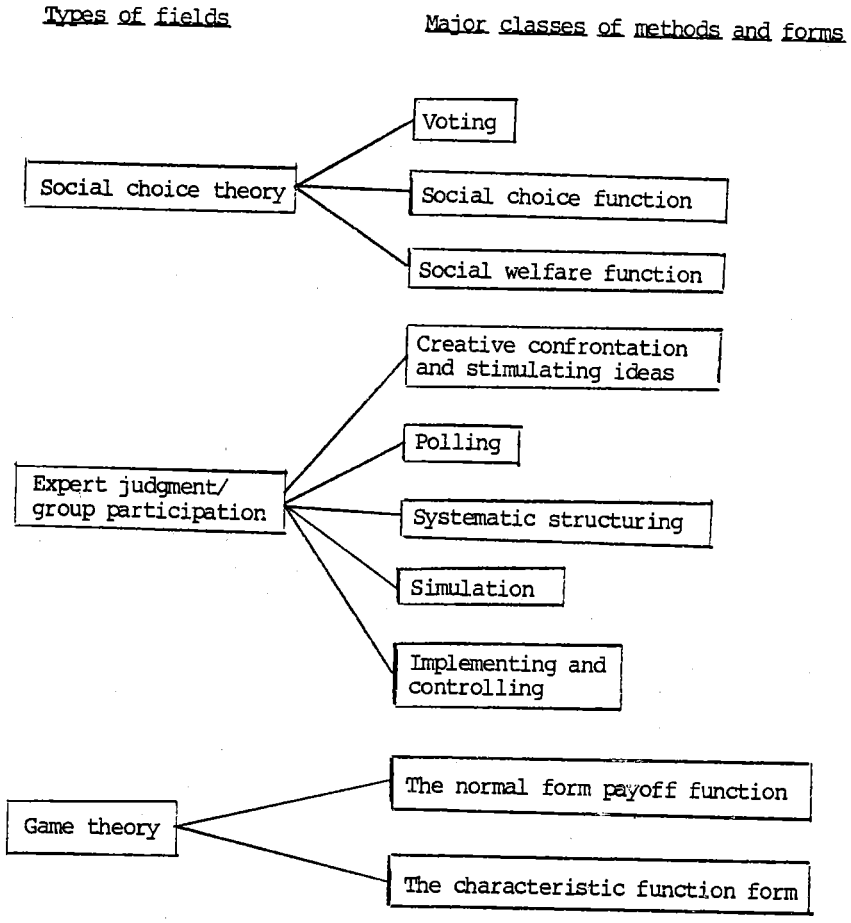

Figure 1.1 A taxonomy of methods for group decision making

democratic society, an expression of the will of the majority. It is a multiple criteria decision making process whenever a voter casts a vote to select a candidate or alternative policy. The candidate's qualifications may be judged by multiple criteria such as trustworthiness and/or honesty, capabilities, general political stance -- conservative, moderate, or liberal -- and positions on specific issues. These criteria are summarized, in a voter's mind, to be a value function (utility function), but in the counting of votes cast, the multiple criteria do not appear explicitly.

However, voting and counting in an electoral system are two different processes, as they are not performed by the same people. The voting process is carried out by all the voters who may be in a country or on a committee. The counting process, on the other hand, is carried out by a small group of selected workers, under expert direction and subject from start to finish to the strictest possible supervision and checking. Obviously the voting process should be kept reasonably simple and straightforward, so as to cause no difficulty to the general run of electors. On the other hand, the primary concern of the counting process is not simplicity but accuracy and effectiveness.

Let us use an example to illustrate the voting and counting processes. Suppose an electoral body of 60 individuals voted for an office holder from a field of three candidates a, b, and c in the following manner:

23 have given the order a P c P b

19 have given the order b P c P a

16 have given the order c P b P a

2 have given the order c P a P b

This is Example 1 of Condorcet (see Section 1.2.1.2 (a) of Part II). The result depends on the methods of voting being employed. Any of the three candidates could be elected: candidate a by the plurality method, candidate b by the second ballot of the majority representation system, and candidate c by the Condorcet principle. This is a clearly undesirable situation since each individual would prefer the method of voting to elect his candidate. In another example, the simple majorities could be intransitive in the situation when x beats y, y beats z, and z beats x. This outcome

is called the paradox of voting. The paradox was known and developed by the Marquis de Condorcet in the eighteenth century, and it is referred to as the Condorcet effect. This problem has been studied since then. As early as 1770, Borda was providing sophisticated studies of voting procedures, and by 1785 Condorcet had sized up many of the analytical problems of majority rule. In the nineteenth century, interest in majority decisions widened, and studies of it attracted such diverse scholars as Laplace in 1814 and Lewis Carroll (i.e., C. L. Dodgson) in 1876 [see Black, B27].

How do we solve this Condorcet effect problem? It is a vital question when dealing with methods of election, particularly the theory of social choice. The question is: what kind of decisions are necessary and sufficient in light of the real world to enable us to derive social orderings of the relevant candidates? The study of the problem, so called the counting process, has been classified in two ways: (1) the social choice function, and (2) the social welfare function. Figure 1.2 presents approaches in social choice theory, which includes voting, social choice function and social welfare functions; these are presented in Part II of this monograph.

Experts judgment/group participation. The problem of group decision making can be broadly classified into two categories in this field: experts judgment, and group participation. The experts judgment process entails making a decision by inventing a new alternative. Specifically, it is concerned with forecasting and involves constructing supplemental objects which may be new designs or new technical solutions. On the other hand, the group participation process entails groups which have common interests, such as a community or an organization, making a decision.

Let us use examples to illustrate the expert judgment and the group participation processes.

First, NASA's Marine Jupiter/Saturn 1977 (MJS 77) project [Dyer and Miles, D29, D30] was to launch two MJS 77 spacecrafts on a pair of trajectories. Before launching, they needed to design the two trajectories, and determine the kinds of experiments to be carried out. There was no past experience to rely on. Therefore, 80 leading scientists (experts) were asked to participate in the decision process.

SOCIAL CHOICE THEORY

1. VOTING

1.1 Nonranked Voting System

1.1.1 One Member Elected From Two Candidates
1.1.2 One Member Elected From Multi-Candidates
(a) The First Past the Post System
(b) Majority Representation System
Repeated Ballots
The Second Ballots

1.1.3 Election of Two or More Members
1.1.3.1 The Single Non-Transferable Vote
1.1.3.2 Multiple Vote
1.1.3.3 Limited Vote
1.1.3.4 Cumulative Vote
1.1.3.5 List Systems
(a) Highest Average
(b) Greatest Reminder
1.1.3.6 Approval Vote

1.2 Preferential Voting System

1.2.1 Simple Majority Decision Rule
1.2.1.1 Two-Alternative Case
1.2.1.2 More Than Two Alternatives Case
(a) Paradox of Voting
(b) The Condorcet Effect

1.3 Nonminority Rule

2. SOCIAL CHOICE FUNCTION

Condorcet Principle

2.1 Condorcet's Function

2.2 Borda's Function

2.3 Copeland's Function

2.4 Nanson's Function

2.5 Dodgson's Function

2.6 Kemeny's Function

2.7 Cook and Seiford's Function

2.8 Fishburn's Function

2.9 Eigenvector Function

2.10 Bernardo's Assignment Approach

2.11 Cook and Seiford's Ordinal Intersection Method

3. SOCIAL WELFARE FUNCTION

3.1 Arrow's Conditions for Social Welfare Function

3.2 Arrow's Possibility Theorem for Two Alternatives

3.3 Arrow's General Possibility Theorem

3.4 Black's Single-Peaked Preferences

3.5 Bowman and Colantoni's Approach

3.6 Goodman and Markowitz's Approach

3.7 Cardinal Social Welfare Function

3.7.1 Value Function for Certainty Case

3.7.2 Utility Function for Uncertainty Case

3.7.2.1 Additive Group Utility Function

3.7.2.2 Multiplicative Group Utility Function

3.7.3.1 Gymnastics Competitions

3.7.3.2 Extended Contribution Rule Method (ECR Method)

Figure 1.2 Approaches in Social Choice Theory

They were divided into eleven science teams, each with different purpose and objective.

Through idea generation activities, they initially generated 2,624 trajectories pairs. Then the team leaders and NASA engineers, through systematic structuring analysis activities, reduced the trajectories to 24 pairs. Through further structuring analysis, these same eleven team leaders and NASA engineers determined the best trajectory pairs. Finally the project was put into action which had needed certain planning and controlling to accomplish it. In this procedure, with a team of experts, the methods of generating ideas, systematic structuring, simulation, and implementing and controlling were used.

Next, as an example let us consider the problem facing a city commission where the issue is to build a downtown mall. The project will affect the community in the future. The city commission has reviewed and consulted with different interest groups and businesses. In the planning procedure, the methods of polling, conference, survey, and systematic structuring are used. In the process of group participation, different points of views are obtained, considered, and then the commission arrives at a final decision.

Figure 1.3 illustrates phases, activities, and methods of experts judgment/group participation, which are presented in Part III of this monograph.

Game theory. Game theory is a mathematical technique used in analyzing conflict-of-interest situations. Game theory, developed by Von Neumann and Morgenstern [V9], is concerned with individuals who are pursuing their own self interest and personal values against other individuals who are pursuing their own self interest and personal values. Game theory is mathematically complex and challenging, but its underlying assumptions are simplistic and its results are difficult to apply in real-life decision problems.

Game theory in this study may be classified into two different forms -- normal form and characteristic function form. The normal form of the game includes a specified number of players, the number of alternative strategies available to each player, and the payoff function of the game. The characteristic function involves

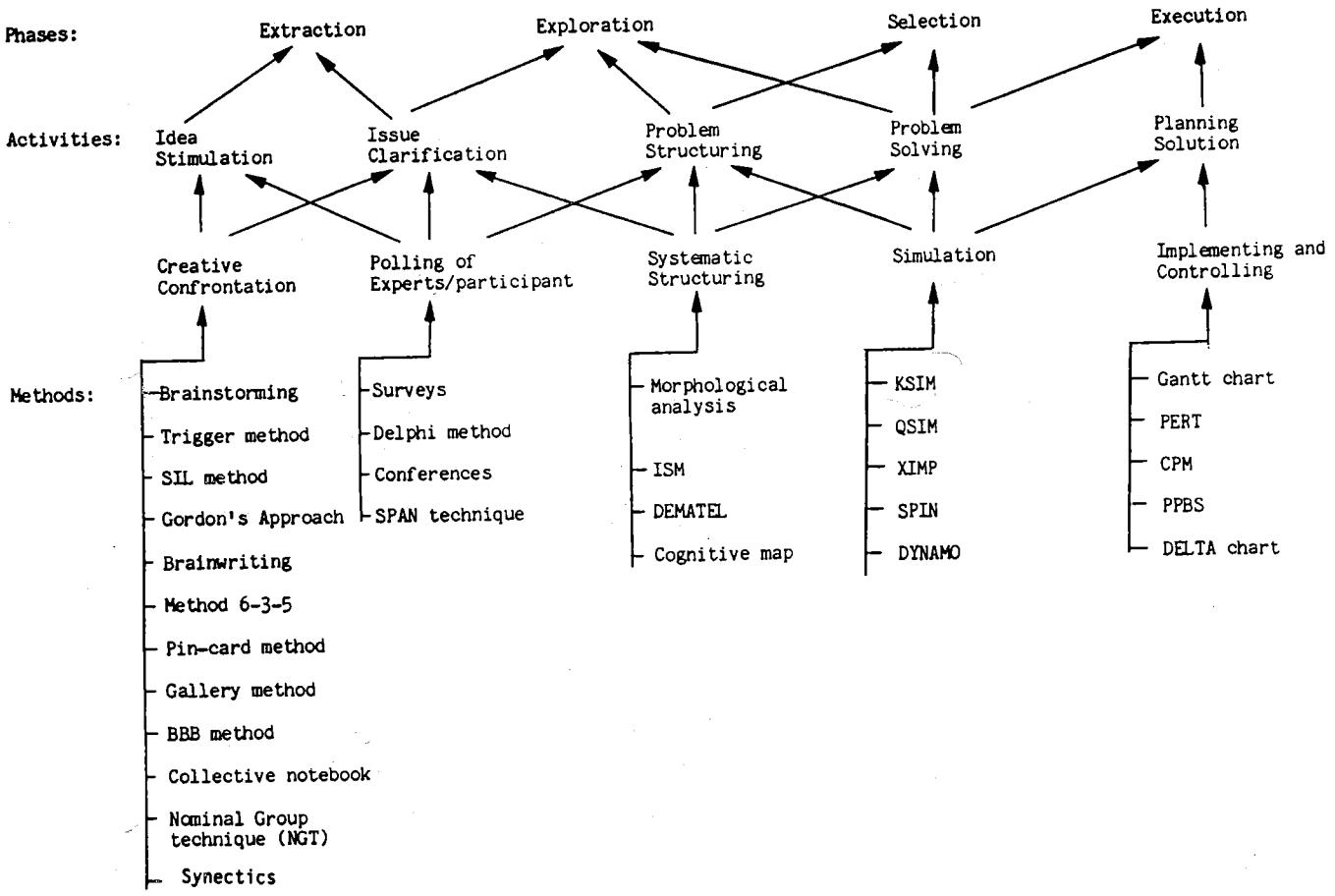

Figure 1.3 Phases, activities and methods of experts judgment/group participation

coalitions. In normal form, however, coalitions are nonexistent. The three basic abstract forms and approaches of game theory are shown in Fig. 1.4. These are presented in Part IV of this monograph.

Objectives of the Present Work:

This work is an introduction to the diversity of group decision making under multiple criteria. It is not a theoretical treatment, but rather a capsule look into the existing methods, their characteristics, and applicability. The many and varied fields of approach to group decision making are systematically classified and simplified for beginners.

A list of more than 800 relevant writings collected from a variety of sources is presented. Significant references on each method or approach are identified for further study.

Because of the broad interdisciplinary character of this field, the literature is diversified in many journals. Although we have tried to give a reasonably thorough list of references, many relevant papers may be inadvertently overlooked or considered not to bear directly on the topics in this study. We apologize to both the readers and the researchers for omitting any relevant ones.

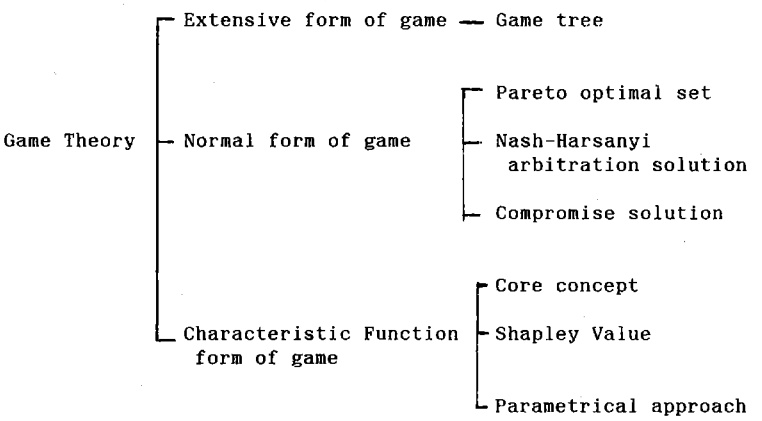

Fig. 1.4 The three basic abstract forms and approaches of game theory

Lecture Notes
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