Lecture Notes in Economics and Mathematical Systems

375

Shu-Jen Chen Ching-Lai Hwang
In Collaboration with Frank P. Hwang

Fuzzy Multiple Attribute Decision Making

Methods and Applications

characteristics and applicability, one may be able to categorize MADM methods into different groups. The result of this classification provides readers with a systematic and overall view of the MADM research field. Various classification schemes have been proposed during the past two decades. We shall present the most dominant ones.

2.2.1 Classification by Information

Hwang and Yoon [H13] classified a group of 17 MADM methods according to the type of information from the decision maker and the salient features of the information. A taxonomy of the methods is shown in Fig. 2.1.

In this classification, the methods were first categorized by the type of information received from the decision maker: no information, information on attributes, or information on alternatives. If no information was given, the methods in this category are dominance, maximin and maximax. If information was given, a subcategory, the salient feature of the received information from the decision maker, was used to further group the methods. The information given may be a standard level of each attribute, such as in the conjunctive or disjunctive method; it may be the ordinal preference of attributes of which, for example, the lexicographic method and Elimination by Aspects (EAB) would apply; it may be a cardinal preference of the attributes of which, for example, the Simple Additive Weighting (SAW) method, Elimination et Choice Translating Reality (ELECTRE) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) apply; or the information may be of the marginal rate of substitution between the attributes where the hierarchical tradeoffs method applies. If the information was given on alternatives, the methods were further subclassified by whether the information was of pairwise preference or the order of pairwise proximity.

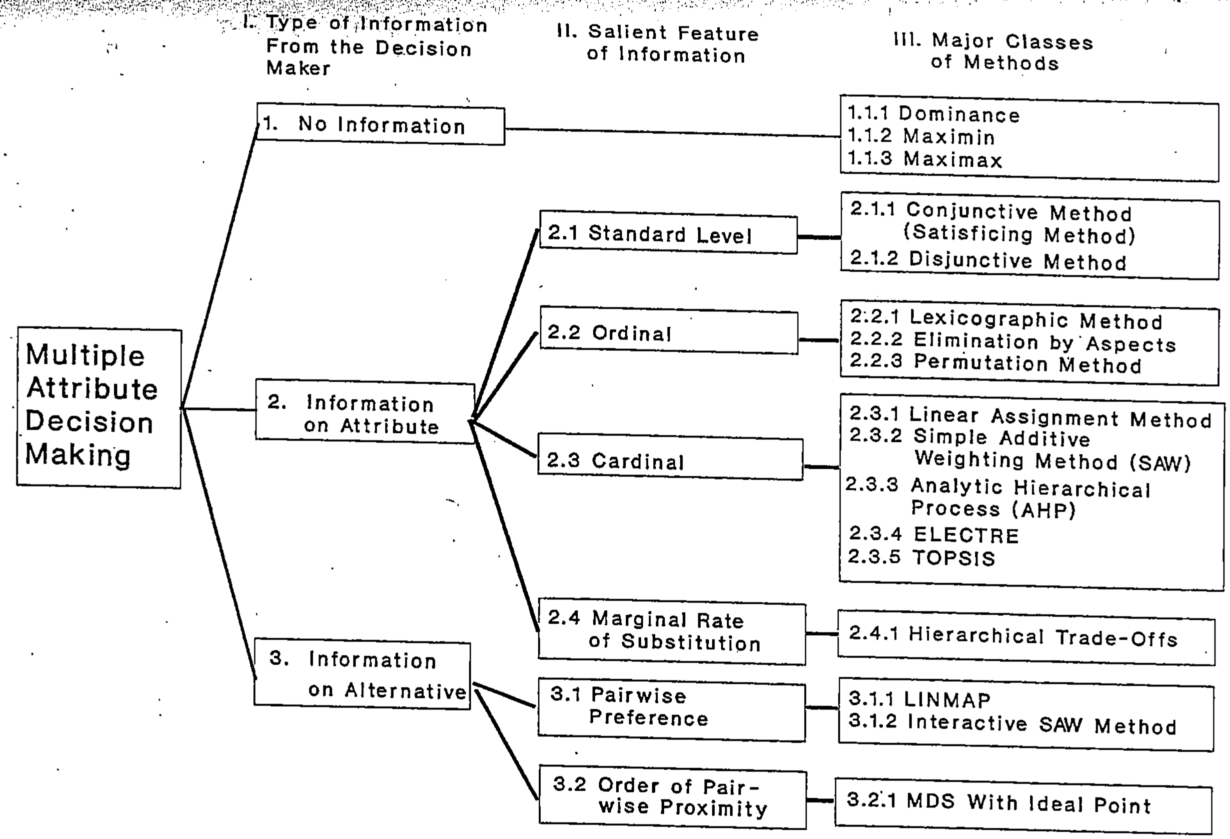

Fig. 2.1 A taxonomy of methods for classical MADM problems [Hwang and Yoon, H13].

The taxonomy of MADM methods by Hwang and Yoon [H13] was modified by Hwang [H15] to Fig. 2.2. Six methods were removed and three new methods were added. The three methods added were the lexicographic semiorder method, the weighted product method, and the distance from target method. The methods removed were the permutation method, the Analytic Hierarchical Process (AHP) method, the Linear Programming Techniques for Multidimensional Analysis of Preference (LINMAP) method, the interactive SAW method and the Multidimensional Scaling (MDS) with ideal point method.

The permutation method was removed because its logic is more difficult for the decision maker to understand, and because the computations are quite involved and not practical for more than about four or five alternatives. The AHP method is not a new technique, but a combination of the eigenvector method and the simple additive weighting method. The hierarchical tradeoffs method requires extensive information from the decision maker, and the approach is more useful for designing an alternative rather than selecting one.

The other three methods removed were LINMAP, the interactive SAW method, and the MDS with ideal point method. These three all belong to the third major branch of the original taxonomy in which the type of information from the DM consists of the information on alternatives. These methods require that the DM indicate his/her preference between two alternatives. This kind of information is far more demanding to assess than the information on attributes, which is one reason why these three methods were removed. The problems which LINMAP and the MDS with ideal point method were designed for involve market research or consumer preference. These methods try to find the reason people buy a particular car; that is, which attributes are most important in influencing the decision. This is a different type of decision making problem.

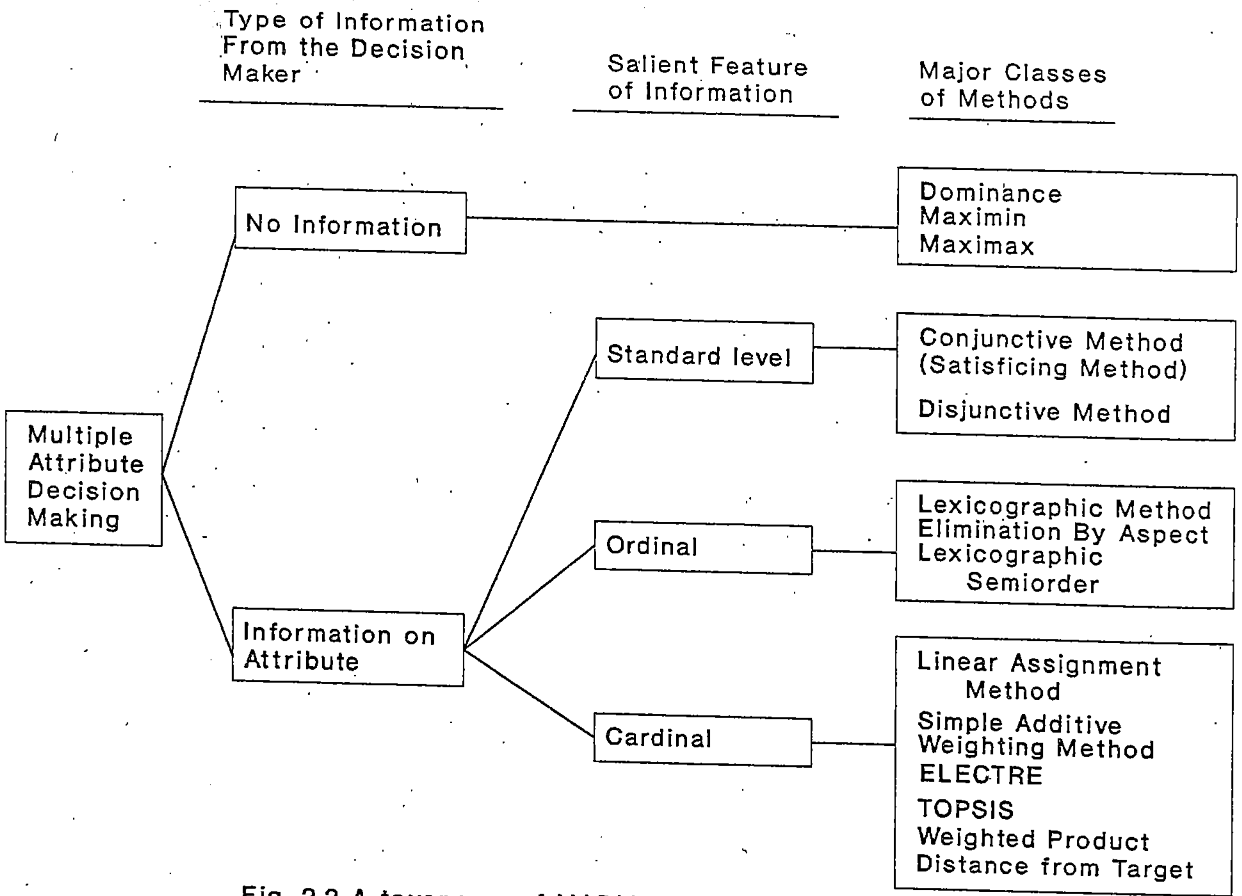

Fig. 2.2 A taxonomy of MADM methods (Hwang [H15]).

2.2.2 Classification by Solution Aimed At

In addition to classifying MADM methods by type of information received from the decision makers, and the salient feature of the information (as in Fig. 2.2), other classification schemes are possible. Another approach (Hwang [H15]) is to classify the methods according to the solution aimed at, as shown in Fig. 2.3. In this classification, if the solution aimed at by the decision maker is to screen, then the dominance method, conjunctive method, or disjunctive method is appropriate. If the solution aimed at is to evaluate, prioritize and select, then maximin, SAW, ELECTRE, or TOPSIS are among the appropriate methods. In some situations, the solution aimed at may be to first screen then evaluate, prioritize and select. In this case, one of the methods for screening can be used for the screening stage, and one of the other methods can be used for the evaluation, prioritization, and selection stage.

2.2.3 Classification by Data Type

One more way of classifying methods is by the data type which the problem contains, as shown in Fig. 2.4 (Hwang [H15]). In this classification, the methods are grouped according to whether they are of data type yes-no where only the dominance, lexicographic, lexicographic semiorder, and EBA method apply; of data type rank where only the dominance, lexicographic, lexicographic semiorder and Linear Assignment Method (LAM) method apply; or of data type numeric where the conjunctive method, SAW, ELECTRE and TOPSIS, for example, apply.

2.3 Description of MADM Methods

The MADM methods to be included are those which are easy to understand and/or easy to apply to real world large size problems. Only the essential ideas will be presented. The details of the

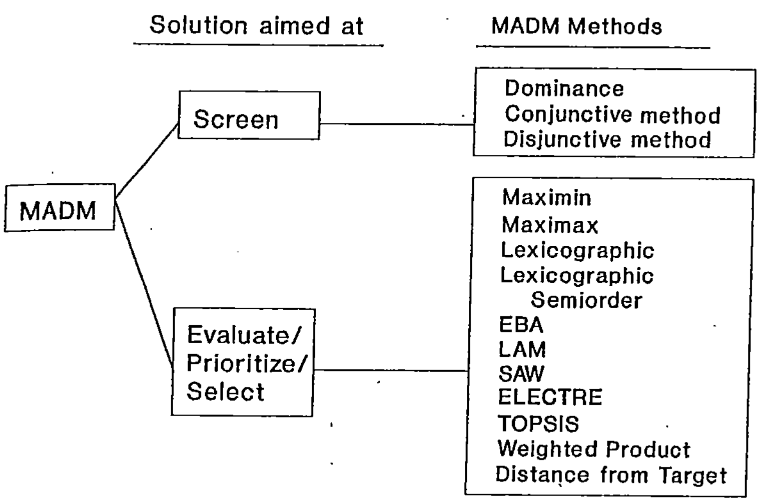

Fig. 2.3 MADM methods classified by solution aimed at (Hwang [H15]).

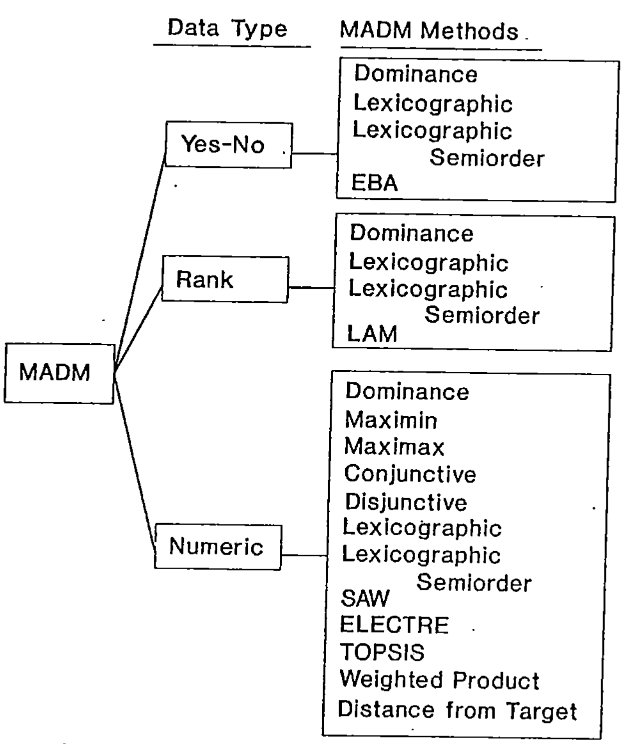

Fig. 2.4 MADM methods classified by data type

methods can be seen in the references mentioned for each method.

The methods are presented in a format which can be used to explain the methods to the user. In the format used, the characteristics of each method are described in the following order: the logic of the method, its basic principle, the step-by-step procedure, any requirements, when it is applicable, its advantages and disadvantages, and its reference.

The methods to be described are listed below in the order of presentation.

1) Dominance
2) Maximin
3) Maximax
4) Conjunctive method (Satisficing method)
5) Disjunctive method
6) Lexicographic method
7) Lexicographic Semicorder
8) Elimination by Aspects (EBA)
9) Linear Assignment Method (LAM)
10) Simple Additive Weighting method (SAW)
11) ELECTRE
12) TOPSIS
13) Weighted Product
14) Distance from Target

Method (1): DOMINANCE

Logic and Basic Principle: An alternative is dominated if there is another alternative which excels it in one or more attributes and equals it in the remaining attributes.

Procedure:

1) Compare the first two alternatives. If one is dominated by the other, discard the dominated one.

2) Next, compare the undiscarded alternatives with the third alternative. Discard any dominated alternatives.

3) Then, introduce the fourth alternative and so on.

4) After (m-1) stages, the nondominated set is determined.

Requirement: None

Applicable when: the solution aimed at is to screen out dominated alternatives.

Advantages: Simple, easy to use and understand.

Disadvantages: Some dominated alternatives, which would get discarded, may actually be better overall than some of the nondominated alternatives.

References: Hwang and Yoon [H13].

Method (2): MAXIMIN

Logic: A chain is only as strong as its weakest link.

Example: An astronaut's life or death in orbit may depend upon his/her worst vital organ.

Basic Principle: The overall performance of an alternative is determined by its weakest or poorest attribute.

Procedure:

1) For each alternative, determine its poorest attribute value.

2) Select the alternative with the best value on the poorest attribute. In mathematical notation, an alternative. A^+, is selected such that

$$A^+ = \{A_i \mid \max_i \min_j x_{ij}\},\ j = 1,2,\ldots,n;\ i = 1,2,\ldots,m.$$

Requirement: All attributes must be measured on a common scale, i.e., attributes should have commensurable units.

Applicable when: the DM is assumed to have a pessimistic nature about the decision making situation.

Advantages: Simple, easy to use and understand.

Disadvantages: Only one attribute is used to represent an alternative. All other (n-1) attributes for a particular alternative are ignored. In other words, the tradeoff among attributes is noncompensatory.

References: Hwang and Yoon [H13], MacCrimmon [M2].

Method (3): MAXIMAX

Logic: An alternative is selected by its best attribute value.

Example: Professional football players are selected based on their best talent: passing, running, kicking, etc.

Basic Principle: The overall performance of an alternative is determined by its best attribute.

Procedure:

1) For each alternative, identify its best attribute value.

2) Select the alternative with the maximum overall best value.

In mathematical notation, an alternative, A^+, is selected such that

$$A^+ = \{A_i \mid \max_i \max_j x_{ij}\},\ j = 1,2,\ldots,n;\ i = 1,2,\ldots,m.$$

Requirement: All attributes must be measured on a common scale, i.e., all attributes should have commensurable units.

Applicable when: the DM is assumed to have an optimistic nature about the decision making situation.

Advantages: Simple, easy to use and understand.

Disadvantages: Only one attribute is used to represent an alternative. All other (n-1) attributes for a particular alternative are ignored. In other words, the tradeoff among attributes is noncompensatory.

References: Hwang and Yoon [H13], MacCrimmon [M2].

Method (4): CONJUNCTIVE METHOD

Logic: An alternative which does not meet the minimal acceptable level for all attributes is rejected.

Example: To obtain a driver's license, one must get an acceptable score on all tests.

Basic Principle: The minimal acceptable levels for each attribute are used to screen out unacceptable alternatives.

Procedure:

1) The DM specifies a minimal acceptable level (cutoff score) for each attribute.

2) For each alternative, determine if the value of each of its attributes equals or exceeds the minimal acceptable level.

3) If so, this alternative is acceptable. Oherwise, it is rejected. Mathematically, A_i is an acceptable alternative only if

$$x_{ij} \geq x_j^o, \; j = 1,2,\ldots,n,$$

where x_j^o is the minimal acceptable level for X_j.

Requirement: A minimal acceptable level on each attribute must be specified.

Applicable when: the solution aimed at is to screen out unacceptable alternatives.

Advantages: Simple, easy to use and understand.

Disadvantages: A candidate with just one unacceptable attribute will be rejected, even if that candidate has high values for all other attributes. In other words, the tradeoff among attributes is noncompensatory.

References: Hwang and Yoon [H13], Dawes [D1a].

Method (5): DISJUNCTIVE METHOD

Logic: A candidate is selected who has an extreme talent in any one attribute.

Example: Professional football players are selected who have an extreme talent in passing, running, kicking etc.

Basic Principle: Desirable levels for each attribute are used to select alternatives which equal or exceed those levels in any one attribute. An alternative is evaluated based on the greatest value (or talent) of an attribute.

Procedure:

1) The DM specifies a desirable level for each attribute.

2) For each alternative, determine if any of its attribute values equals or exceeds the desirable level.

3) If any do, the alternative is acceptable. Otherwise, it is rejected. Mathematically, A_i is an acceptable alternative only if

$$x_{ij} \geq x_j^o, \; j = 1, \text{ or } 2 \text{ or } \ldots \text{ or } n,$$

where x_j^o is a desirable level of x_j.

Requirement: A minimal acceptable (desirable) level for each attribute must be specified.

Applicable when: the solution aimed at is to screen out unacceptable alternatives.

Advantages: Simple, easy to use and understand.

Disadvantages: Alternatives who are good in all attributes but lack an exceptional one will not be selected. The tradeoff among attributes is noncompensatory.

Reference: Hwang and Yoon [H13], Dawes [D1a].

<u>Method</u> (6): LEXICOGRAPHIC METHOD

<u>Logic</u>: In some decision making situations a single attribute seems to predominate.

<u>Example</u>: The "buy the cheapest" rule is one in which price is the most important attribute to the DM.

<u>Basic Principle</u>: To compare the alternatives in the order of the important attributes.

<u>Procedure</u>:

1) Compare all alternatives with respect to the most important attribute. Select the alternative with the highest value on that attribute.

2) If there are several alternatives with the highest value, compare those tied alternatives with respect to the next most important attribute and select the alternative with the highest value in that attribute.

3) Proceed in this manner until only one alternative is left or until all attributes have been considered.

<u>Requirement</u>: The attributes must be ranked in terms of importance.

<u>Applicable when</u>: attributes have a dominating relationship such that $w_1 >>> w_2 >>> \ldots >>> w_n$ where w_i is the weight of the ith most important attribute.

<u>Advantages</u>: Simple, easy to use and understand.

<u>Disadvantages</u>: The tradeoff among attributes is noncompensatory.

<u>References</u>: Hwang and Yoon [H13].

Method (7): LEXICOGRAPHIC SEMIORDER METHOD

Logic: In some decision making situations a single attribute seems to predominate but allow bands of imperfect discrimination so that one alternative is not judged better just because it has a slightly higher value on the predominated attribute.

Procedure:

1) Compare all alternatives with respect to the most important attribute. Select the alternative(s) with the highest value on that attribute or with a value not significantly lower than the highest value.

2) If more than one alternative is selected, compare those tied alternatives with respect to the next most important attribute and select the alternative(s) with the highest or near highest value in that attribute.

3) Proceed in this manner until only one alternative is left or until all attributes have been considered.

Requirement: The attributes must be ranked in terms of importance. Also a tolerance value must be specified on each attribute, indicating the amount of difference from the best value which is not considered significant.

Applicable when: attributes have a dominating relationship such that $w_1 >>> w_2 >>> \ldots >>> w_n$, where w_i is the weight of the ith most important attribute.

Advantages: Simple, easy to use and understand.

Disadvantages: The tradeoff among attributes is noncompensatory.

References: Hwang and Yoon [H13], Luce [L9], Tversky [T14].

Method (8): ELIMINATION BY ASPECTS (EBA)

Logic and Basic Principle: The elimination process is governed by the successive selection of aspects (attributes). Alternatives are compared one attribute at a time and eliminated from consideration if they do not pass a yes-no or minimum acceptabale level.

Procedure:

1) The DM specifies minimum cutoffs for each attribute.

2) Starting with the attribute that has the most discrimination power in a probabilistic mode, eliminate all alternatives which do not pass the yes-no or minimum cutoff for that attribute.

3) Proceed attribute by attribute in order of the discrimination power in a probabililstic mode until only one alternative is left or until all attributes have been considered.

Requirment: The attributes are ranked in terms of their discrimination power in a probabilistic mode.

Advantages: Simple, easy to use and understand.

Disadvantages: A candidate with just one unacceptable attribute will be rejected even if that candidate has high values for all other attributes. In other words, the tradeoff among attributes is noncompensatory.

References: Hwang and Yoon [H13], Tversky [T14].

Method (9): LINEAR ASSIGNMENT METHOD (LAM)

Logic and Basic Principle: An alternative which has many high ranked attributes should be ranked high.

Procedure:

1) Rank the alternatives for each attribute.

2) Assign an importance weight to each attribute.

3) Create a square (m x m) nonnegative matrix Π whose element Π_{ik} represents the score of alternative A_i on the kth attributewise ranking. The score Π_{ik} is the summation of the weights of all attributes where A_i is ranked k.

4) Use the linear assignment method to assign a rank to each alternative such that the summation of the scores for that assignment is maximized.

Requirement: None

Applicable when: ordinal data is given as the score of the alternative on each attribute.

Advantages: Simple, easy to use and understand. It requires less effort to collect data as compared to methods requiring cardinal data.

Disadvantages: The actual cardinal difference between alternatives on each attribute is not considered. Thus, an alternative ranked first on an attribute could have a cardinal score of 100, and one ranked second could have a high score of 99. Yet on a different attribute, an alternative ranked first may have a score of 100, while one ranked second only a score of 50.

References: Hwang and Yoon [H13], Bernardo and Blin [B9b].

Method (10): SIMPLE ADDITIVE WEIGHTING METHOD (SAW)

Logic and Basic Principle: The overall score of an alternative is computed as the weighted sum of the attribute values.

Procedure:

1) For each alternative, compute a score by multiplying the scale rating of each attribute by its importance weight and summing these products over all attributes.

2) Select the alternative with the highest score. Mathematically, the most preferred alternative, A^*, is selected such that

$$A^* = \{A_i \mid \max_i \sum_{j=1}^{n} w_j x_{ij} / \sum_{j=1}^{n} w_j\}$$

where x_{ij} is the outcome of the ith alternative about the jth attribute with a numerically comparable scale, w_j is the importance weight of the jth attribute.

Requirement: The attributes must be both numerical and comparable. The decision maker assigns importance weights to attributes.

Advantages: The best known and most widely used method. Simple. easy to use and understand. The tradeoff among attributes is compensatory.

Disadvantages: If attributes are complementary (a high score on one attribute always occurs with a high score on another attribute), the computed score violates the assumption of separable utility of each attribute.

References: Hwang and Yoon [H13], MacCrimmon [M2].

Method (11): ELECTRE (Elimination et Choice Translating Reality)

Logic: The concept of an outranking relationship is used, which says that even though two alternatives A_k and A_l do not dominate each other mathematically, the DM accepts the risk of regarding A_k as almost surely better than A_l.

Procedure:

Because the procedure is rather lengthy, it will not be presented here. Those interested can refer to the references shown below, specifically, Hwang and Yoon [H13].

Advantages: The tradeoff among attributes is compensatory. It fully utilizes the information contained in the decision matrix.

Disadvantages: Only a partial prioritization of alternatives is computed. As the number of alternatives increases, the amount of calculations rises quite rapidly. Computational procedures are quite elaborate.

References: Hwang and Yoon [H13], Benayoun, Roy, and Sussman [B9a], Roy [R10].

Method (12): TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)

Logic and Basic Principle: The chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution.

Procedure:

1) Calculate the normalized decision matrix. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n.$$

2) Calculate the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as:

$$v_{ij} = w_j r_{ij}, \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n,$$

where w_j is the weight of the jth attribute and $\sum_{j=1}^{n} w_j = 1$.

3) Determine the ideal and negative-ideal solution.

$$A^* = \{(\max_i v_{ij} \mid j \in J),\ (\min_i v_{ij} \mid j \in J') \mid i = 1,2,\ldots,m\}$$
$$= \{v_1^*,\ v_2^*,\ \ldots,\ v_j^*,\ \ldots,\ v_n^*\},$$

$$A^- = \{(\min_i v_{ij} \mid j \in J),\ (\max_i v_{ij} \mid j \in J') \mid i = 1,2,\ldots,m\}$$
$$= \{v_1^-,\ v_2^-,\ \ldots,\ v_j^-,\ \ldots,\ v_n^-\},$$

where $J = \{j=1,2,\ldots,n \mid j \text{ associated with benefit criteria}\}$

where $J' = \{j=1,2,\ldots,n \mid j \text{ associated with cost criteria}\}$

4) Calculate the separation measures. The separation between each alternative can be measured by the n-dimensional Euclidean distance. The separation of each alternative from the ideal one is then given as:

$$S_{i^*} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, \quad i=1,2,\ldots,m.$$

Similarly, the separation from the negative-ideal solution is given as:

$$S_{i^-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \quad i=1,2,\ldots,m.$$

5) Calculate the relative closeness to the ideal solution. The relative closeness of A_i with respect to A^* is defined as:

$$C_{i^*} = S_{i^-} / (S_{i^*} + S_{i^-}), \quad 0 < C_{i^*} < 1, \quad i=1,2,\ldots,m.$$

6) Rank the preference order.

Requirement: The attributes must be both numerical and comparable.

Advantages: Simple, easy to use and understand. The tradeoff among attributes is compensatory.

References: Hwang and Yoon [H13].

Method (13): WEIGHTED PRODUCT METHOD

Logic: In order to penalize alternatives with poor attribute values more heavily, a product instead of a sum of the values is made across the attributes.

Procedure:

1) For each alternative, raise the scale rating of each attribute to a power equal to the importance weight of the attribute. Then multiply the resulting values over all attributes.

2) Select the alternative with the highest product.

Mathematically, the most preferred alternative, A^*, is selected such that

$$A^* = \{A_i \mid \max_i \left(\prod_{j=1}^{n} (x_{ij})^{w_j} \right)\}$$

where x_{ij} is the outcome of the ith alternative about the jth attribute, with a numerically comparable scale, and w_j is the normalized importance weight of the jth attribute.

Requirement: The attributes must be both numerical and comparable.

Applicable when: the DM wishes to avoid alternatives with poor attribute values.

Advantages: Simple, easy to use and understand. The tradeoff among attributes is compensatory.

References: Easton [E1].

Method (14): DISTANCE FROM TARGET METHOD

Logic: For some attributes, the best value may be located in the middle of the attribute range.

Example: In buying a house the number of rooms should be neither too many nor too few.

Basic Principle: The alternative which has the shortest distance from the target alternative is selected.

Procedure:

1) For each alternative, compute the deviation from the target.

2) Select the alternative with the shortest distance value.

Mathematically, an alternative, A^*, is selected with the shortest distance

$$d_i = \sqrt{\sum_{j=1}^{n} w_j^2 \, (x_{ij} - t_j)^2}, \quad i=1,2,\ldots,m,$$

where x_{ij} is the outcome of the ith alternative about the jth attribute with a numerically comparable scale; t_j and w_j are the target level and the normalized weight of the jth attribute, respectively.

Requirement: A target level must be specified on each attribute. The attributes must be both numerical and comparable.

Applicable when: the DM has in mind a set of target levels on each attribute. Also, the attributes do not necessarily have a monotonically increasing or monotonically decreasing utility. That is, for "benefit" attributes, bigger is not necessarily better or for "cost" attributes, smaller better. A value in the middle may be preferred here.

Advantages: Simple, easy to use and understand. The tradeoff among attributes is compensatory.

References: Easton [E1].