Report for Q3 and Q4

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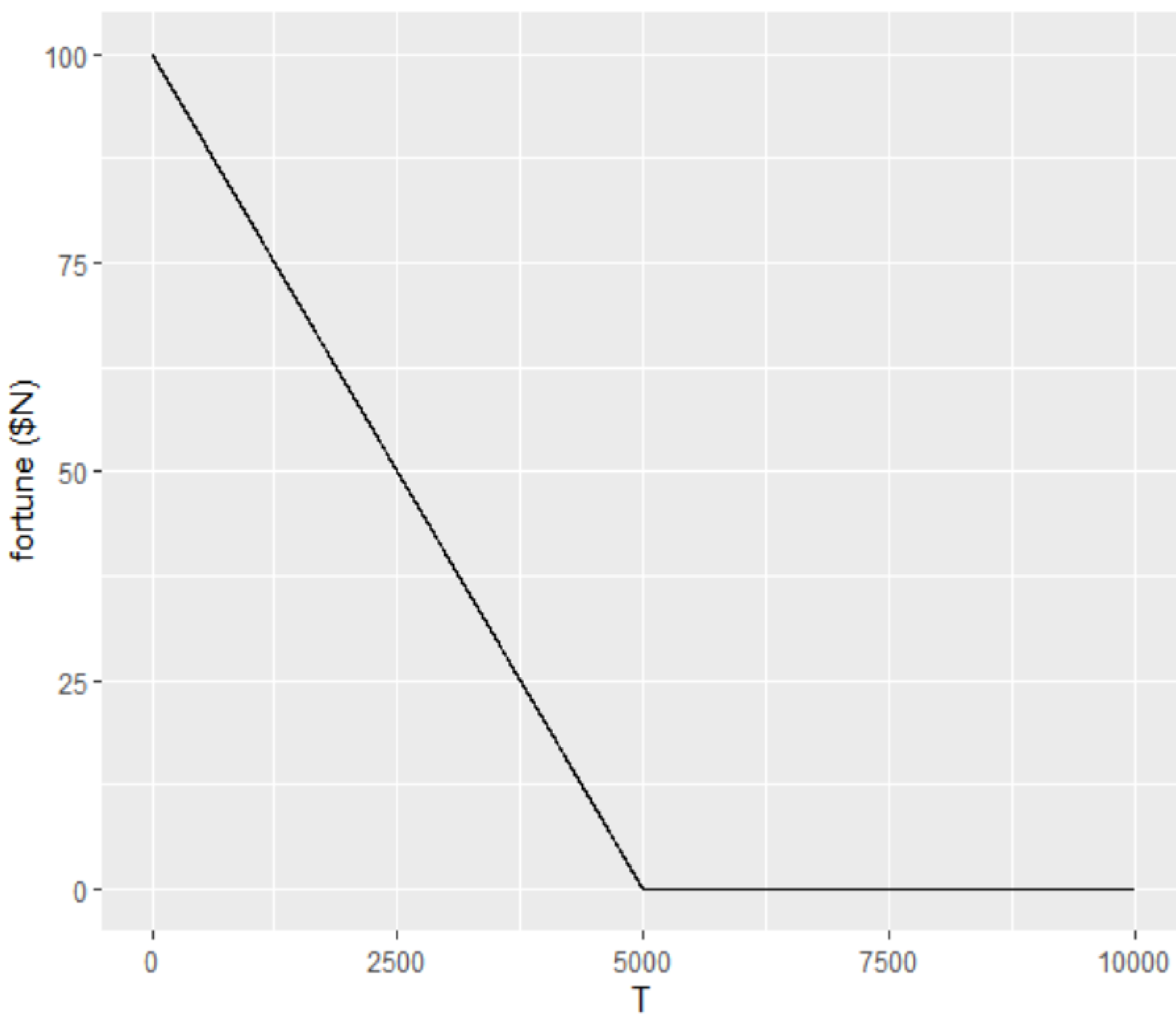
Q3

Q3(1)

The probability that he do not run out of money after 100,1,000,10,000 , and 100,000 times are (approximately) 1, 0.99, 0.085, 0 respectively.

(This answer is obtained by Monte Carol Simulation, so may have minor error to the accurate value.)

Q3(2)

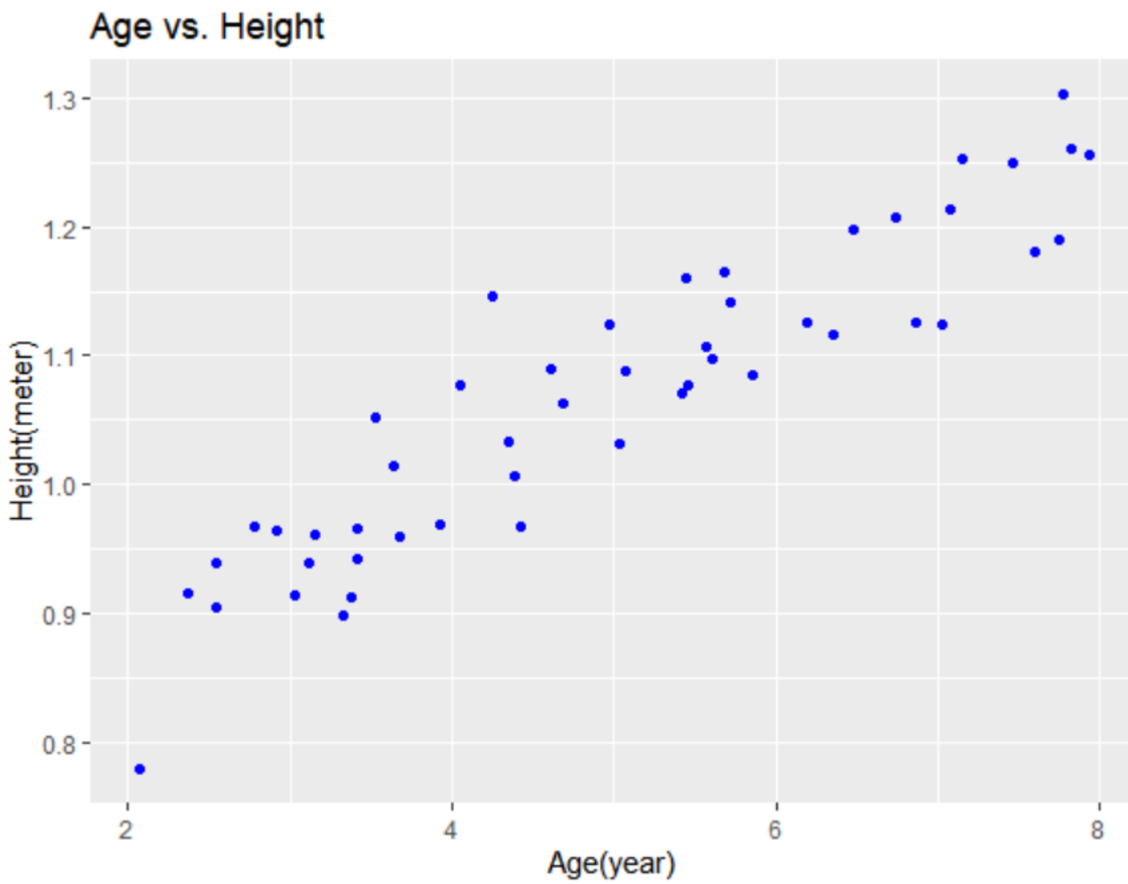


My observation: The more he gamble, the less money he will have on average.(i.e. the money he have is a decreasing liner function to the gamble times T until he run out of money. He will eventually end up with no money.)

Q4

Q4(1)

Our sample looks like this,



to find the linear regression line is to find a line which minimize the sum of errors.

for each point, error $d_i = y_i - (\widehat{\beta_1}x_i + \widehat{\beta_0})$.

Sum of error square $D = (y_i - (\widehat{\beta_1}x_i + \widehat{\beta_0}))^2$

To minimize D, we shall use calculus.
Find the partial derivate of $\widehat{\beta_0}$ and $\widehat{\beta_1}$

$$\frac{\partial D}{\partial \widehat{\beta_0}} = \sum_{i=1}^{n} -2(y_i - \widehat{\beta_0} - \widehat{\beta_1}x_i)$$

$$\frac{\partial D}{\partial \widehat{\beta_1}} = -2(\sum_{i=1}^{n} x_i\, y_i - \widehat{\beta_0}\sum_{i=1}^{n} x_i - \widehat{\beta_1} x_i^2)$$

let $\frac{\partial D}{\partial \widehat{\beta_0}}$ and $\frac{\partial D}{\partial \widehat{\beta_1}}$ be 0, $n\bar{x} = \sum_{i=1}^{n} x_i$ and $n\bar{y} = \sum_{i=1}^{n} y_i$. (I think unbiased estimation is also ok here (as slides shows), anyway it do not affect the result)

We have:

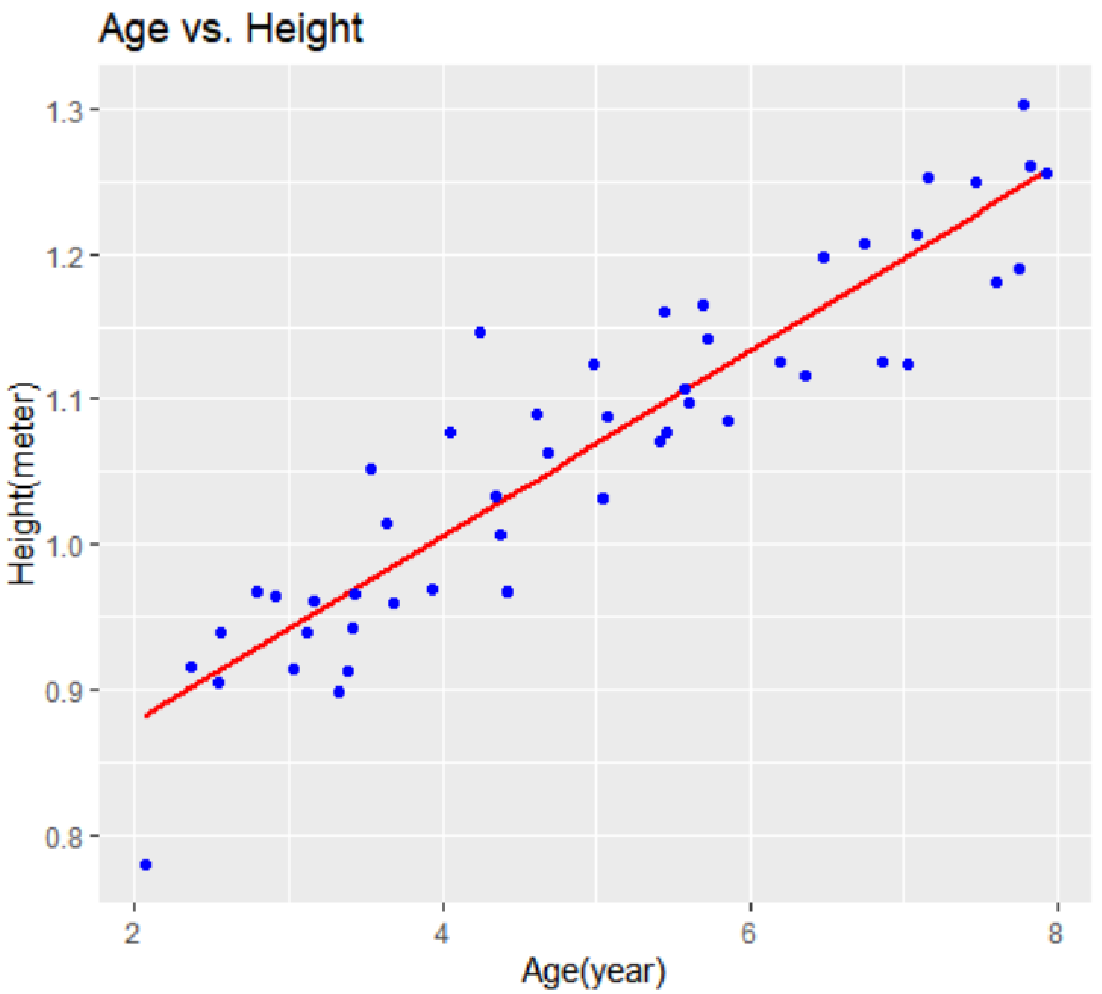
$$\sum_{i=1}^{n} x_i\, y_i - \bar{y}\sum_{i=1}^{n} x_i + \widehat{\beta_1}\bar{x}\sum_{i=1}^{n} x_i - \widehat{\beta_1}\sum_{i=1}^{n} x_i^2 = 0$$

and $\widehat{\beta_0} = \bar{y} - \widehat{\beta_1}\bar{x}$

Finally, we can get: $\widehat{\beta_0} = \bar{y} - \widehat{\beta_1}\bar{x}$

and $\widehat{\beta_1} = \frac{\sum_i^n = 1(x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n = 1(x_1 - \bar{x})^2}$

Q4(2)



Q4(3)

the model we get in Q4(2) is $y = 0.75016 + 0.06388x$.

To predict height, just simply use this model with different x value.
So, for a 3.5-yr-old boy, his expected height is $0.75016 + 0.06388 * 3.5 = 0.97374$ meters.
and for an 8-yr-old boy, it's $0.75016 + 0.06388 * 8 = 1.2612$ meters