#

**Assignment 1**

# Question 1

## (1.a)

Two coins taken out can be : 2 one-dollar, 2 five-dollar, 2 ten-dollar,1 one-dollar and 1 five-dollar,1 five-dollar and 1 ten-dollar, 1 one-dollar and 1 ten dollar.  
So we can get Sample space :

## (1.b) Let vector denotes the outcome of one expriment, then it is a discrete random variable.  
for example stands for is one-dollar and is five-dollar.  
Then there will be possible expriments if sequence is considered.  
if , then there are three possible outcomes .  
- for (5,1), there are expriments. - for (10,1), there are expriments. - for (10,5), there are expriments.

Let be the event that the value of is larger than , then ## (1.c)   
1. for , only if the coin we draw is ten-dollar can we satisfy the requirement.  
We can easily get . 2. for , it implies that first draw is not ten-dollar.  
The requirement can be met if our outcome is (1,10) or (5,5) or (5,10)  
then,

1. for , it excludes , so first draw cannot be (10), and the second draw cannot be (1,10) (5,5) (5,10).  
   The outcome should be   
   then,

So, we have

## (1.d)

They are **not** independent.  
Especially, suppose we draw 10-dollar coin at first, then we will have and .

**So, X and Y are not independent**. ## (1.e)

1. for ,
2. for ,

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 100 | 121 | 144 | 169 | 196 | 225 | 256 | 289 | 324 | 361 |
|  |  |  |  |  |  |  |  |  |  |  |

## (1.f)

can be 1,2,3,4,5,6,7,8,9,10  
1.

1. for ,
2. for

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Y,X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |  |  |
| 12 |  |  |  |  |  |  |  |  |  |  |  |
| 13 |  |  |  |  |  |  |  |  |  |  |  |
| 14 |  |  |  |  |  |  |  |  |  |  |  |
| 15 |  |  |  |  |  |  |  |  |  |  |  |
| 16 |  |  |  |  |  |  |  |  |  |  |  |
| 17 |  |  |  |  |  |  |  |  |  |  |  |
| 18 |  |  |  |  |  |  |  |  |  |  |  |
| 19 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

# Question2

## (2.a)

Proof:

Note that , then for any event ,

Also note that for any , we have ,  
Then for any event E, (i.e. they are disjoint)  
By the distributive law,

## (2.b)

Let denote the event that the sample student is outstanding, and denote that the student come from school A, B, C, D respectively.  
Then , it can be easily derived that:

By the Law of total Probabiliy,

## (2.c)

We already have   
By the Baye’s Theroem,

# Question 3

## (3.a)

Proof Markov Inequality for continuous random variable:  
for any

So we have

## (3.b)

Using Chebyshev’s Inequality, it can be derived that:

Since ,

## (3.c) Let denotes the expectation of our sample, then we have

Then,

If , Then we have equation (by the symmetric property of normal distribution)

(1)

Where for a standard normal distribution.  
It can be obtained by checking the table that ,  
By equation , we can get , where shoulud be an integer.

**Thus, at least people whould have to be sampled to satisfy the requirement.**

**(3.d)**

By the Law of Large Number and Central Limit Theorem,  
 approximately follows

Then

To get a 95% confidence interval,   
By checking the standard Normal distribution table, we have   
So, the 95% confidence interval of :

## (3.e)

We have   
Null hypothesis   
Then p-value:

So we will reject at level of significance   
we will also reject at level of significance