DECONVOLUTION OF ULTRASOUND IMAGES

Weiqi Jiang (20730283)

ABSTRACT

Ultrasound diagnosis has been widely used because of its low-cost, real-time and non-invasive characteristics. Compared to other medical images, ultrasound images have a relative low resolution. The main reason for this is that the point spread function convolves with the reflection of the detected tissue, plus the inevitable noise of the environment, which cause the images blurred. Thus, we need to take advantage of deconvolution methods to get the closest estimation of the original image. This report realizes some deconvolution methods and analyze the influence of different selection of parameters.

***Index Terms*—** Deconvolution, Ultrasound images, Fourier, Wavelet

1. INTRODUCTION

Ultrasound images are the representation of scatters in the human tissue. Before applying deconvolution methods to improve the quality and resolution of ultrasound images, a degradation model is needed. The model used in this report is showed as below:

$$y(n) = x(n) * h(n) + \gamma(n) \tag{1}$$

Where x is the original signal, y is the degraded signal, h represents the point spread function (PSF) and γ is the additive noise, * represents the convolution function.

There are two main class to get the restoration. The first is non-blind deconvolution method, which means the we estimate the PSF first, then using some classical deconvolution methods to get the restored images. The second is blind deconvolution methods, getting restored image directly from degraded image without estimating point spread function.

2. ITERATIVE BILIND DECONVOLUTION

Iterative blind deconvolution is a blind deconvolution method, it gains the estimation of restored images and correspond PSF iteratively. According to the reference [1], we can setting the support region of PSF to be 15×15, which improve the efficient greatly. There are some constrain conditions. Power constraint means mainly that the value of every pixel cannot be negative and the point spread function has a constant power. Frequency domain constraint can be divided into three conditions:

If the estimator of PSF, $|\widetilde{H_\iota}(u,v)| \geq |Y(u,v)|$.

$$X_{i+1}(u,v) = (1-\beta)\widetilde{X_\iota}(u,v) + \beta \frac{Y(u,v)}{\widetilde{H_\iota}(u,v)} \tag{2}$$

Where β is the frequency domain constraint parameter. It controls the convergence speed.

If $|Y(u,v)| < noise\ level.$

$$X_{i+1}(u,v) = \widetilde{X_\iota}(u,v) \tag{3}$$

If $|\widetilde{H_\iota}(u,v)| < |Y(u,v)|$.

$$\frac{1}{X_{i+1}(u,v)} = \frac{1-\beta}{\widetilde{X_\iota}(u,v)} + \beta \frac{\widetilde{H_\iota}(u,v)}{Y(u,v)} \tag{4}$$

Under those constraint conditions, using the least square method, we can get the estimation of the point spread function of ultrasound images, the K-th iteration formula is as follows:

$$\widetilde{H_K}(u,v) = \frac{Y(u,v)\widetilde{X_{k-1}^*}(u,v)^2}{|\widetilde{F_{k-1}}(u,v)|^2 + \alpha/|\widetilde{H_{k-1}}(u,v)|^2} \tag{5}$$

$$\widetilde{X_k}(u,v) = \frac{Y(u,v)\widetilde{H_{k-1}^*}(u,v)^2}{|\widetilde{H_{k-1}}(u,v)|^2 + \alpha/|\widetilde{X_{k-1}}(u,v)|^2} \tag{6}$$

3. FOURIER-WAVELET DECONVOLUTION

Processing signal in Fourier domain is the most common used method in signal processing filed. The discrete Fourier transform for formula (1) is as follows:

$$Y(u,v) = H(u,v)X(u,v) + \Gamma(u,v) \tag{7}$$

Where Y, H, K and Γ are N point discrete Fourier transform forms for y, h, x and γ. If we use the inverse operator to do the filtering, When H (u, v) is very small or zero in some certain frequency points and noise may be amplified unboundedly at these points, which leads to the results of the loss of validity of deconvolution and becomes an ill-condition problem[2].

Thus, a shrinkage coefficient is needed to deal with the ill-posed problem. The regularized filter of Fourier deconvolution has a general formula:

$$G(u,v) = \frac{H(u,v)^*}{|H(u,v)|^2 + \Lambda(u,v)} \tag{8}$$

Where $\Lambda(u,v) \geq 0$ is the regularization term.

Tikhonov Deconvolution proposed by [3] uses regularization term:

$$\Lambda(f_k) = \tau = \alpha \frac{N\sigma^2}{\|y-\mu(y)\|_2^2} \tag{9}$$

Where $\mu(y) = \sum_n y(n)/N$. N is the number of pixels. The optimal value of α can be obtained by solve the cost function below[4]:s

$$\sum_{k=-\left(\frac{N}{2}\right)+1}^{\frac{N}{2}} \frac{|H(f_k)|^2}{|H(f_k)|^2+\tau} \frac{1}{|H(f_k)|} \left|H(f_k)\hat{X}(f_k) - Y(f_k)\right|^2 \tag{10}$$

$$\tau = \alpha \frac{N\sigma^2}{|y-\sum_n y(n)/N|_2^2} \tag{11}$$

The value of α which makes the cost function output minimum is the optimal value. It can be gained through iteration.

However, after Fourier domain deconvolution, the details of image may lose. Thus scholars proposed Fourier-Wavelet Regularization Deconvolution(ForWaRD) method[4]. This method combines Fourier transform and wavelet transform[5], can remove the noise further while preserving the details.

According to the reference [6], the noise standard deviation can be computed from the formula below:

$$\sigma_j = Median(abs(cD_j))/0.6745 \tag{12}$$

Through experiments, I found that the threshold computation method proposed in [6] makes the threshold relatively high, so that some details may lose. Threshold can be modified to preserve some details of images, Let T become the symbol of hierarchical threshold.

T in reference[6]:

$$T = \sigma_j \sqrt{2log_e N} \tag{13}$$

Modified T1:

$$T1 = \sigma_j \sqrt{2log_e N}/sqrt(j) \tag{14}$$

Modified T2:

$$T2 = \sigma_j \sqrt{2log_e N}/sqrt(1+sqrt(j)) \tag{15}$$

After denoising in wavelet domain, we apply wavelet wiener filtering, the coefficient is as follows:

$$\lambda_j^w = \frac{|w_j|^2}{|w_j|^2 + {\sigma_j}^2} \tag{16}$$

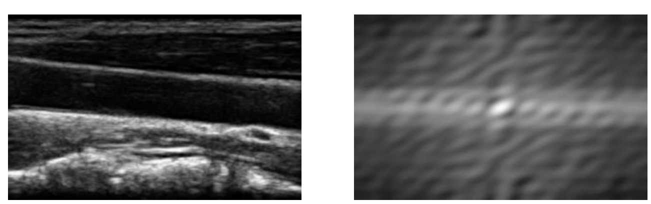
The whole process of ForWaRD is concluded as followed:

1. Set value of α, and do Tikhonov shrinkage
2. Wavelet threshold estimation
3. Wavelet wiener filtering
4. Computing the cost function, if the α is the optimal one, output restored image. Else, go to step 1.

4. EXPERIMENTAL RESULTS

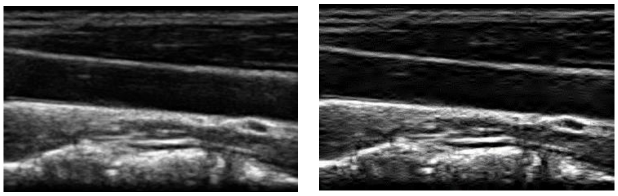
This section presents several experiments conducted on ultrasound images. Those images are obtained from online open access database. Those images are common carotid artery of ten volunteers (mean age 27.5 ± 3.5 years) with different weight (mean weight 76.5 ±9.7 kg).

First, we use some other algorithm to estimate the point spread function. The result is shown in fig 1(b). Then adopting the Fourier-Wavelet Regularization Deconvolution method, given the prior knowledge about PSF. Applying different threshold shrinkage methods and different thresholds, the results are shown in fig.2 and fig.3 below:

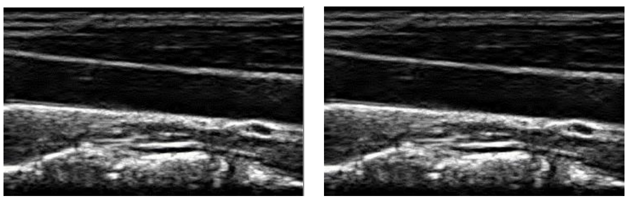


(a) (b)

Fig. 1. Sample images. (a)degraded image. (b) PSF estimation

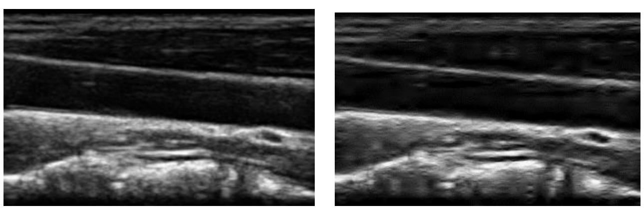


(a) (b)

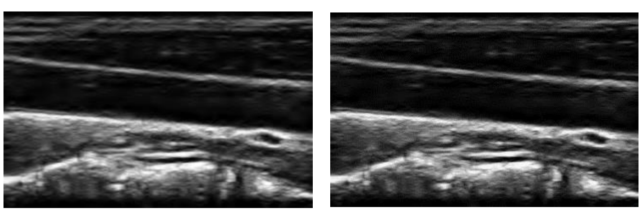


(c) (d)

Fig. 2. Hard thresholding images. (a) degraded image. (b) result of threshold T. (c) result of threshold T1. (d) result of threshold T2.



(a) (b)

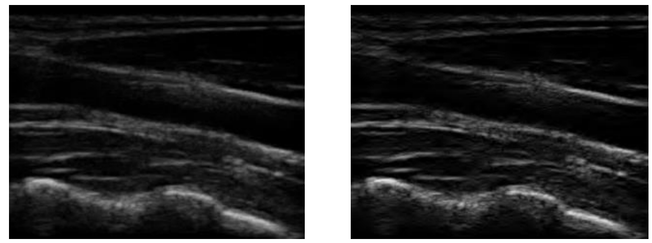


(c) (d)

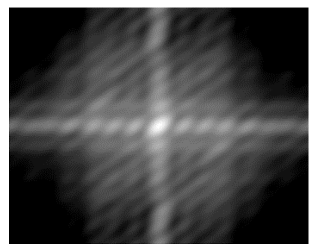
Fig. 3. Soft thresholding images. (a) degraded image. (b) result of threshold T. (c) result of threshold T1. (d) result of threshold T2.

From fig.2 and fig.3, the result of hard thresholding has a higher contrast and better visuality. Soft thresholding has less contrast but it has better noise suppression performance. In general, it is a trade off between keeping details and removing noise. In this report, we choose hard thresholding. From those result images, we can see that there exists some slight difference in the results between different threshold computation methods. we take noise removal and details keeping both into consideration, thus, threshold T1 is the optimal one, because it has a reasonable noise removal perfomance and keep details as much as possible.

Using another image to further test the algorithm. It shows that this algorothm can deblur image and remove noise reasonably. The result is shown in fig.4:



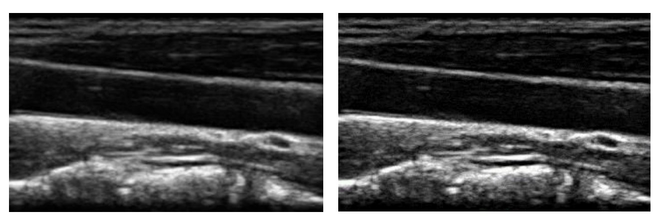
(a) (b)



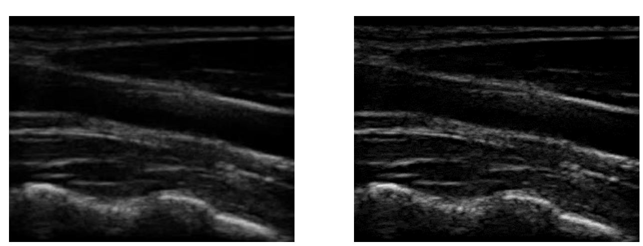
(c)

Fig. 4. Another carotid artery ultrasound image. (a) degraded image. (b) resorted image(using hard thresholding and T1 threshold). (c)coresspond PSF estimation.

Also, the result of the iterative blind deconvolution method is shown in the fig 5. we can conclude that the IBD method has almost the same result as the ForWaRD method.



(a) (b)



(c) (d)

Fig. 5. Results of IBD. (a)degraded image 1. (b) restored image 1. (c)degraded imge 2. (d) restored image 2.

Running time for two methods are shown in table I.

TABLE I Runing Time Of Two Methods

Running time(s)	IBD(mean+std)	ForWaRD(mean+std)
Image 1	3.7149 ± 0.0822	0.9725 ± 0.0134
Image 2	4.2756 ± 0.0357	1.0699 ± 0.0143

5. CONCLUSION

This report presents the classical Fourier-Wavelet Regularization Deconvolution(ForWaRD) method and proposes two modified thresholds based on it, as well as the iterative blind deconvolution methods. After conducting some experiments on ultrasound image of carotid artery, we choose hard thresholding method and T1 as our threshold. Using some images to test the performance of those two methods, it shows reasonable results.

The ForWaRD has more time efficiency according to the table I, which is a great advantage. It means this method can be used in real time application. However, it needs the prior knowledge about the ultrasound images, mostly, it needs the estimation of PSF. In reality, the estimation of PSF is not a thing we always know. So, this drawback restrains its application. Besides that, the performance of this method rely greatly on the accurancy of the PSF estimation. The estimation algorithm needs to be chosen very carefully.

The advantage of the iterative blind deconvolution is that it does not need any prior knowledges about images, the only thing it needs is the degraded images. However, due to the computational complexity, it can be only adopted in off-line applications.

There are still some future works need to do:

1. Ultrasound images of different human tissues should be introduced. This report only tests the performance of this algorithm for human carotid artery due to the limitation of database.
2. Cause we can only get the degraded images from the open access database, we do not have original images to do full-reference quality assessment. And due to time limit, we have not found out a purpose no-reference image evaluation metric.
3. The performance of those two algorithms mainly depends on the accurancy of PSF estimation. Thus, the PSF estimation algorithm need to be improved to get a better result.

6. REFERENCES

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