Removing the impact of the balun in reflectometry measurements for HERA EoX project.

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1. The S-matrix description of the balun.

We first discuss how measurements of the S-parameters of the three unbalanced ports of a balun may be used to determine and undo its impact on measurements of the differential S-parameters of an antenna. A balun may be described as a three port system with the unbalanced port numbered on and the balanced port formed from two unbalanced ports which we will label 2 and 3. Incoming waves at each port are related to outgoing waves by the S-matrix.

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{1}$$

or

$$\mathbf{b} = \mathbf{S}\mathbf{a} \tag{2}$$

Since we are ultimately attempting to measure the S-parameters of the differential antenna, it is convenient for us to work in the differential and common-mode basis for ports 2 and 3. We define the differential and common mode incoming and outgoing waves as

$$a_c = \frac{1}{\sqrt{2}}(a_2 + a_3) \tag{3}$$

$$a_d = \frac{1}{\sqrt{2}}(a_2 - a_3) \tag{4}$$

$$b_c = \frac{1}{\sqrt{2}}(b_2 + b_3) \tag{5}$$

$$b_d = \frac{1}{\sqrt{2}}(b_2 - b_3) \tag{6}$$

The transformation matrix between the un-balanced and balanced basis is given by the matrix

$$\mathbf{M} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \tag{7}$$

The S-matrix in this basis may be expressed as the transformation

$$\begin{pmatrix} b_1 \\ b_c \\ b_d \end{pmatrix} = \begin{pmatrix} S_{11} & S_{1c} & S_{1d} \\ S_{c1} & S_{cc} & S_{cd} \\ S_{d1} & S_{dc} & S_{dd} \end{pmatrix} \begin{pmatrix} a_1 \\ a_c \\ a_d \end{pmatrix} \tag{8}$$

or

$$\mathbf{b}' = \mathbf{S}'\mathbf{a}' \text{ where } \mathbf{b}' = \mathbf{Mb}\ ,\ \mathbf{a}' = \mathbf{Ma}\ ,\ \text{and } \mathbf{S}' = \mathbf{MSM}^{-1} \tag{9}$$

We can apply the matrix transformation in the third equation of 9 to derive the differential S-matrix of our balun. Here is the mathematica code

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Mmatrix = {{1, 0, 0}, {0, 1/Sqrt[2], 1/Sqrt[2]}, {0, 1/Sqrt[2], -1/
    Sqrt[2]}};
Smatrix = {{s11, s12, s13}, {s21, s22, s23}, {s31, s32, s33}};
MatrixForm[FullSimplify[Mmatrix.Smatrix.Inverse[Mmatrix]]]
```

which gives us

$$\begin{pmatrix} S_{11} & S_{1c} & S_{1d} \\ S_{c1} & S_{cc} & S_{cd} \\ S_{d1} & S_{dc} & S_{dd} \end{pmatrix} = \begin{pmatrix} S_{11} & \frac{S_{12}+S_{13}}{\sqrt{2}} & \frac{S_{12}-S_{13}}{\sqrt{2}} \\ \frac{S_{21}+S_{31}}{\sqrt{2}} & \frac{1}{2}\left(S_{22}+S_{23}+S_{32}+S_{33}\right) & \frac{1}{2}\left(S_{22}-S_{23}+S_{32}-S_{33}\right) \\ \frac{S_{21}-S_{31}}{\sqrt{2}} & \frac{1}{2}\left(S_{22}+S_{23}-S_{32}-S_{33}\right) & \frac{1}{2}\left(S_{22}-S_{23}-S_{32}+S_{33}\right) \end{pmatrix} \tag{10}$$

Having measured the unbalanced S-parameters of the balun, we may obtain the differential and common mode S-parameters from the single-port S-parameters with equation 10.

2. Measurements of the balun as a three-port system

3. Characterization of the balun using back-to-back baluns.

If one assumes that balun has good common-mode rejection, it is possible to characterize the balun through a two-port measurement of two baluns attached back-to-back on their differential port. In this measurement, the VNA has the RF-out arm attached to the unbalanced port of one balun and the RF-in arm on the other. The VNA measures both the $S'_{11} \equiv R^m$ of the back to back baluns and $S'_{21} \equiv T^m$. We illustrate the path of an inserted signal at the unbalanced port of the first balun in Fig. **??**. The reflected signal used to determine Γ_m is equal to the sum of the initial reflection off of S_{11} plus the component of the signal that is transmitted to the interface between the two baluns, is reflected back, and transmitted back out of port-1, picking up a factor of $S_{1d}S_{dd}S_{d1}$. A component of this signal is not transmitted but is rather re-reflected within the balun-balun interface, picking up an additional factor of S_{dd}^2 before being transmitted back and re-reflected. The resulting series of reflection gives us

$$\begin{aligned} R^m &= S_{11} + S_{1d}S_{dd}S_{d1} + S_{1d}S_{dd}S_{dd}^2S_{d1} + \cdots + S_{1d}S_{dd}S_{dd}^{2m}S_{d1} \\ &= S_{11} + \frac{S_{1d}S_{dd}S_{d1}}{1 - S_{dd}^2} \end{aligned} \tag{11}$$

The transmitted signal used to determine T^m results from a component that is transmitted through the first balun and the second balun, $S_{1d}S_{d1}$ plus a component that is transmitted through the first

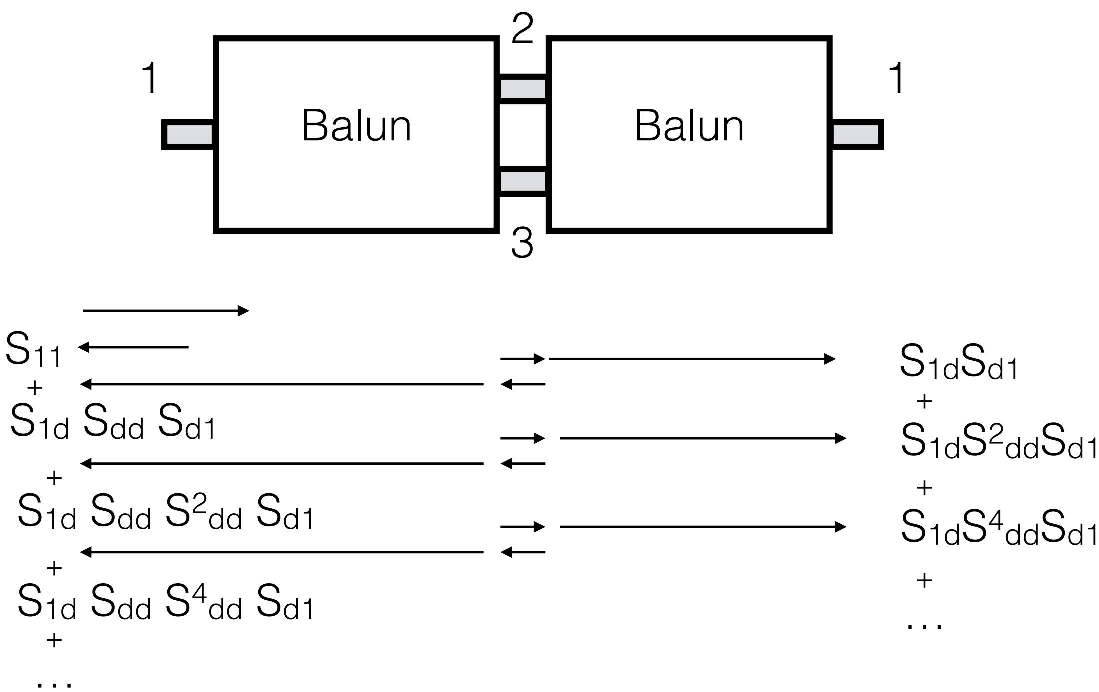

Fig. 1.— An illustration of the wave components arising at the input and output unbalanced ports of back-to-back baluns which give rise to equations 11 and 12

balun, reflected between the baluns, and re-transmitted, picking up a factor of S_{dd}^2, and the sum of re-reflected components, which each pick up a factor of S_{dd}^{2m}, for m re-reflections. This sums to be

$$\begin{aligned} T^m &= S_{1d}S_{d1} + S_{1d}S_{dd}^2 S_{d1} + \cdots + S_{1d}S_{dd}^{2m} S_{d1} \\ &= \frac{S_{1d}S_{d1}}{1 - S_{dd}^2} \end{aligned} \tag{12}$$

If we measure S_{11} using the three-port technique, we can also subtract it to determine S_{dd}.

$$S_{dd} = \frac{R^m - S_{11}}{T^m} \tag{13}$$

We can also detrmine $S_{1d}S_{d1}$

$$S_{1d}S_{d1} = \frac{\left(R^m - S_{11}\right)\left(1 - S_{dd}^2\right)}{S_{dd}} \tag{14}$$

4. The impact of the balun on antenna reflectometry.

4.1. Assuming no common-mode reflections

To simplify things, we assume that $S_{12} = -S_{13}$ and $S_{21} = -S_{23}$ (we check this in section) so that $S_{1c} = S_{c1} = 0$. Our goal is to determine Γ, the differential reflection coefficient of the antenna. These assumptions on the balun can also be relaxed as long as the antenna itself has negligible reflection coefficient $\Gamma_c = 0$. Thus the assumptions of this analysis are $S_{1c} = S_{c1} = 0$ or $\Gamma_c = 0$.

The reflection coefficient we measure, we shall call Γ^m. Here we compute Γ^m in terms of the balun's $S - parameters$ and Γ. We consider an input wave a_m from the VNA on the unbalanced-port 1 of the balun. The VNA measures b_m at the output of port-1 and divides by a_m to determine Γ^m. b_m is contributed to from an initial reflection off of port-1 given by the S_{11} of the balun along components of the input wave that are transmitted through the balun to the antenna and subsequently reflected.

$$b_m = S_{11}a_m + \text{subsequent reflections.} \tag{15}$$

What are these subsequent reflections? The component of the wave that is not immediately reflected at port-1 of the balun is transmitted into both the common and differential mode. We will ignore common-mode for now so that a signal is incident from the balun onto the antenna terminals that is given by

$$S_{d1}a_m \tag{16}$$

This wave is reflected off of the antenna and back onto the differential port of the balun with a factor of the Γ acquired. A component of this wave is transmitted into port-1 with a total complex amplitude of

$$S_{d1}\Gamma S_{1d}a_m \tag{17}$$

so that our total reflected wave has an amplitude of

$$b_m = S_{11} a_m + S_{d1} \Gamma S_{1d} a_m + \dots \tag{18}$$

we are not done because we have still not included the component of the wave that was reflected off of the antenna that did not transmit through the balun to port-1. This component is re-reflected back onto the antenna and returns to the differential port of the balun with an additional factor of ΓS_{dd} some of it will be transmitted to port-1 with a factor of S_{1d} and another component will be re-reflected to pick up another factor of ΓS_{dd} before undergoing the same partial transmission and re-reflection. The sum of these reflections between the differential balun-port and the antenna terminals adds an infinite geometrical sum to b_m.

$$\begin{aligned} b_m &= S_{11} a_m + S_{d1} \Gamma S_{1d} a_m + S_{d1} \Gamma S_{dd} \Gamma S_{d1} a_m + \cdots + S_{d1} \Gamma \left(\Gamma S_{dd} \right)^m S_{1d} a_m \\ &= S_{11} a_m + \frac{S_{d1} \Gamma S_{1d}}{1 - S_{dd} \Gamma} a_m \end{aligned} \tag{19}$$

Thus the measured reflection coefficient is

$$\Gamma^m = S_{11} + \frac{S_d 1 \Gamma S_{1d}}{1 - S_{dd} \Gamma} \tag{20}$$

The antenna can be de-imbedded from the balun by solving equation 20 for Γ.

$$\Gamma = \frac{\Gamma^m - S_{11}}{S_{1d} S_{d1} + S_{dd} \left(\Gamma^m - S_{11} \right)} \tag{21}$$

Both $S_{1d} S_{d1}$ and S_{dd} can be obtained either through back-to-back balun measurements ore the three-port balun technique.