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# Functions

**Table 17**
Mathematical formulation and properties of unimodal, multimodal and composition functions.

| Functions | Dim | Domain | Global opt |
|---|---|---|---|
| $TF_1(x) = \sum_{i=1}^{d} x_i^2$ | 50 | [−100,100] | 0 |
| $TF_2(x) = \sum_{i=1}^{d} \lvert x_i \rvert + \prod_{i=1}^{d} \lvert x_i \rvert$ | 50 | [−100,100] | 0 |
| $TF_3(x) = \sum_{i=1}^{d} (\sum_{j=1}^{d} x_j)^2$ | 50 | [−100,100] | 0 |
| $TF_4(x) = \mathrm{Max}\{\lvert x_i \rvert, 1 \le i \le d\}$ | 50 | [−100,100] | 0 |
| $TF_5(x) = \sum_{i=1}^{d-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ | 50 | [−30,30] | 0 |
| $TF_6(x) = \sum_{i=1}^{d-1} (\lfloor x_i + 0.5 \rfloor)^2$ | 50 | [−100,100] | 0 |
| $TF_7(x) = \sum_{i=1}^{d} i x_i^4 + random[0,1]$ | 50 | [−1.28,1.28] | 0 |
| $TF_8(x) = \sum_{i=1}^{d} -x_i \sin(\sqrt{\lvert x_i \rvert})$ | 50 | [−500,500] | $-418.98 \times d$ |
| $TF_9(x) = \sum_{i=1}^{d} [x_i^2 - 10\cos(2\pi x_i) + 10]$ | 50 | [−5.12,5.12] | 0 |
| $TF_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + e$ | 50 | [−32,32] | 0 |
| $TF_{11}(x) = \frac{1}{4000}\sum_{i=1}^{d} x_i^2 - \prod_{i=1}^{d}\cos(\frac{x_i}{\sqrt{i}}) + 1$ | 50 | [−600,600] | 0 |
| $TF_{12}(x) = \frac{\pi}{d}\{10\sin(\pi y_1) + \sum_{i=1}^{d-1}(y_i - 1)^2[1 + 10\ sin^2(\pi y_{i+1})] + (y_d - 1)^2\} + \sum_{i=1}^{d} u(x_i, 10, 100, 4)$ | 50 | [−50,50] | 0 |
| $TF_{13}(x) = 0.1\{\sin^2(3\pi x_1) + \sum_{i=1}^{d-1}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)] + (x_d - 1)^2[1 + \sin^2(2\pi x_d)]\} + \sum_{i=1}^{d} u(x_i, 5, 100, 4)$ | 50 | [−50,50] | 0 |
| $TF_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$ | 2 | [−65,65] | 1 |
| $TF_{15}(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$ | 4 | [−5,5] | 0.00030 |
| $TF_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$ | 2 | [−5,5] | −1.0316 |
| $TF_{17}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})cosx_1 + 10$ | 2 | [−5,5] | 0.398 |
| $TF_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times$ $[30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$ | 2 | [−2,2] | 3 |
| $TF_{19}(x) = -\sum_{i=1}^{4} c_i\ exp(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2)$ | 3 | [0,1] | −3.86 |
| $TF_{20}(x) = -\sum_{i=1}^{4} c_i\ exp(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2)$ | 6 | [0,1] | −3.32 |
| $TF_{21}(x) = -\sum_{i=1}^{5}[(X - a_i)(X - a_i)^T + c_i]^{-1}$ | 4 | [0,10] | −10.1532 |
| $TF_{22}(x) = -\sum_{i=1}^{7}[(X - a_i)(X - a_i)^T + c_i]^{-1}$ | 4 | [0,10] | −10.4028 |
| $TF_{23}(x) = -\sum_{i=1}^{10}[(X - a_i)(X - a_i)^T + c_i]^{-1}$ | 4 | [0,10] | −10.5363 |

(*continued on next page*)

Results for unimodal, multimodal and composition functions.

| | Function | | MPA | PSO | GA | GSA | CS | SSA | CMA-ES | SHADE | LSHADE-cnEpSin |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Unimodal | TF1 | Ave | 3.27E-21 | 0.0409 | 1.095 | 0.0034 | 210.64 | 0.0037 | 8.27E-15 | 1.08E-08 | 2.19E-04 |
| | | Std | 4.61E-21 | 0.0416 | 0.4896 | 0.0189 | 81.505 | 0.00974 | 5.76E-15 | 1.17E-08 | 1.21E-04 |
| | TF2 | Ave | 1.57E-12 | 0.0659 | 0.106 | 0.0806 | 15.98 | 5.0487 | 1.28E-06 | 0.1226 | 0.04134 |
| | | Std | 1.42E-12 | 0.0864 | 0.0498 | 0.3802 | 4.788 | 2.013 | 2.92E-06 | 0.1854 | 0.02237 |
| | TF3 | Ave | 0.0864 | 4236.3 | 25,187.3 | 1313.88 | 10,412.38 | 4343.27 | 9.170 | 265.12 | 70.118 |
| | | Std | 0.1444 | 1217.9 | 5243.43 | 343.116 | 2456.305 | 2136.39 | 6.533 | 127.43 | 34.618 |
| | TF4 | Ave | 2.6E-08 | 9.335 | 35.619 | 6.410 | 18.507 | 15.055 | 1.44E-04 | 1.644 | 3.1933 |
| | | Std | 9.25E-09 | 1.0119 | 9.4072 | 1.535 | 2.463 | 3.195 | 7.00E-05 | 0.535 | 0.9194 |
| | TF5 | Ave | 46.049 | 310.39 | 715.98 | 76.561 | 27,288.5 | 434.43 | 52.11 | 60.96 | 59.253 |
| | | Std | 0.4219 | 430.60 | 634.71 | 41.64 | 15,589.9 | 457.70 | 23.15 | 35.535 | 29.012 |
| | TF6 | Ave | 0.398 | 0.0589 | 0.925 | 2.21E-12 | 218.17 | 0.0021 | 5.98E-15 | 8.32E-09 | 2.94E-04 |
| | | Std | 0.1914 | 0.1217 | 0.5063 | 5.91E-13 | 53.864 | 0.0030 | 5.36E-15 | 1.00E-08 | 2.71E-04 |
| | TF7 | Ave | 0.0018 | 0.0665 | 0.1130 | 0.0926 | 0.4055 | 0.2807 | 0.0320 | 0.0294 | 0.00993 |
| | | Std | 0.0010 | 0.0123 | 0.0355 | 0.0322 | 0.1313 | 0.0911 | 0.0077 | 0.0100 | 0.00306 |
| Multimodal (High dimensional) | TF8 | Ave | −13,594.1 | −10,815.3 | −17,911.6 | −3570.52 | −11,942.8 | −12,232.6 | −11,670.6 | −14,832.5 | −15,928.4 |
| | | Std | 811.3 | 992.1 | 343.1 | 592.0 | 343.1 | 1063.0 | 884.3 | 418.8 | 516.8 |
| | TF9 | Ave | 0.000 | 78.42 | 35.61 | 32.36 | 220.86 | 78.79 | 30.91 | 101.54 | 76.052 |
| | | Std | 0.000 | 16.44 | 8.971 | 7.055 | 22.055 | 25.18 | 5.383 | 14.874 | 9.15 |
| | TF10 | Ave | 9.69E-12 | 1.204 | 0.1844 | 8.94E-07 | 9.493 | 3.479 | 11.019 | 0.190 | 0.00363 |
| | | Std | 6.13E-12 | 0.729 | 0.1487 | 1.54E-07 | 2.0936 | 0.8281 | 9.795 | 0.448 | 0.00192 |
| | TF11 | Ave | 0.000 | 0.0128 | 0.6561 | 26.479 | 3.067 | 0.0905 | 2.08E-10 | 0.0027 | 0.00424 |
| | | Std | 0.000 | 0.0130 | 0.1881 | 5.7472 | 0.782 | 0.0407 | 1.51E-10 | 0.0051 | 0.00638 |
| | TF12 | Ave | 0.0085 | 0.0319 | 0.0344 | 1.0151 | 11.209 | 8.541 | 2.99E-13 | 0.0187 | 0.00208 |
| | | Std | 0.0052 | 0.0560 | 0.0776 | 0.5386 | 7.5438 | 2.556 | 1.97E-13 | 0.0635 | 0.01135 |
| | TF13 | Ave | 0.4901 | 0.419 | 0.189 | 10.25 | 1306.48 | 59.895 | 4.31E-12 | 0.00183 | 0.00711 |
| | | Std | 0.1932 | 0.5814 | 0.0972 | 6.335 | 3931.79 | 16.745 | 3.44E-12 | 0.0041 | 0.00797 |
| Multimodal (Fixed-dimensional) | TF14 | Ave | 0.9980 | 2.1825 | 0.9980 | 3.7182 | 0.9980 | 1.0311 | 8.1094 | 0.9980 | 0.9980 |
| | | Std | 2.47E-16 | 2.0085 | 8.84E-12 | 2.678 | 5.46E-16 | 0.1815 | 5.9456 | 3.38E-16 | 0.0000 |
| | TF15 | Ave | 3.07E-04 | 5.61E-04 | 2.69E-03 | 2.05E-03 | 3.97E-04 | 8.40E-04 | 1.09E-02 | 1.76E-03 | 3.07E-04 |
| | | Std | 4.09E-15 | 4.38E-04 | 4.84E-03 | 6.64E-04 | 1.05E-04 | 2.81E-04 | 1.87E-02 | 5.06E-03 | 1.34E-19 |
| | TF16 | Ave | −1.0316 | −1.0316 | −1.0316 | −1.0316 | −1.0316 | −1.0316 | −1.0316 | −1.0316 | −1.0316 |
| | | Std | 4.46E-16 | 6.64E-16 | 2.39E-08 | 2.10E-15 | 4.79E-16 | 1.48E-14 | 6.77E-16 | 0.0000 | 6.51E-16 |
| | TF17 | Ave | 0.3979 | 0.3979 | 0.3979 | 0.3979 | 0.3979 | 0.3979 | 0.3979 | 0.3979 | 0.3979 |
| | | Std | 9.12E-15 | 0.000 | 1.90E-06 | 6.14E-16 | 7.23E-14 | 5.62E-14 | 0.0000 | 1.12E-16 | 0.0000 |
| | TF18 | Ave | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 5.7000 | 3.0000 | 3.0000 |
| | | Std | 1.95E-15 | 1.38E-15 | 2.45E-07 | 7.00E-14 | 1.85E-15 | 2.46E-13 | 1.48E+01 | 4.52E-16 | 1.92E-15 |
| | TF19 | Ave | −3.8628 | −3.8628 | −3.8628 | −3.8628 | −3.8628 | −3.8628 | −3.8628 | −3.8628 | −3.8628 |
| | | Std | 2.42E-15 | 2.68E-15 | 2.85E-08 | 9.33E-15 | 2.48E-15 | 3.96E-14 | 2.71E-15 | 2.71E-15 | 2.71E-15 |
| | TF20 | Ave | −3.3220 | −3.2625 | −3.2705 | −3.3220 | −3.3220 | −3.2344 | −3.2903 | −3.2824 | −3.3220 |
| | | Std | 1.14E-11 | 6.05E-02 | 5.99E-02 | 1.56E-13 | 1.01E-07 | 5.87E-02 | 5.35E-02 | 5.70E-02 | 1.36E-15 |
| | TF21 | Ave | −10.1532 | −5.3010 | −6.1531 | −6.5834 | −10.1532 | −9.9837 | −6.7946 | −9.7358 | −10.1532 |
| | | Std | 2.53E-11 | 2.9288 | 3.6282 | 3.6765 | 7.06E-07 | 1.3240 | 3.6860 | 1.6200 | 6.39E-15 |
| | TF22 | Ave | −10.4029 | −7.0716 | −7.9827 | −10.4029 | −10.4029 | −9.3507 | −9.2089 | −10.1484 | −10.1532 |
| | | Std | 2.81E-11 | 3.6840 | 3.4924 | 3.18E-13 | 1.07E-04 | 2.569 | 2.7439 | 1.3943 | 6.39E-15 |
| | TF23 | Ave | −10.5364 | −7.2467 | −7.7652 | −10.5364 | −10.5364 | −9.8704 | −9.2797 | −10.5364 | −10.5364 |
| | | Std | 3.89E-11 | 3.6582 | 3.7265 | 3.07E-13 | 9.60E-05 | 2.2936 | 2.8975 | 9.03E-15 | 2.47E-15 |

(*continued on next page*)

The 23 functions are divided into 3 groups. First group contains 12 functions, which are F1, F3, F4, F5, F6, F7, F8, F9, F10, F11, F12, and F13. (First group mainly contains unimodal and multimodal high-dimensional functions) The second group consists of 7 functions, named F2, F14, F16, F17, F18, F19, and F20. The last group consists of 4 functions, i.e., F15, F21, F22, and F23. (Second group and third group mainly contain multimodal fixed dimensional functions)

## Conclusion

- SSGA converges quickly on the first group of functions, followed by Lamarck, then Baldwin, but the quality of the final results produced by all three is comparable in the long run, i.e., if SSGA produces results with an accuracy of 0.01, then so do Lamarck and Baldwin. (First group mainly contains unimodal and multimodal high-dimensional functions)
- In the second group, Lamarck and Baldwin initially performed significantly better than SSGA, but their subsequent performance was similar to that of the first group, with SSGA converging fastest, followed by Lamarck and Baldwin, producing results of comparable quality in the long run.
- In the third group, the SSGA converged quickly, but Lamarck and Baldwin are able to find smaller solutions in the long run. (Second group and third group mainly contain multimodal fixed dimensional functions)

In terms of convergence speed, SSGA converges fast, Lamarck is second and Baldwin is third. In terms of the quality of the results produced, Lamarck and Baldwin perform better on multimodal fixed dimensional functions. Either they find significantly better results initially (second group), or the quality of the final results found is better, i.e., they find smaller results (third group).

## Sampling

Baldwin and Lamarck need to compute f.() twice during each iteration(one individual has a genotype and a phenotype), but SSGA only needs to compute f.() once (one individual only has one genotype). So, for a given budget, say budget=10,000, this means that SSGA can perform 10,000 iterations, but Baldwin and Lamarck can only perform 5000 iterations. SSGA takes the best solution every 50 iterations, but Baldwin and Lamarck take the best solution

every 25 iterations.1 In this case, the final number of data points sampled is the same. We gave all 20 parameter combinations 20 runs. (Note: for the first generation, the use of f.() for initialization is not counted. Because the parameter : number of individuals is not fixed in 20 parameter combinations.)

## First Group

Figure 1 shows the Budget-Best solution curve for F8, F9 in first group. Figure 2 shows the Budget-Best solution curve for F4, F10in first group. Figure 3 shows the Budget-Best solution curve for F1, F3, F5, F6, F7, F11, F12 and F13 in first group.

*Figure 1 Budget convergence for part of functions of first group, budget = 10,000*

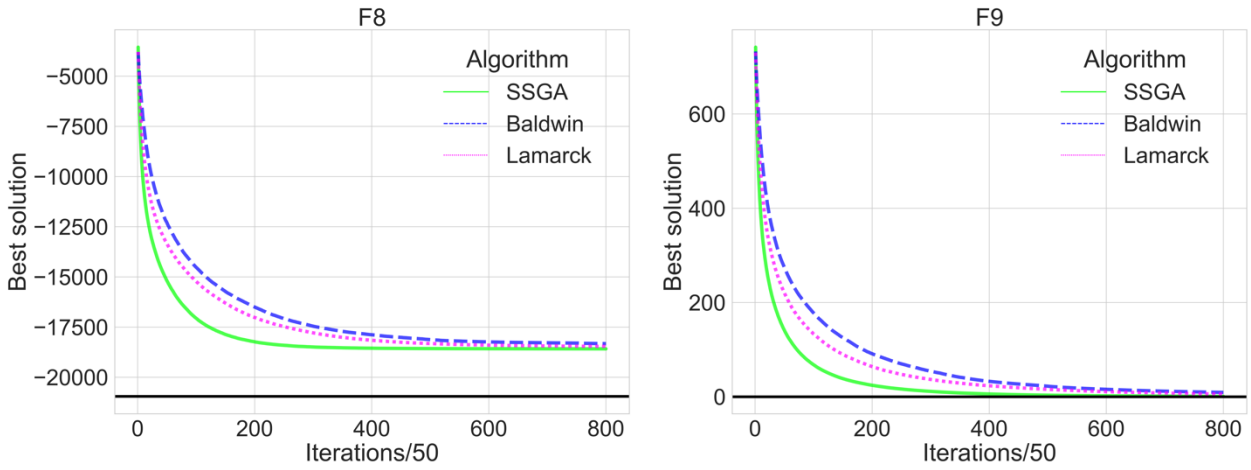



*Figure 2 Budget convergence for part of functions of first group, budget = 10,000*

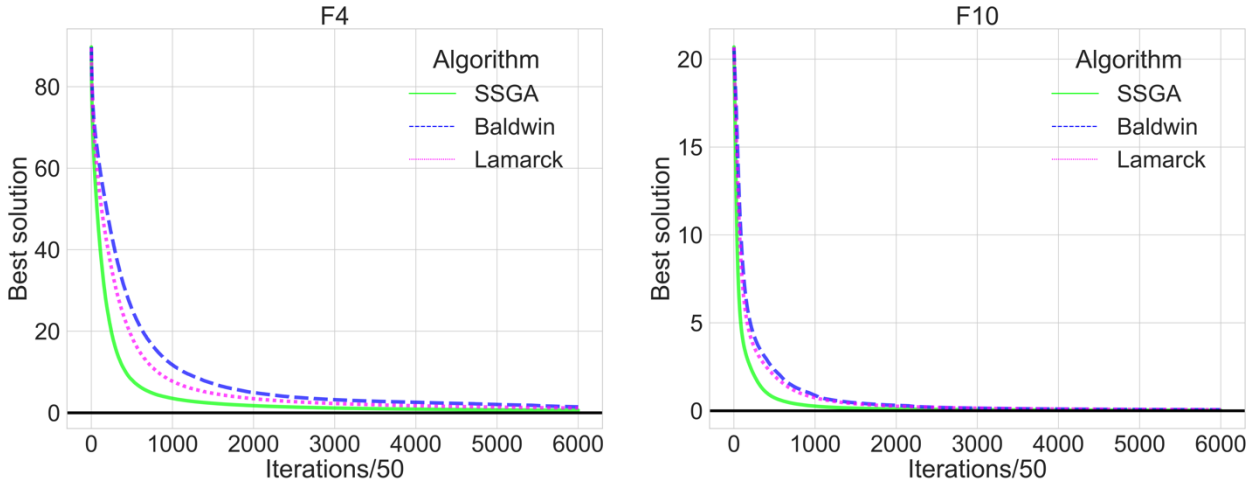
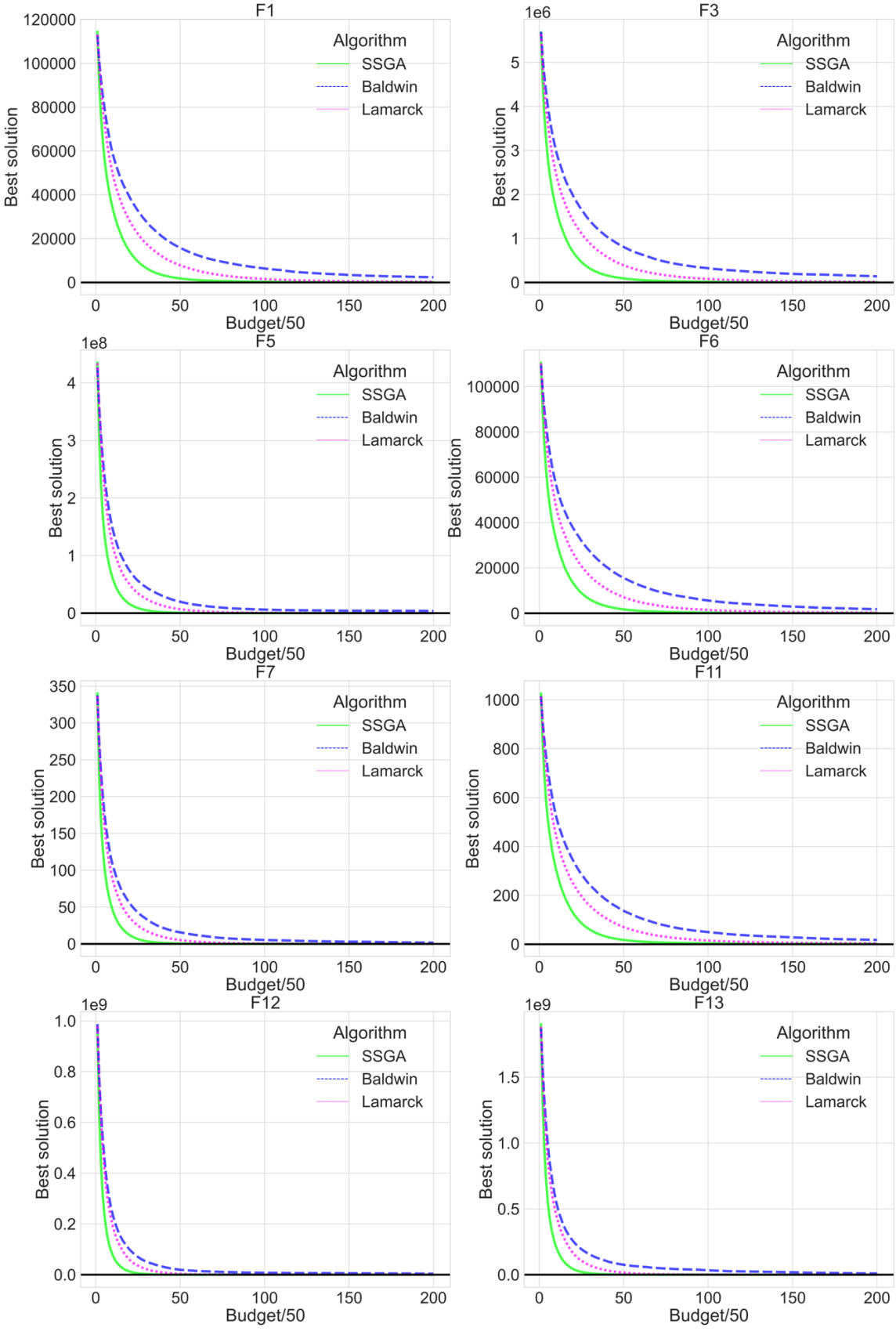


*Figure 3 Budget convergence for part of functions of first group, budget = 10,000*



## Second group

Figure 4 shows Budget-Best solution curve for the second group.

Figure 5 shows Budget-Best solution curve for the second group with less budgets.

Figure 6 shows the first 50 budgets for the second group.

*Figure 4 Budget convergence for second group, budget = 10,000*

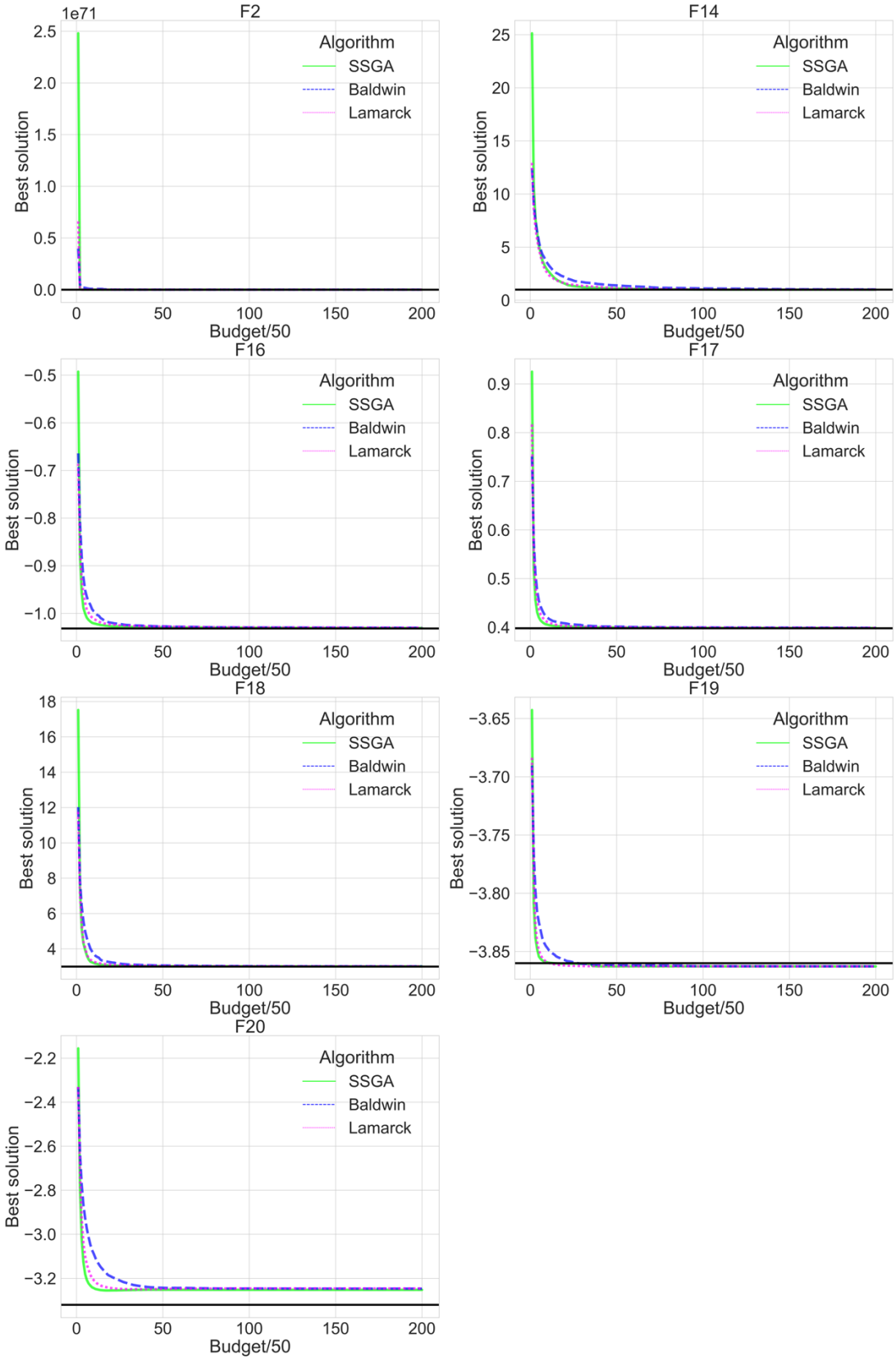


*Figure 5 Budget convergence for second group, budget = 1,000*

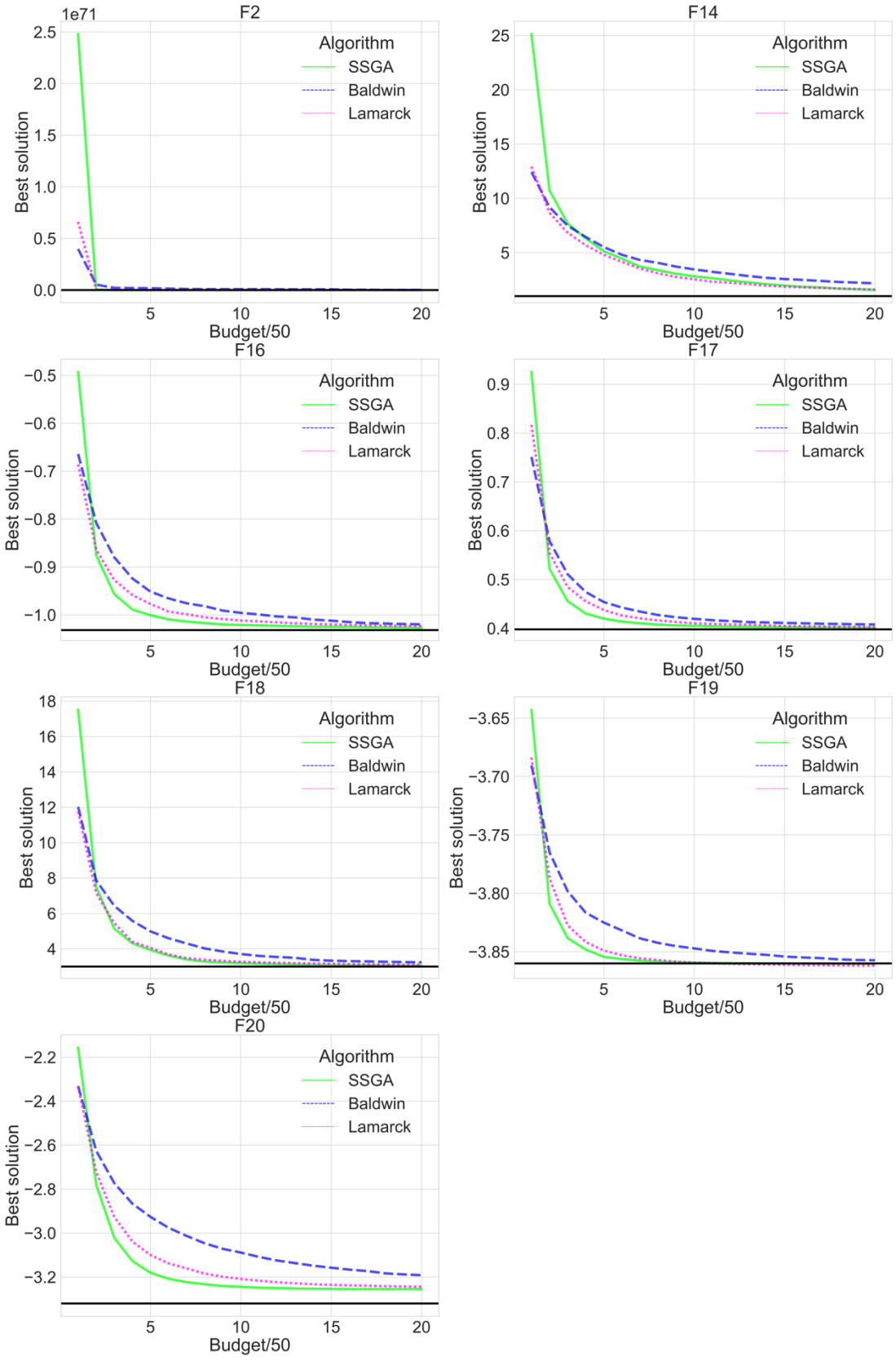
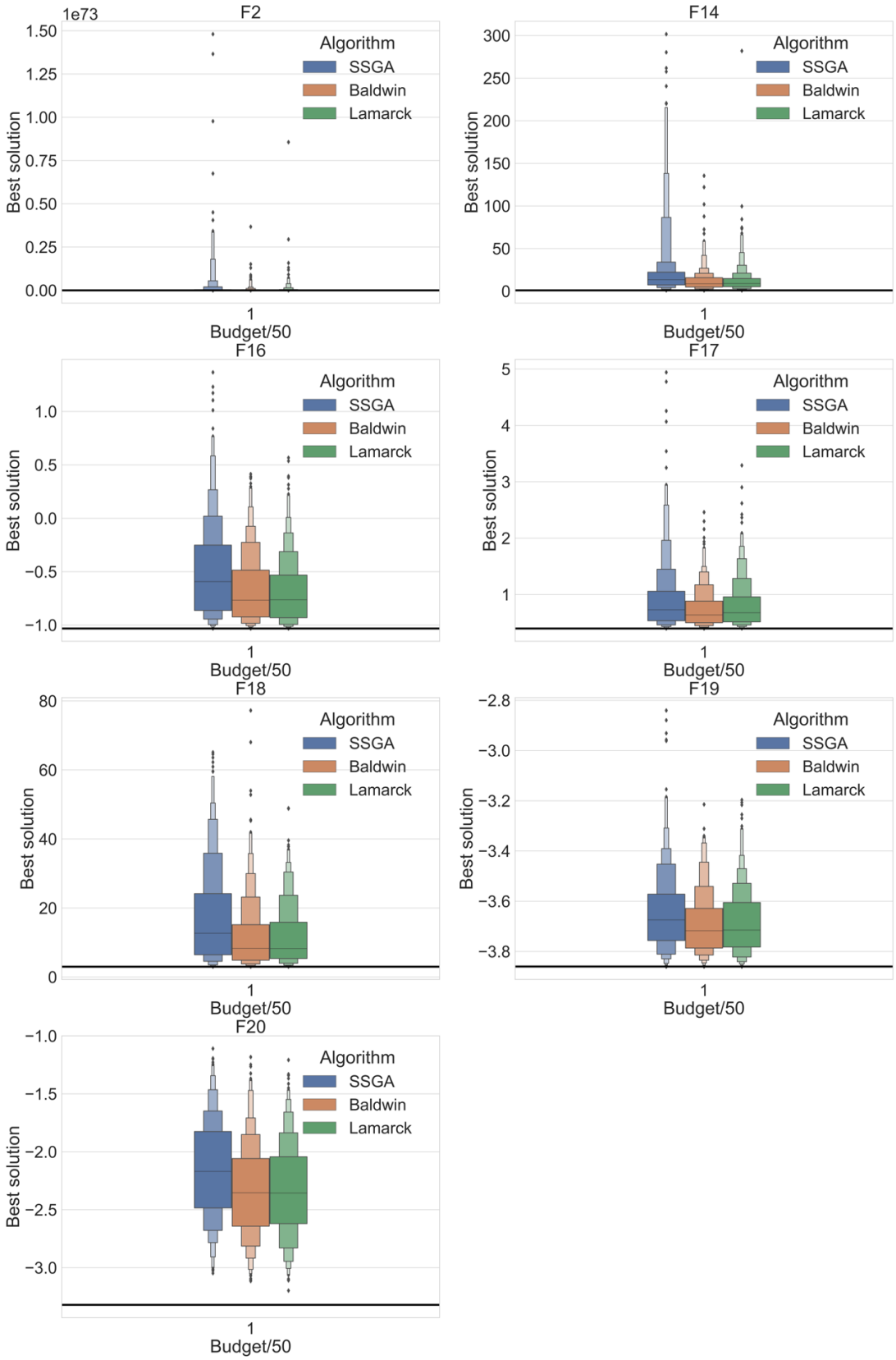


*Figure 6 first 50 budgets for second group*



## Third group

Figure 7 shows Budget-Best solution curve for the third group.

*Figure 7 Budget convergence for the third group*