Efficient Learning and Planning with Compressed Predictive States

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Problem

Goal is to construct RL agents capable of *agnostic* learning, i.e. learning and planning in complex partially observable domains without any prior knowledge (no known state-space, transition probabilities etc.).

Contribution

Developed a novel, efficient model-based RL algorithm for *agnostic* learning and planning. The learning algorithm combines predictive state representations (PSRs) [1] and random projections [5] in order to:

- Regularize the model.
- Increase computational efficiency of learning.

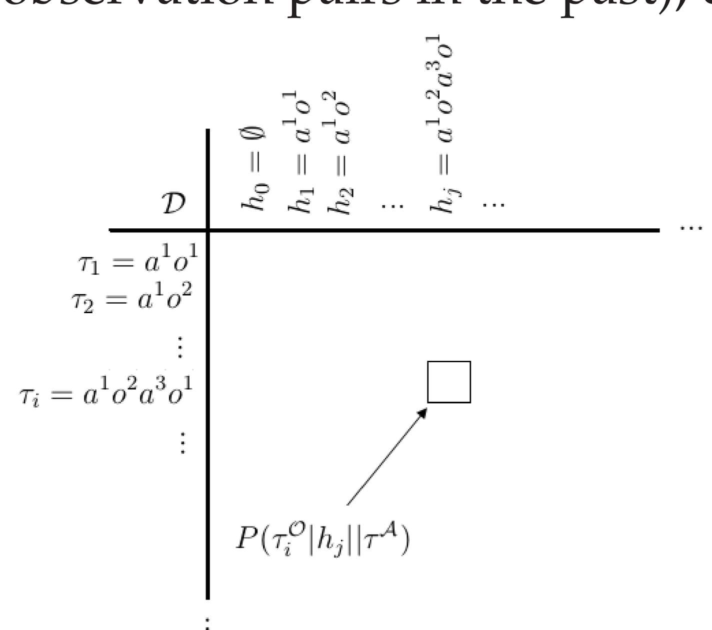
The planning algorithm uses a modified version of fitted-Q value iteration [3].

PSR Models

Predictive state representations model dynamical systems using only observable events [1].

- More general than hidden state learning approaches (e.g. EM with HMMs).
- Immune to local minima (e.g. TPSRs [2]).

Goal is learning the conditional probabilities of tests (sequences of action-observation pairs in the future), denoted τ_i, given histories (sequences of action-observation pairs in the past), denoted h_j.



Exploiting Sparsity

We say $\mathcal{D}$ is k sparse if $\forall \mathbf{c_i} \in \mathcal{D} : ||\mathbf{c_i}||_0 \leq k$.
Only a subset of tests possible given any history.
With sparsity condition, most of the information is preserved after **random projections**.
We compress $\mathbf{Y}^{m \times n}$ to $\mathbf{X}^{d \times n}$, where $d \ll m$:

$$\mathbf{X} = \Phi \mathbf{Y}$$

$\Phi^{d \times m}$ is a Johnson-Lindestrauss projection.

CPSR Learning Algorithm

Let $\mathcal{T}$ denote the set of all defined tests and $\mathcal{H}$ the set of all possible histories. Define the *observable matrices*:

- $\mathcal{P}_{\mathcal{T},\mathcal{H}}$: joint probability of all tests $\tau_i \in \mathcal{T}$ and histories $h_j \in \mathcal{H}$.
- $\mathcal{P}_{\mathcal{T},ao,\mathcal{H}}$: joint probability of tests $\tau_i \in \mathcal{T}$ and histories $h_j \in \mathcal{H}$ with action-observation pair ao prepended to each test.
- $\mathcal{P}_{\mathcal{H}}$: marginal probability for all $h_j \in \mathcal{H}$.

Obtain **compressed estimates** of the observable matrices: $\Phi_{\mathcal{T}} \hat{\mathcal{P}}_{\mathcal{T},\mathcal{H}} \Phi_{\mathcal{H}}^T$, $\hat{\mathcal{P}_{\mathcal{H}}} \Phi_{\mathcal{H}}^T$, $\Phi_{\mathcal{T}} \hat{\mathcal{P}}_{\mathcal{T},ao,\mathcal{H}} \Phi_{\mathcal{H}}^T$.

↓

Use **regression** on the **compressed estimates** to build **compact model**:

$$\mathbf{c_1} = (\Phi_{\mathcal{T}} \hat{\mathcal{P}}_{\mathcal{T},ao,\mathcal{H}} \Phi_{\mathcal{H}}^T) \mathbf{1}_d$$

$$\mathbf{C}_{ao} = (\Phi_{\mathcal{T}} \hat{\mathcal{P}}_{\mathcal{T},ao,\mathcal{H}} \Phi_{\mathcal{H}}^T)(\Phi_{\mathcal{T}} \hat{\mathcal{P}}_{\mathcal{T},\mathcal{H}} \Phi_{\mathcal{H}}^T)^+ \ \forall ao$$

$$\mathbf{c}_\infty = \hat{\mathcal{P}_{\mathcal{H}}} (\Phi_{\mathcal{T}} \hat{\mathcal{P}}_{\mathcal{T},\mathcal{H}} \Phi_{\mathcal{H}}^T)^+$$

- $\mathbf{c_1}$ is the initial **predictive model state**.
- $\mathbf{C}_{ao}$ are **update operators**.
- $\mathbf{c}_\infty$ is a **normalizer**.

Fitted-Q Planning

Define the **action-value (Q) function**:

$$Q : \mathbf{C} \times \mathcal{A} \rightarrow \mathbb{R}$$

$\mathbf{C}$ is the space of CPSR model states and $\mathcal{A}$ the set of actions. $Q(\mathbf{c}, a)$ is expected return given by taking action $a \in \mathcal{A}$ in predictive state $\mathbf{c} \in \mathbf{C}$. We estimate $\hat{Q}(\mathbf{c}, a)$ iteratively [3].

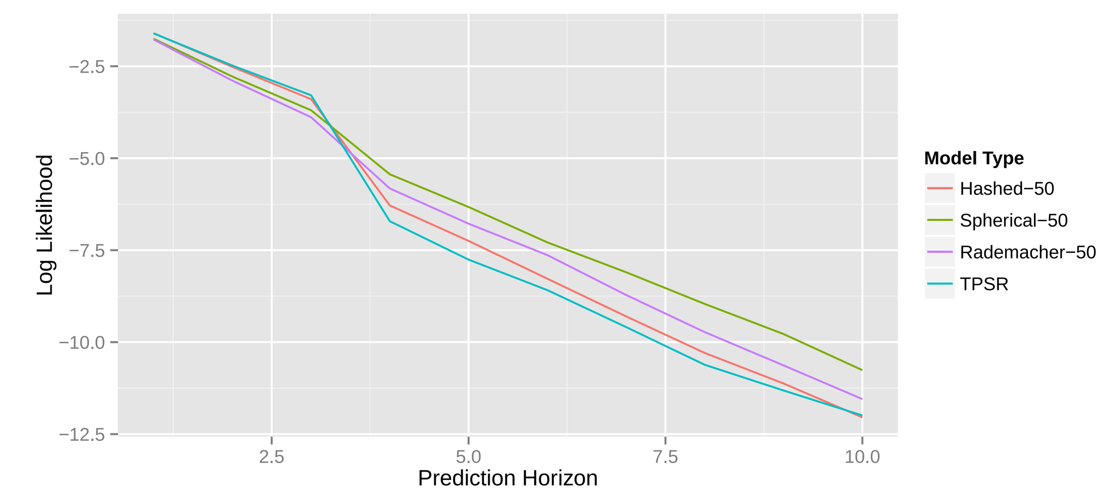
Build train set, $\mathbb{T} = \{(y^l, i^l), l = 1, ..., |\mathcal{Z}|\}$. $i^l = (\mathbf{c}_t^l, a_t^l)$, $y^l = r_t^l + \gamma \max_a \hat{Q}_{k-1}(\mathbf{c}_t^l, a)$.

↓ ↑

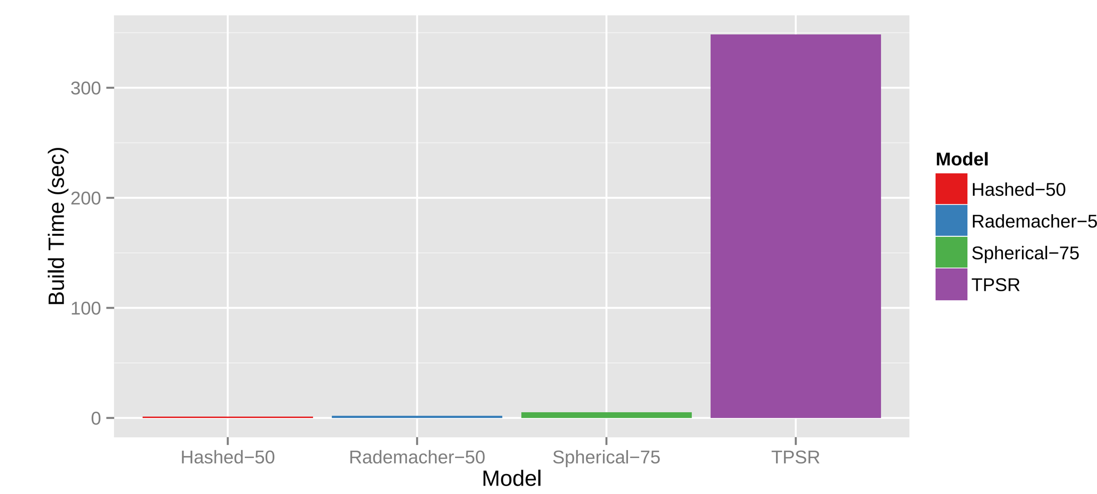
Use random forests on $\mathbb{T}$ to obtain $\hat{Q}_k$.

Empirical Results

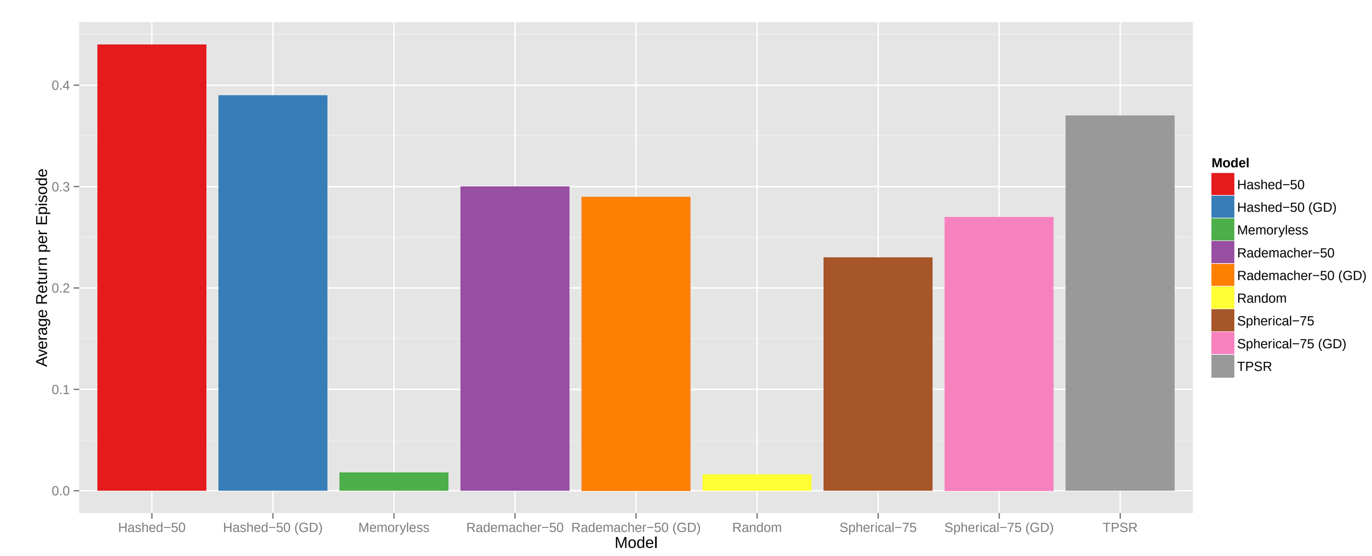
We examine the performance of different CPSRs and some baselines on two domains: *ColoredGridWorld*, a 47 state maze with 4 (noisy) actions and 81 observations; and *S-PocMan*, a partially observable variant of the video-game PacMan with approximately 10^{56} states, 4 actions, and 2^8 observations [4].



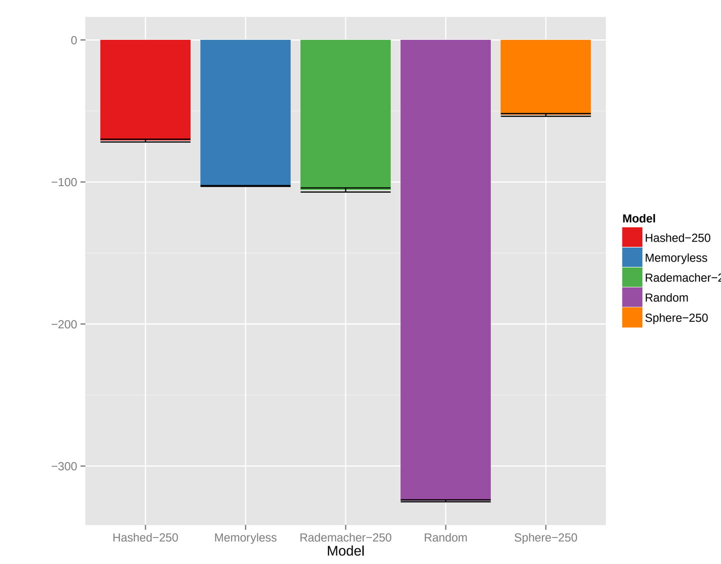
Model quality of compressed and uncompressed PSRs on *ColoredGridWorld* domain.



Empirical runtimes for constructing model of *ColoredGridWorld* domain



Average return achieved by planners in the *ColoredGridWorld* domain using uncompressed and compressed PSRs, as well as results from a memoryless controller baseline.



Average return achieved by planners in the *S-PocMan* domain.

Theoretical Results

With probability no less than $1 - \delta$ we have:
$\|\mathbf{C}_{ao}(\Phi \mathcal{P}_{\mathcal{Q},h}) - \Phi \mathcal{P}_{\mathcal{Q},ao,h}\|_{\rho(\mathbf{x})}$
$\leq \sqrt{d} \epsilon(|\mathcal{H}|, |\mathcal{Q}|, d, L_{ao}, \sigma_{ao}^2, \delta/d)$
where L_{ao} is a function of the solution's norm and σ_{ao} the induced noise after projection

$\Leftrightarrow$

With **high probability** the compressed model has **error proportional to** $\sqrt{d}$ (where d is the compressed dimension) times the usual error of performing **compressed regression** (which is bounded [5]).

Discussion

- Accuracy of CPSRs competitive with uncompressed PSRs.
- Compressed learning has far lower computational cost and regularizes the learned model.
- CPSRs facilitate model-based RL in complex partially observable domains that are intractable for uncompressed PSR learning.

References

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