

2/8/2023
Wednesday

Symbol: Any character that belongs to a to z (or) 0-9 (or) any special characters like , , # , id , (,) ...

Alphabet (Σ): It is denoted by ' Σ '. It is a finite set of elements where those elements are nothing but symbols

Eg: $\Sigma_1 = \{a, b\}$, $\Sigma_2 = \{0, 1\}$, $\Sigma_3 = \{(,)\}$

String (W): It is a sequence of characters formed by given alphabet.

- Input symbols are alphabet elements \rightarrow set of symbols
- It is denoted by ' W '

Note: $|W| \Rightarrow$ • Magnitude of string (or) length of string
• It returns no. of characters in the string

Empty String (ϵ): A string whose length is equal to 'zero'.
i.e., $|W| = 0$

- It is represented by " ϵ " \rightarrow read as (Epsilon)

Note: Q) $w = \text{abaab}$ then $|W| = ?$

A) length of string (W) = 5

$$\Rightarrow |W| = 5$$

Language (L): It is represented by ' L '. and It is a set of strings formed over given alphabet symbol (Σ).

Eg 1: $\Sigma = \{0, 1\}$ design a language ' L ' which contains strings of length exactly 2. $\textcircled{A} L = \{00, 01, 10, 11\}$.

Eg2: $\Sigma = \{a, b\}$ design a Language which contains strings starting with 'a'.

Sol: $L = \{ a, aa, ab, aaa, aab, aba, abb, aaaa, \dots \}$

NOTE: Language set can be a finite set (or) infinite set.

Eg-3: Design a Language which contains even length strings ($\Sigma = \{a, b\}$)

Sol: $L = \{ \epsilon, aa, ab, ba, bb, aaaa, abaa, abba, abbb, bbbb, \dots \}$
 \downarrow
empty string $\rightarrow |w| = 0^{\text{even}}$

CLOSURES

1) Positive closure (+): $\Sigma = \{a, b\}$
 $\Sigma^+ = \{ a, b, aa, ab, ba, bb, \dots \}$

- It is combination of all strings that are formed by using given alphabet symbols.

2) Kleene closure (*): $\Sigma = \{a, b\}$
 $\Sigma^* = \{ \epsilon, a, b, aa, ab, ba, bb, \dots \}$

- It is combination of all strings that are formed by using given alphabet symbols including " ϵ ".

NOTE: Positive closure never contains an empty string i.e., " ϵ ". but Kleene closure must contain an empty string (ϵ).

★ Relationship b/w L^+ and L^* :

$$L^* = L^+ \cup \{E\}$$

• $L^* = L^+ + \{E\}$. , Here $+$ and \cup are same .

$$\rightarrow L_1 = \{a, b\} \quad L_2 = \{c, d\}$$

$$L_1 \cdot L_2 = \{ac, ad, bc, bd\}$$

$$\rightarrow L = \{a, b\} .$$

$$L^2 = L \cdot L = \{aa, ab, ba, bb\} . \text{ similarly } L^3 = L^2 \cdot L \text{ and } \dots$$

Q.1) $\Sigma = \{a, b\}$. Design a language which contains strings starting with b .

Sol: $L_1 = \{bab, ba, baa, bb, bbb, baaa, baba, \dots\}$

Q.2) $\Sigma = \{a, b\}$, Design a language which contains strings ending with 'b' .

Sol: $L_2 = \{ab, bb, aab, abb, bbb, baab, \dots\}$

Q.3) Design a language which contains strings which 'b' ($\Sigma = \{a, b\}$) .

Sol: $L = L_1 \cup L_2$.

→ Exercise :

$$L_1 = \{abc, bba, abb, abab\}$$

$$L_2 = \{aa, bc, dd, bb\}$$

$$L_1 \cdot L_2 = \{abcaa, abcbc, abcd, abcb, bbaaa, bbabc, bbadd, bbabb, abbaa, abbba, abbdd, abbbb, ababaa, ababbc, ababdd, ababbb\}$$

$$L_2 \cdot L_1 = \{aaabc, aabba, aaabb, aabab, bcabc, bcbba, bcabb, bcabab, ddabc, ddbba, ddabb, ddabab, bbabc, bbbba, bbabb, bbabab\}$$

$$L_1^+ =$$

$$L_1^* =$$

Grammar (G):

- It is a four tuple
- It is represented by $G(V, T, P, S)$

Here, V = finite set of variables (or) non-terminals

T = finite set of Terminals

P = Finite set of Productions .. where it is of the form $\alpha \rightarrow \beta$

Production (or) Grammar rule

S = It is starting variable in the given Productions

Eg:

$$A \rightarrow abB$$

$$B \rightarrow bD/f$$

$$D \rightarrow d$$

$B \rightarrow bD/f$
$\Rightarrow B \rightarrow bD$
$\Rightarrow B \rightarrow f$

$$V = \{ A, B, D \}$$

$$T = \{ a, b, d, f \}$$

$$P = \{ A \rightarrow abB, B \rightarrow bD, B \rightarrow f, D \rightarrow d \}$$

$$S = A$$

NOTE :-

- The ultimate goal of any grammar is to produce strings. If in case a Grammar is not able to produce a string, then G is called Useless grammar.