

The Ball Problem - Editorial

Difficulty:

Medium - Hard

Prerequisites:

Combinatorics

Divide and Conquer - ([Tutorial](#))

Fast Fourier Transform - ([Tutorial](#))

Generating Function - ([Tutorial](#))

Problem in Brief:

Given an Array A, and a number K, count the different possible ways to choose K elements from A. Two ways are considered different if count of elements is different in them for any element.

Editorial:

We can convert this problem to a combinatorial one. Notice that the equation $X_1 + X_2 + \dots + X_{200000} = N$ where X_i is the number of balls with color i that we choose is the solution to our problem.

If we could choose as many balls of any color that we wanted then the solution would be $C(N + K - 1, K)$ where $C(i, j)$ is the binomial coefficient which is the number of ways to choose j objects from the given i objects.

But we have a restriction:

Let L_i be the count of ball colored i in the given N balls.

The restriction $X_i \leq L_i$ should be followed for all i colors.

To overcome this restriction we will use Generating Functions.

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The solution to our problem is the coefficient of X^K in the following polynomial:

$$(X^0 + X^1 + X^2 + X^3 + \dots X^{L-1}) * (X^0 + X^1 + X^2 + X^3 + \dots X^{L-2}) * (X^0 + X^1 + X^2 + X^3 + \dots X^{L-3}) * \dots (X^0 + X^1 + X^2 + X^3 + \dots X^{L-200000})$$

Now to actually find this polynomial we will use Fast Fourier Transform. Notice that if we multiply the polynomials 1 by 1, The complexity will become $O(N^2 * \log(N))$ which is not useful for us.

To find this polynomial efficiently we will use Divide And conquer:

For a given (L, R)

We will find polynomials (L, mid) and (mid + 1, R)

And multiply them using Fast Fourier Transform to obtain our answer.

In the base case we will return the polynomial with all coefficients = 1 till $\text{freq}[\text{elem}]$ where elem is the value of the element being considered in the base case and $\text{freq}[x]$ is the number of times x appears in the N balls.

We finally return the Kth coefficient in polynomial of (1, 200000) as our answer.

Time Complexity:

$$O(N * \log^2(N))$$

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