Wedding Crasher Tutorial

Pre-requisites:

This problem will concepts of divisibility to solve. ake sure your basics of divisibility and number theory are clear before attempting this problem. The following links can help you learn the prerequisites:

- http://www.tutorialspoint.comprime_numbers_factors_and_multiplesdivisibility_rules_f/
- http://www.topcoder.comcommunitydatasciencedatasciencetutorialsprimenumber/
- http://www.topcoder.comcommunitydatasciencedatasciencetutorialsmathematicsf/

Problem Description:

In this problem, we have to determine if $5^n + 1$ divides $5^m + 1$ or not
Now, as per the constraints, $0 \leq n, m \leq 10^3$. This makes it infeasible to calculate the modulus of two numbers as large as 5^10^3. Moreover, we have to calculate it for 'N' such pairs where $1 \leq N \leq 10^5$. The challenge is to determine the divisibility of so many large numbers in practical time.

Difficulty Level:

Medium - Hard

Editorial:

This question involves a concept of number theory and once that concept is understood, the solution becomes pretty easy.

Let's begin with the concept first:

If n is an odd integer,
$a^n + b^n = (a+b)(a^{n-1} - a^{n-2} b + \ldots - ab^{n-2} + b^{n-1})$

Therefore, $a^n + b^n$ is always divisible by a+b if n is odd.

Now, moving to the problem,
if n|m
=> m = nk

now, $5^m + 1 = 5^{n.k} + 1$

$= (5^n)^m + 1$

$= (5^n)^m + 1^m$
$= (5^n + 1)$ (_____)

therefore, if n|m => $5^n + 1 \mid 5^m + 1$

Now, all we need to compute is whether n | m or not, because as per the constraints of the problem, computing whether $5^n + 1 \mid 5^m + 1$ or not directly would be infeasible.

Complexity of solution:

The complexity of the solution will be the same as that of finding whether two numbers are divisible or not, which can be solved in constant time. For N test cases, the complexity becomes O(N).