

Valid Line Tutorial

Pre-requisites:

This problem will concepts of gcd, divisibility and dynamic programming to solve. Make sure your basics of gcd, euclidean algorithm, number theory and dynamic programming are clear before attempting this problem. The following links can help you learn the prerequisites:

- <https://www.topcoder.com/community/data-science/data-science-tutorials/prime-numbers-factorization-and-euler-function/>
- <https://www.topcoder.com/community/data-science/data-science-tutorials/mathematics-for-topcoders/>
- <http://www.geeksforgeeks.org/basic-and-extended-euclidean-algorithms/>
- <http://www.virtualnerd.com/pre-algebra/factors-fractions-exponents/prime-factorization-greatest-common-factor/greatest-common-factor/greatest-common-factor-two-numbers>
- <http://www.geeksforgeeks.org/recursion/>
- <https://www.topcoder.com/community/data-science/data-science-tutorials/an-introduction-to-recursion-part-1/>
- <https://www.topcoder.com/community/data-science/data-science-tutorials/an-introduction-to-recursion-part-2/>
- <https://www.topcoder.com/community/data-science/data-science-tutorials/dynamic-programming-from-novice-to-advanced/>

Problem Description:

In this problem, out of the given set of numbers x_1, x_2, \dots, x_n , we have to find the length of the largest sequence possible $\langle a_1, a_2, a_3, \dots, a_k \rangle$ such that it satisfies the following conditions:

1. $\gcd(a_i, a_{i+1}) > 1$
2. $a_i < a_{i+1}$

Difficulty Level:

Easy-Medium

Editorial:

The main idea is DP. Let's define $dp[a]$ as the maximal value of the length of the valid sequence whose last element is a , and define $d[i]$ as the (maximal value of $dp[a]$ where a is divisible by i).

You should calculate $dp[a]$ in the increasing order of a . The value of $dp[a]$ is (maximal value of $d[i]$ where i is a divisor of a) + 1. After you calculate $dp[a]$, for each divisor i of a , you should update $d[i]$ too.

Note that there is a corner case. When the set is $\{1\}$, you should output 1.

Complexity of solution:

This algorithm works in $O(n \log n)$ because the sum of the number of the divisor from 1 to n is $O(n \log n)$.