Data structures and Algorithms

Trees

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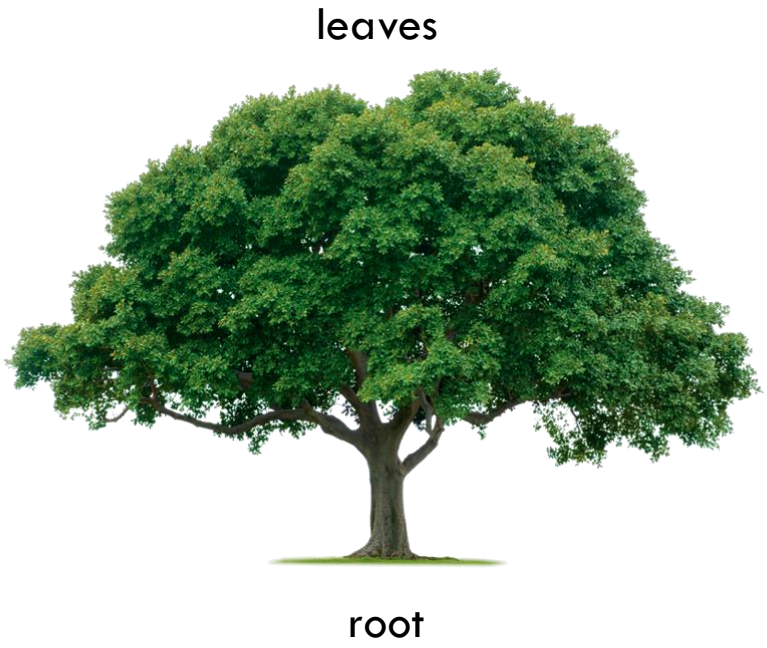
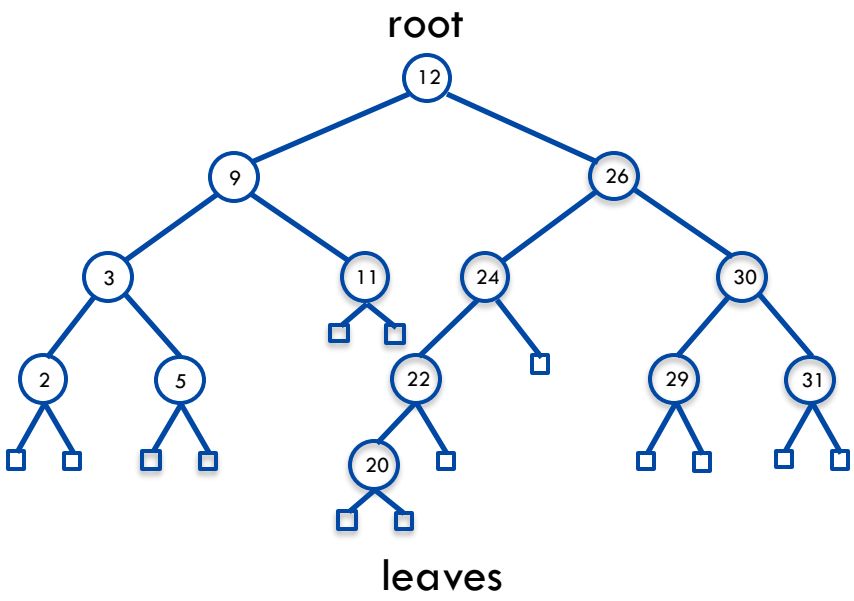
Agenda: Trees

- Definition and terminology
- Applications
- Tree ADT
- Tree traversal algorithms
- Binary trees
- Implementing trees
- Recursive code on trees

Trees



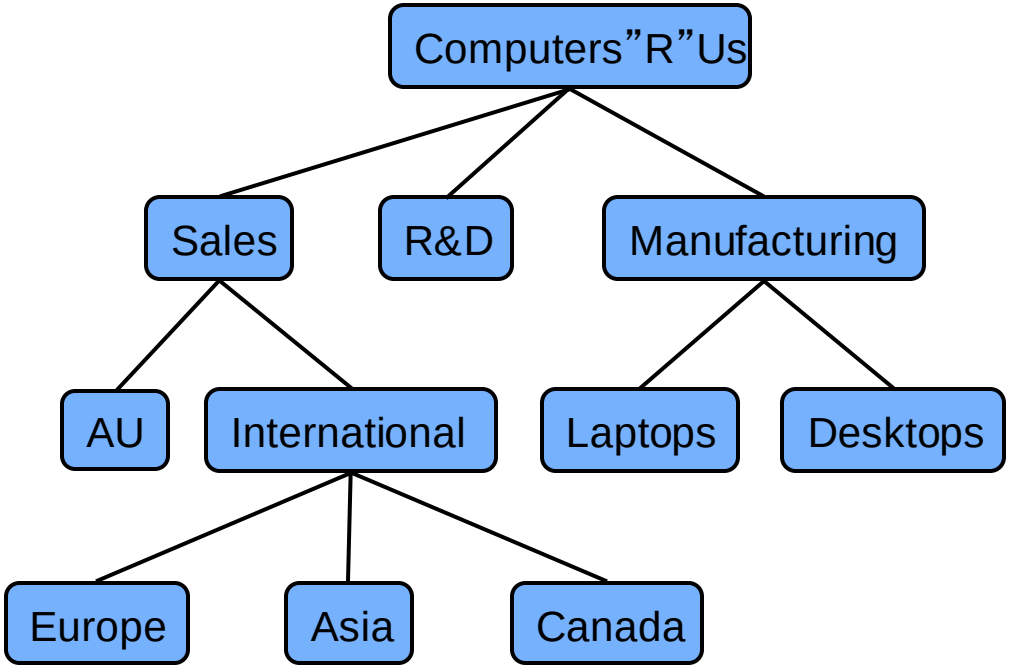


What is a Tree

In computer science, a tree is an abstract model of a hierarchical structure

A tree consists of nodes with a parent-child relation

- if u is parent of v, then v is a child of u
- a node has at most **one** parent in a tree
- a node can have zero, one or more children



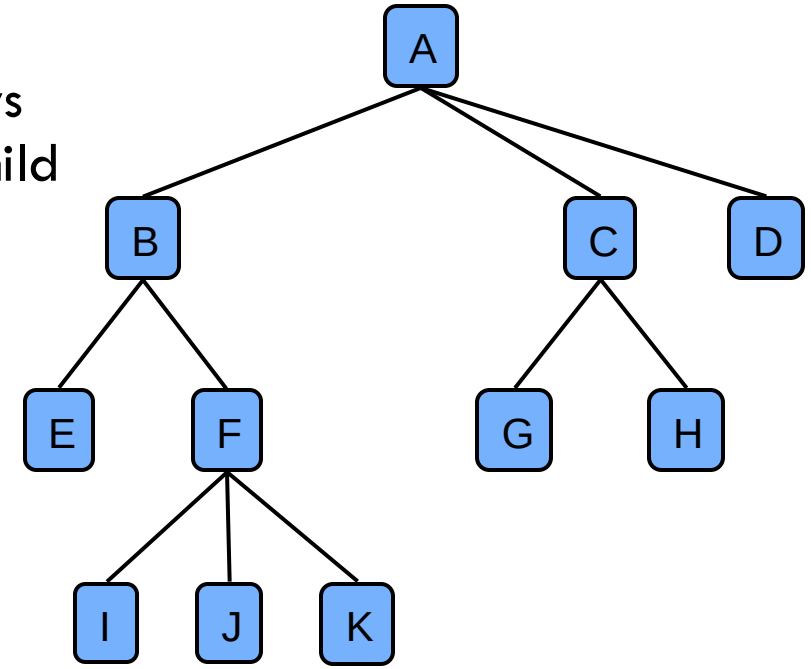
Applications:

- Organization charts
- File systems
- Phrase structure

Formal definition

A tree T is made up of a set of nodes endowed with parent-child relationship with following properties:

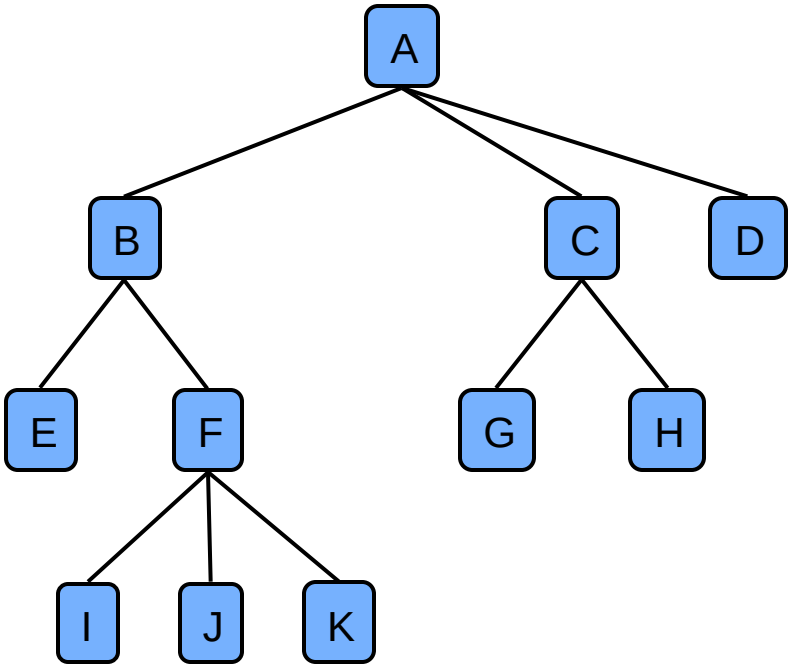
- If T is non-empty, it has a special node called the root that has no parent
- Every node v of T other than the root has a unique parent
- Following the parent relation always leads to the root (i.e., the parent-child relation does not have "cycles")



Tree Terminology

Depending on where they are in the tree, we classify nodes into:

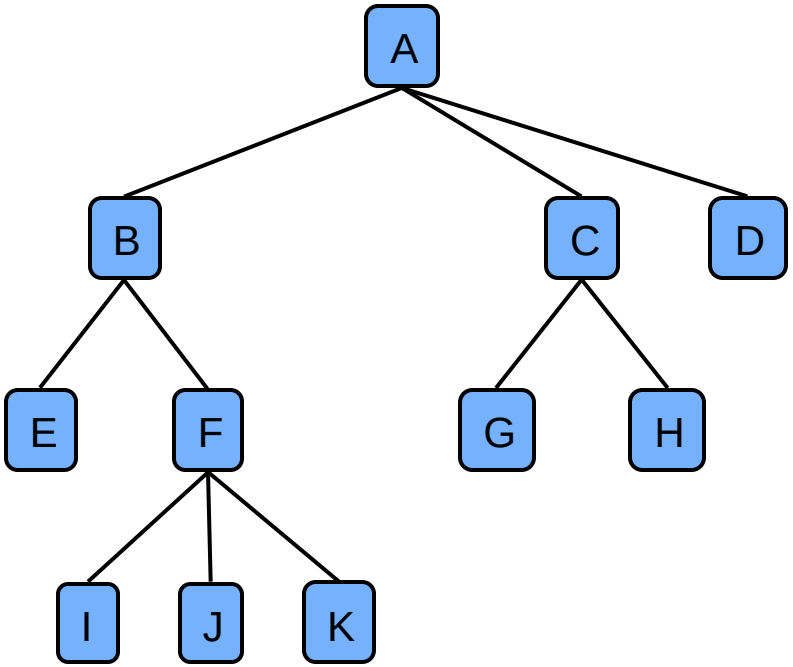
- Root: node without parent (e.g., A)
- Internal node: node with at least one child (e.g., A, B, C, F)
- External/leaf node: node without children (e.g., E, I, J, K, G, H, D)



Tree Terminology

We can extend the parent-child relation to capture indirect relations:

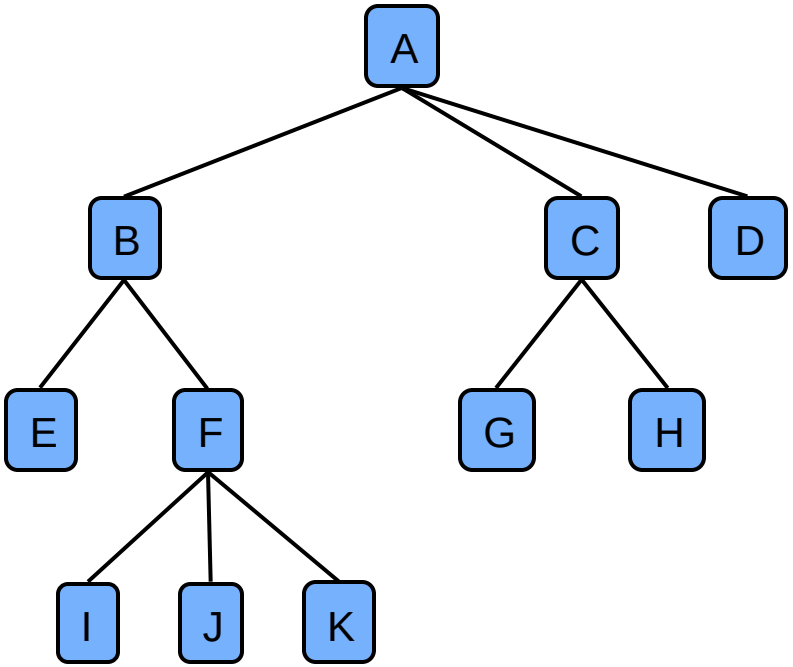
- Ancestors: parent, grandparent, great-grandparent, etc. (e.g., ancestors of F are A, B)
- Descendants: child, grandchild, great-grandchild, etc. (e.g., descendants of B are E, F, I, J, K)
- Two nodes with the same parent are siblings (e.g., B and D)



Tree Terminology

More fine-grained location concepts:

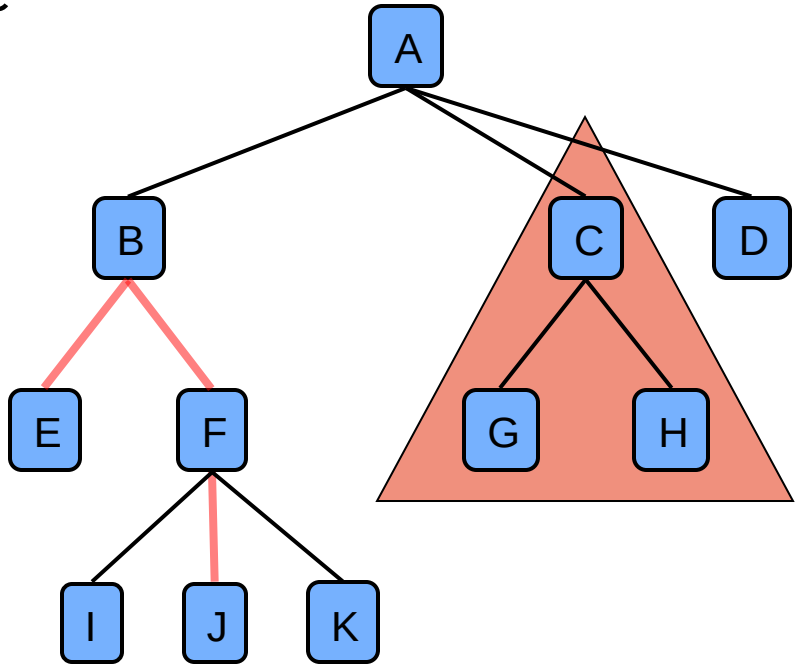
- Depth of a node: number of ancestors not including itself (e.g., depth(F) = 2)
- Level: set of nodes with given depth (e.g., {E, F, G, H} are level 2)
- Height of a tree: maximum depth (e.g., 3)



Tree Terminology

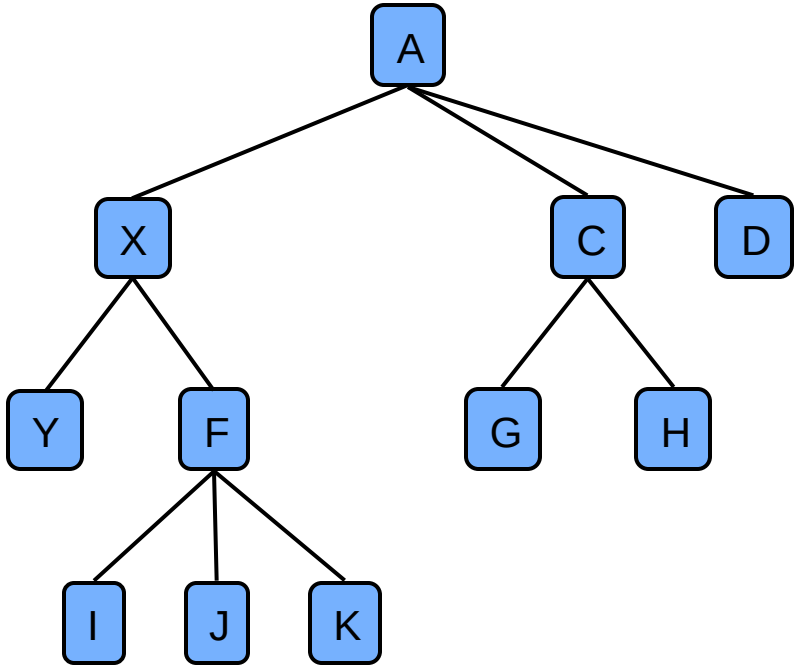
Substructures of a tree:

- Subtree: tree made up of some node and its descendants. (e.g., subtree rooted at C is {C, G, H})
- Edge: pair of nodes (u, v) such that one is the parent of the other
- Path: sequence of nodes such that 2 consecutive nodes in the sequence have an edge (e.g., <E, B, F, J>).



Tree facts

- If node X is an ancestor of node Y, then Y is a descendant of X.
- Ancestor/descendant relations are transitive
- Every node is a descendant of the root
- There may be nodes where neither is an ancestor of the other
- Every pair of nodes has at least one common ancestor.
- The lowest common ancestor (LCA) of x and y is a node z such that z is the ancestor of x and y and no descendant of z has that property



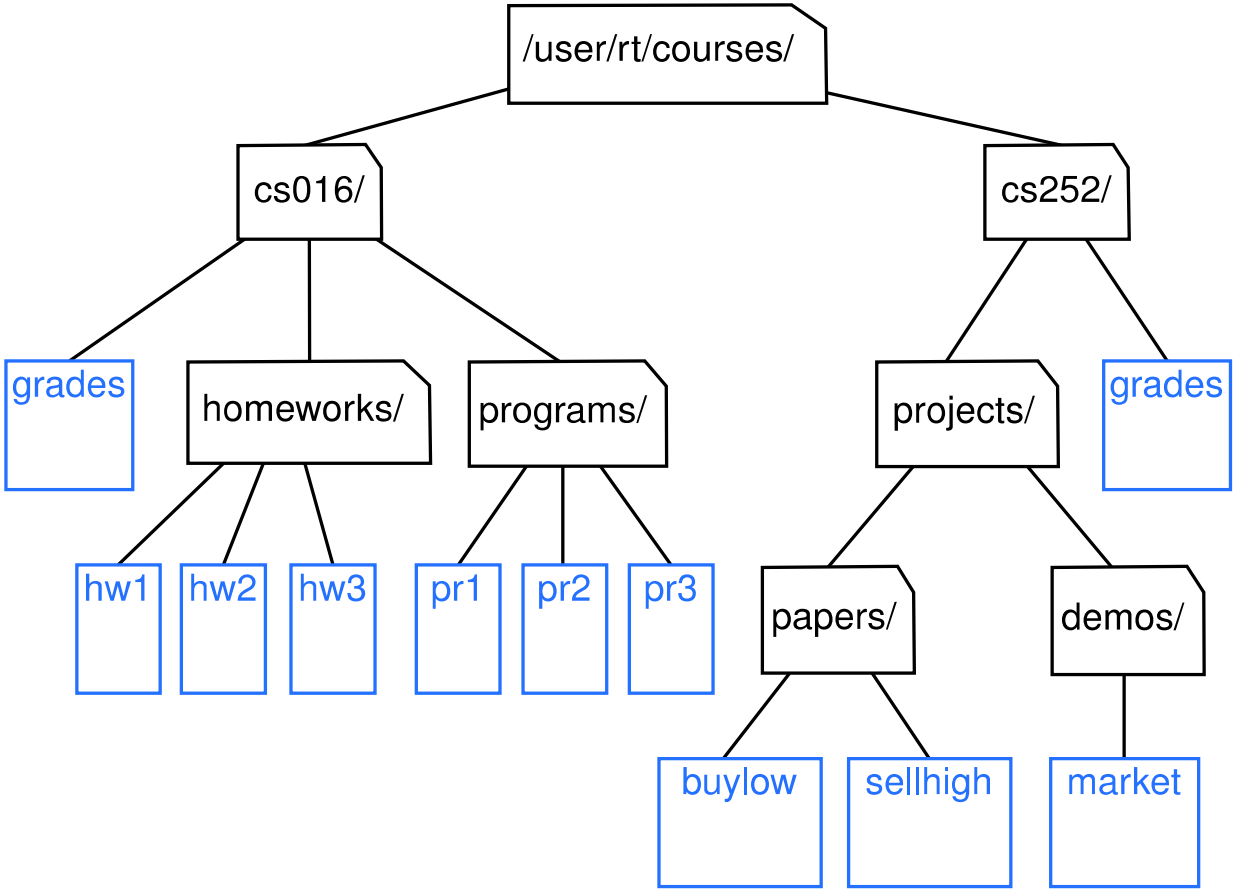
Ordered Trees

Sometimes order of siblings matter

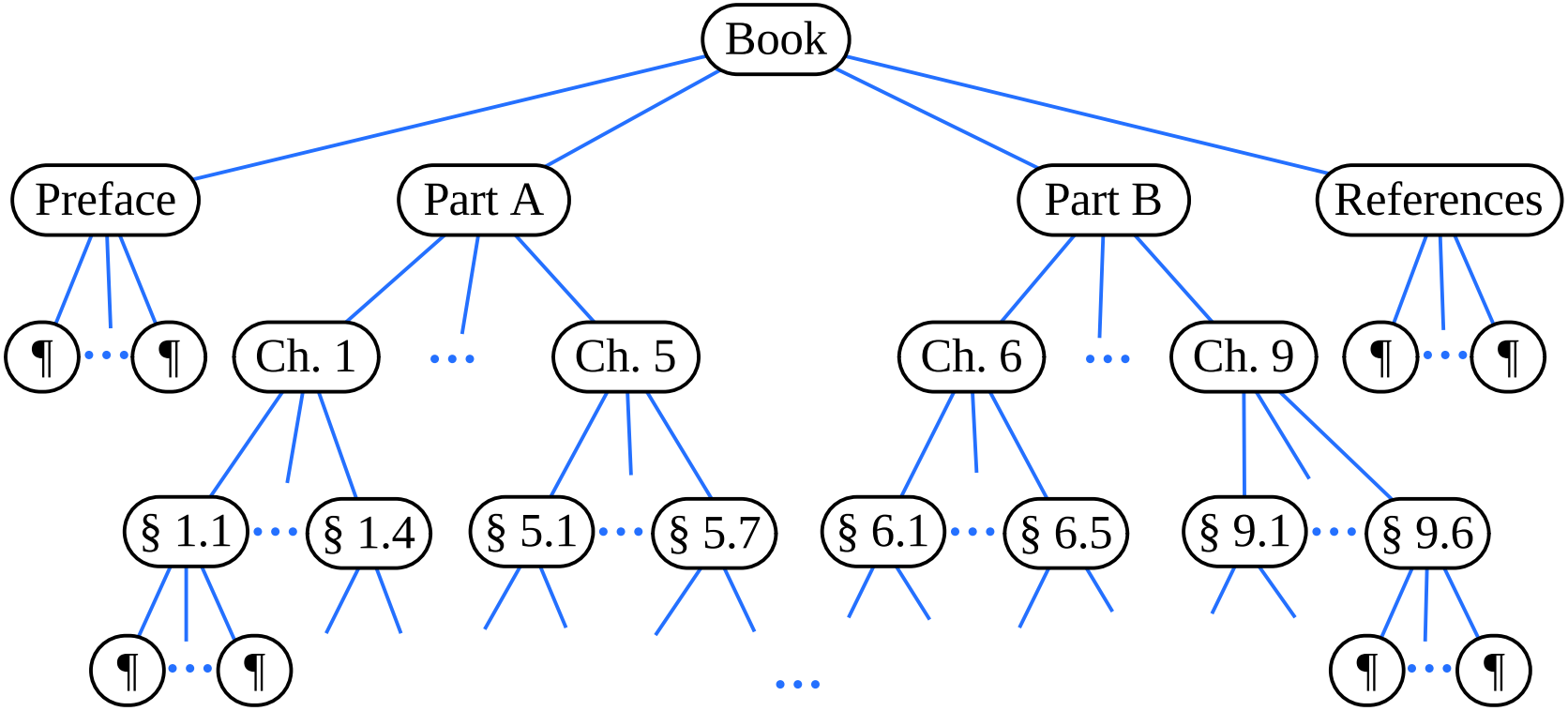
In an ordered tree there is a prescribed order for each node's children

In a diagram this ordering is usually represented by the left to right arrangement of the nodes

Application: OS file structure



Application: Document structure



Tree ADT

- Position as Node abstraction
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - Iterator iterator()
  - Iterable positions()
- Access methods:
  - Position root()
  - Position parent(p)
  - Iterable children(p)
  - Integer numChildren(p)

- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Additional update methods may be defined by data structures implementing the Tree ADT

Node object

Node object implementation typically has the following attributes:

- value: the value associated with this Node
- children: set or list of children of this Node
- parent: (optional) the parent of this Node

```
def is_external(v)
    # test if v is a leaf
    return v.children.is_empty()

def is_root(v)
    # test if v is the root
    return v.parent = null
```

Traversing trees

A traversal visits the nodes of a tree in a systematic manner

When traversing a simpler structure like a list there is one natural traversal strategy (forward or backwards)

Trees are more complex and admit more than one natural way:

- pre-order
- post-order
- in-order (for binary trees)

Preorder Traversal

To do a preorder traversal starting at a given node, we visit the node before visiting its descendants

```
def pre_order(v)
    visit(v)
    for each child w of v
        pre_order(w)
```

If tree is ordered visit the child subtrees in the prescribed order

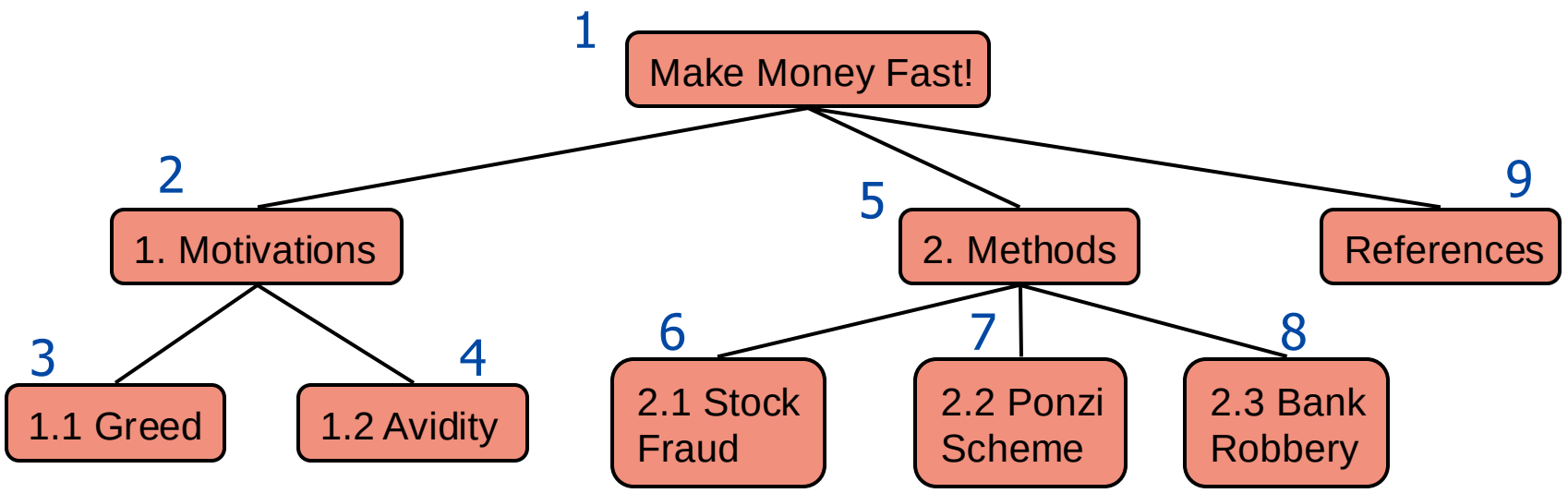
Visit does some work on the node:

- print node data
- aggregate node data
- modify node data

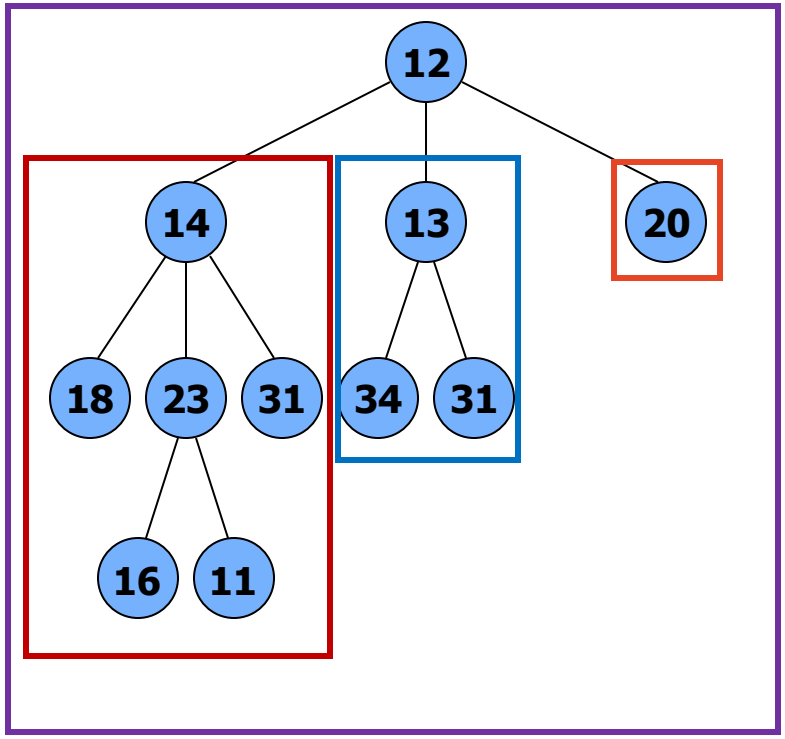
Preorder Traversal Example

Nodes are numbered in the order they are visited when we call pre_order() at the root

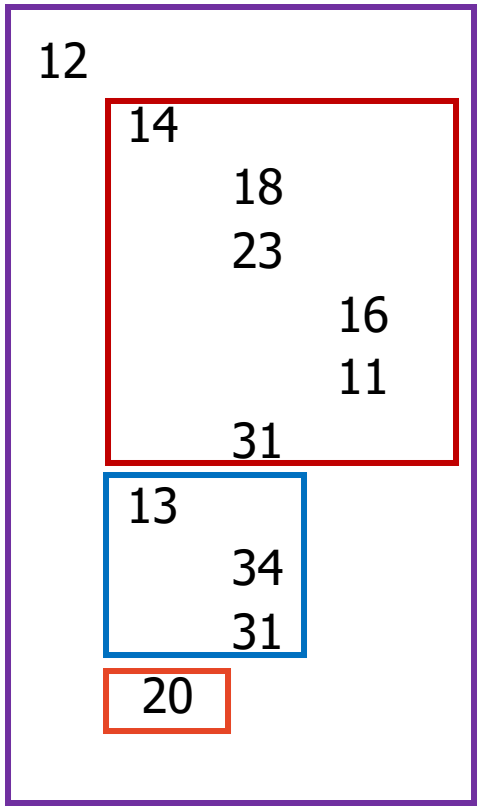
```
def pre_order(v)
    visit(v)
    for each child w of v
        pre_order(w)
```



Preorder Traversal Example



visit order



Preorder traversal of subtree

Postorder Traversal

To do a postorder traversal starting at a given node, we visit the node after its descendants

If tree is ordered visit the child subtrees in the prescribed order

Visit does some work on the node:

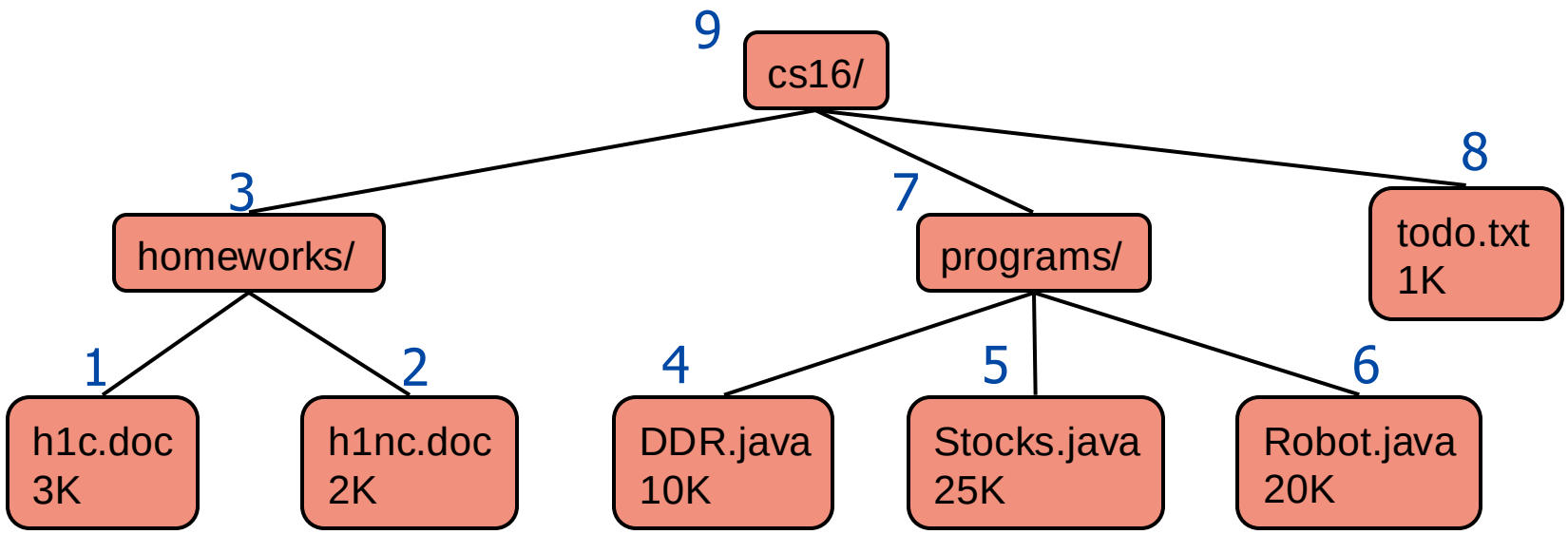
- print node data
- aggregate node data
- modify node data

```
def post_order(v)
    for each child w of v
        post_order(w)
    visit(v)
```

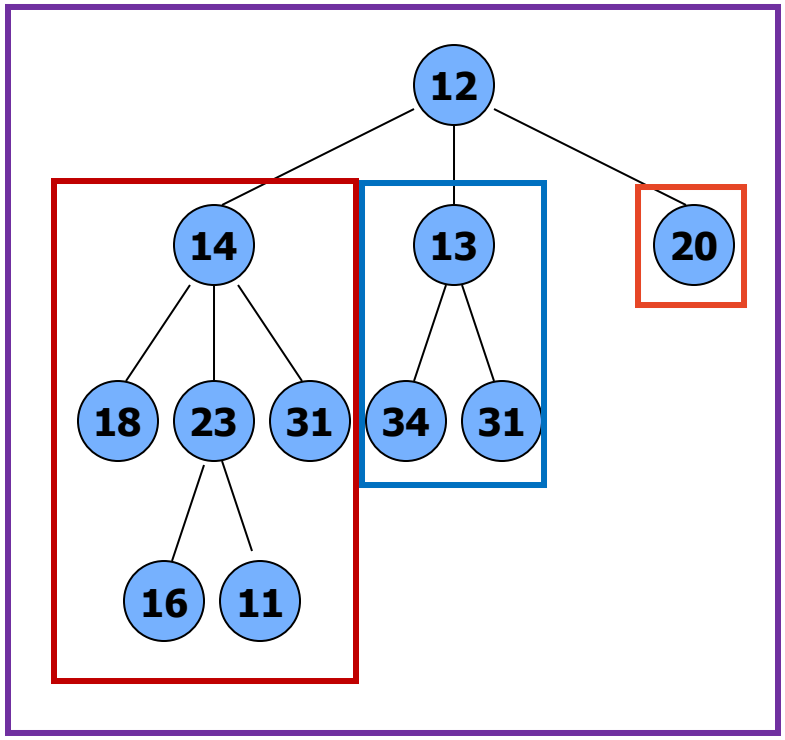
Postorder Traversal

Nodes are numbered in the order they are visited when we call post_order() at the root

```
def post_order(v)
    for each child w of v
        post_order(w)
    visit(v)
```



Traversing in postorder



visit order

18
16
11
23
31
14
34
31
13
20
12

Postorder traversal of subtree

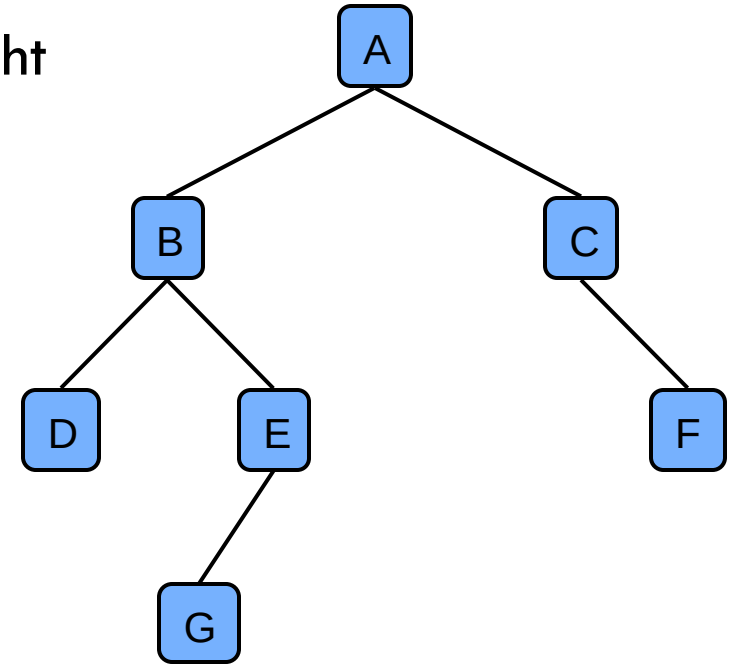
Binary Trees

A binary tree is an ordered tree with the following properties:

- Each internal node has at most two children
- Each child node is labeled as a left child or a right child
- Child ordering is left followed by right

The right/left subtree is the subtree root at the right/left child.

We say the tree is proper if every internal node has two children

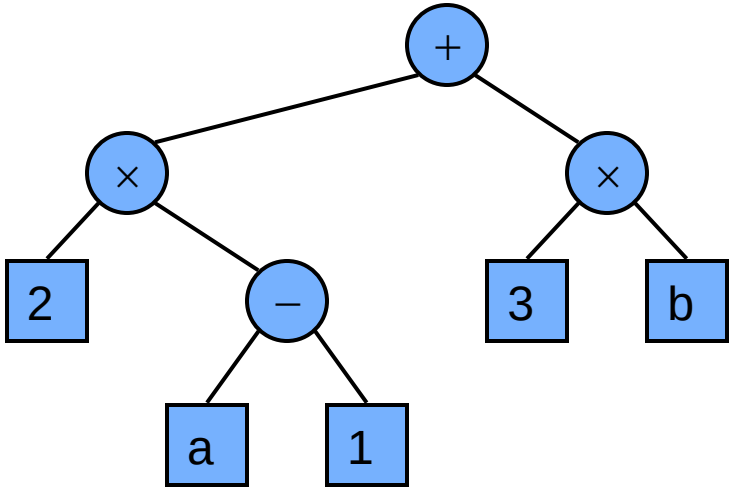


Binary tree application: Arithmetic expression tree

Binary tree associated with an arithmetic expression

- internal nodes: operators
- external nodes: operands

Example: Arithmetic expression tree for $(2 \times (a - 1) + (3 \times b))$

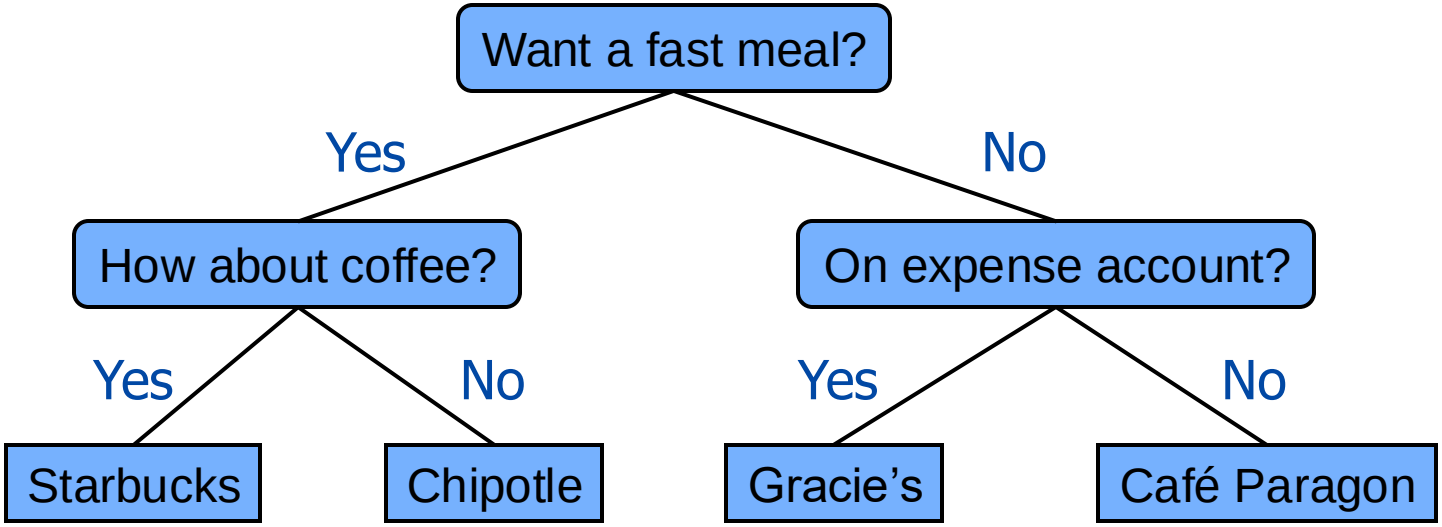


Binary tree application: Decision trees

Tree associated with a decision process

- internal nodes: questions with yes/no answer
- external nodes: decisions

Example: dining decision



Binary Tree Operations

- A binary tree extends the Tree operations, i.e., it inherits all the methods of a tree.
- Update methods may be defined by data structures implementing the binary tree
- Additional methods:
  - position leftChild(p)
  - position rightChild(p)
  - position sibling(p)

return null when there is no left, right, or sibling of p, respectively

Node object

Node object implementation typically has the following attributes:

- value: the value associated with this Node
- left: left child of this Node
- right: right child of this Node
- parent: (optional) the parent of this Node

```
def is_external(v)
    # test if v is a leaf
    return v.left = null and v.right = null
```

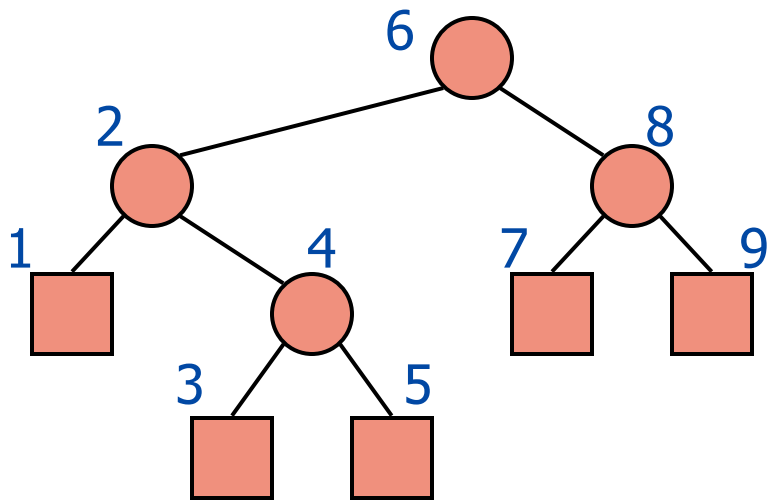
Inorder Traversal

To do an inorder traversal starting at a given node, the node is visited after its left subtree but before its right subtree

Visit does some work on the node:

- print node data
- aggregate node data
- modify node data

```
def in_order(v)
    if v.left ≠ null then
        in_order(v.left)
    visit(v)
    if v.right ≠ null then
        in_order(v.right)
```

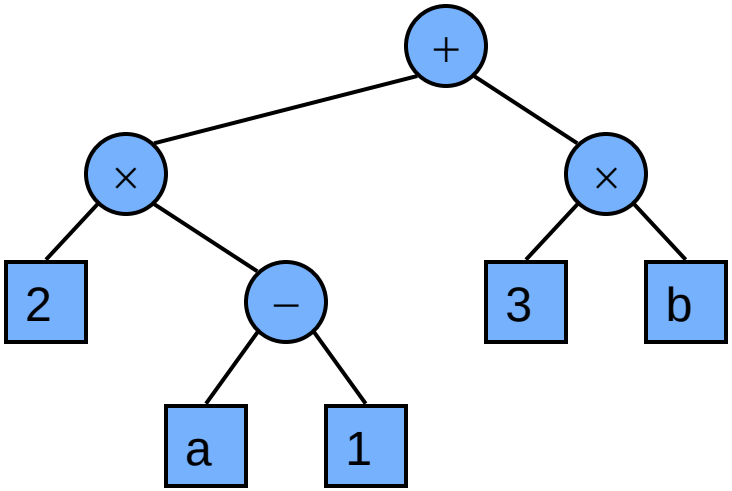


Print Arithmetic Expressions

Extended inorder traversal:

- print operand or operator when visiting node
- print "(" before left subtree
- print ")" after right subtree

```
def print_expr(v)
    if v.left ≠ null then
        print("(")
        print_ expr(v.left)
    print(v.element)
    if v.right ≠ null then
        print_expr(v.right)
        print (")")
```



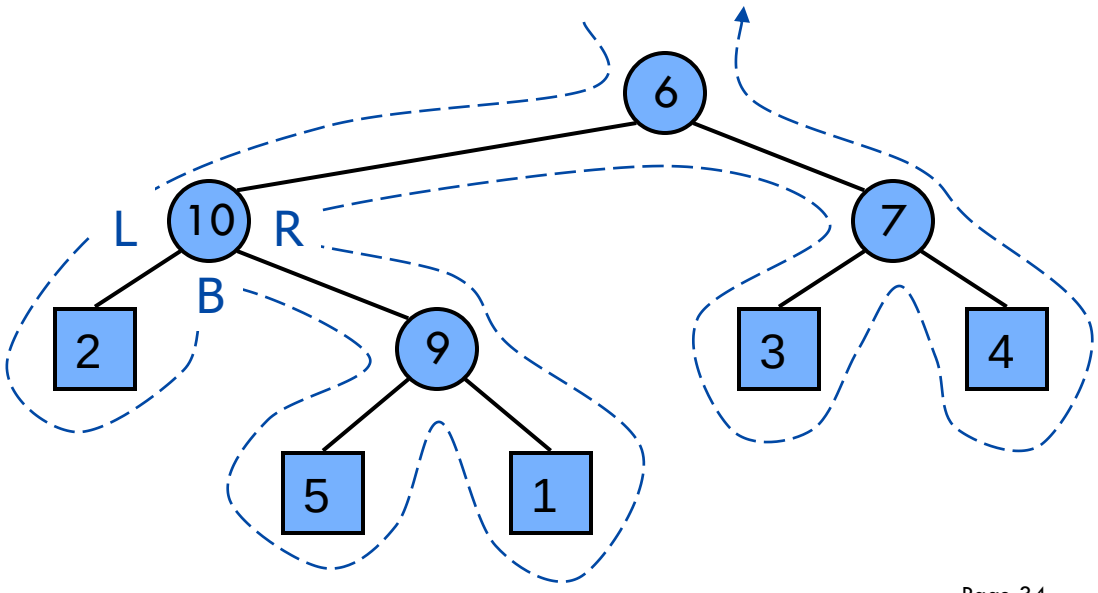
$$((2 \times (a - 1)) + (3 \times b))$$

Euler Tour Traversal

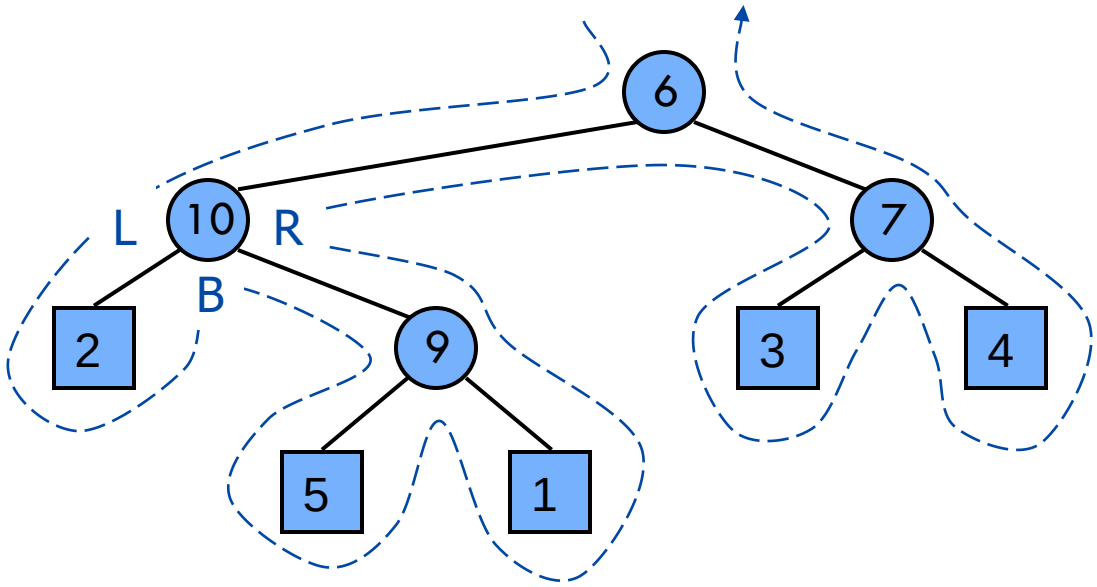
Generic traversal of a binary tree. Includes as special cases the preorder, postorder and inorder traversals

Walk around the tree, keeping the tree on your left, and visit each node three times:

- on the left (preorder)
- from below (inorder)
- on the right (postorder)



Euler Tour Traversal



6,10,2,2,2,10,9,5,5,5,9,1,1,1,9,10,6,7,3,3,3,7,4,4,4,7,6

Preorder (first visit): 6, 10, 2, 9, 5, 1, 7, 3, 4

Inorder (second visit): 2, 10, 5, 9, 1, 6, 3, 7, 4

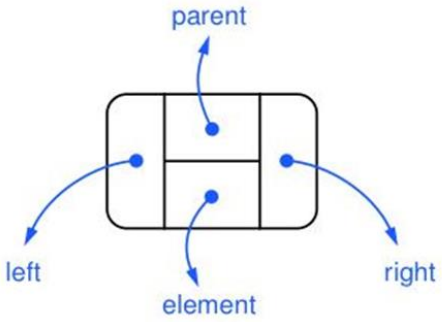
Postorder (third visit): 2, 5, 1, 9, 10, 3, 4, 7, 6

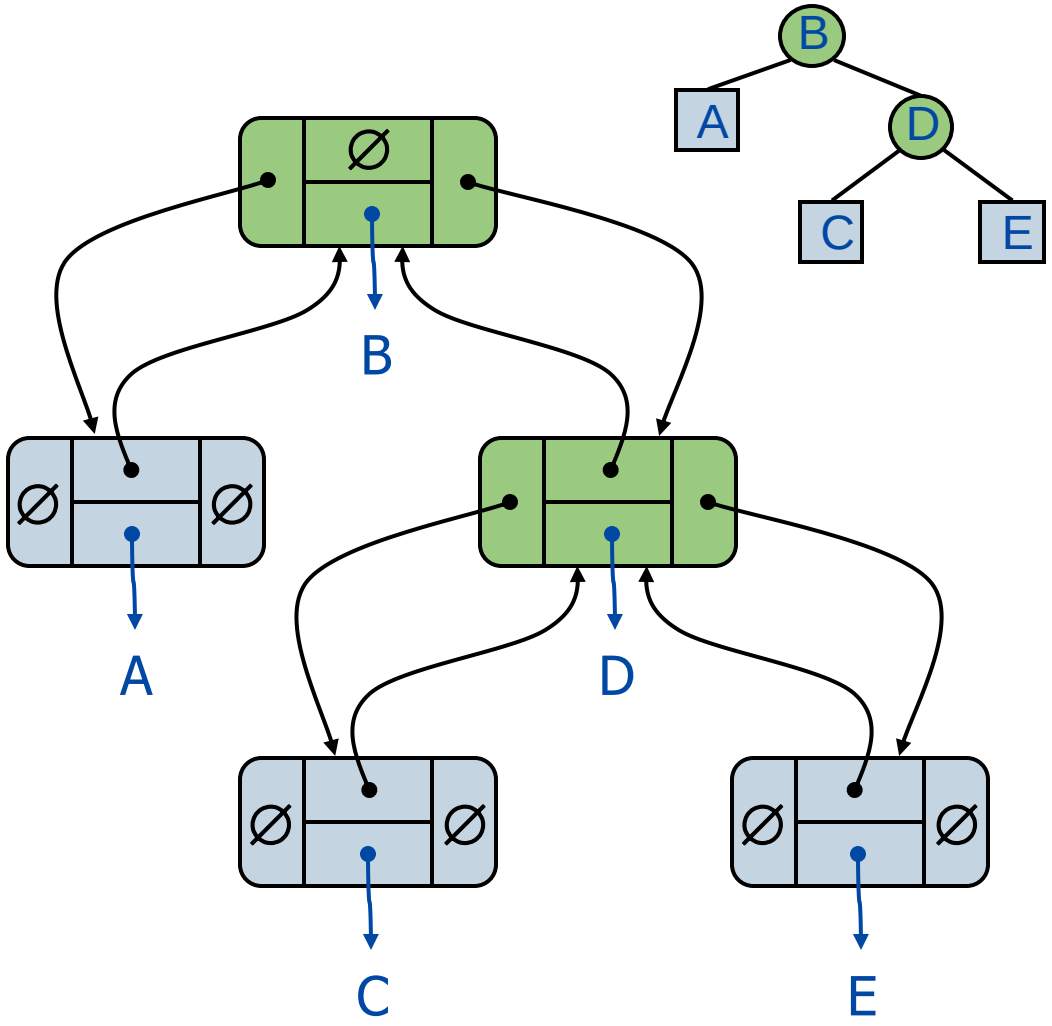
Linked Structure for Binary Trees

A node is represented by an object storing

- Element
- Parent node
- Left child node
- Right child node

Node objects implement the Position ADT



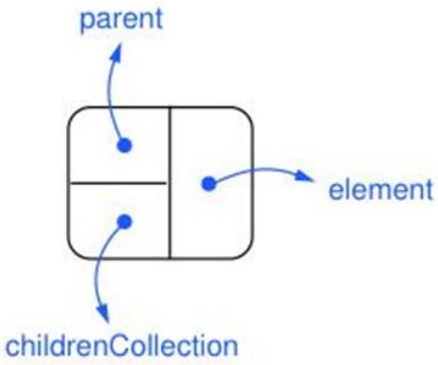


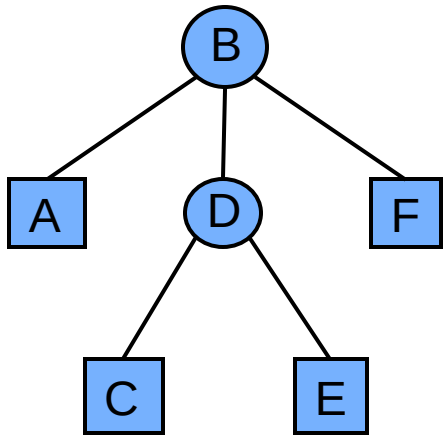
Linked Structure for General Trees

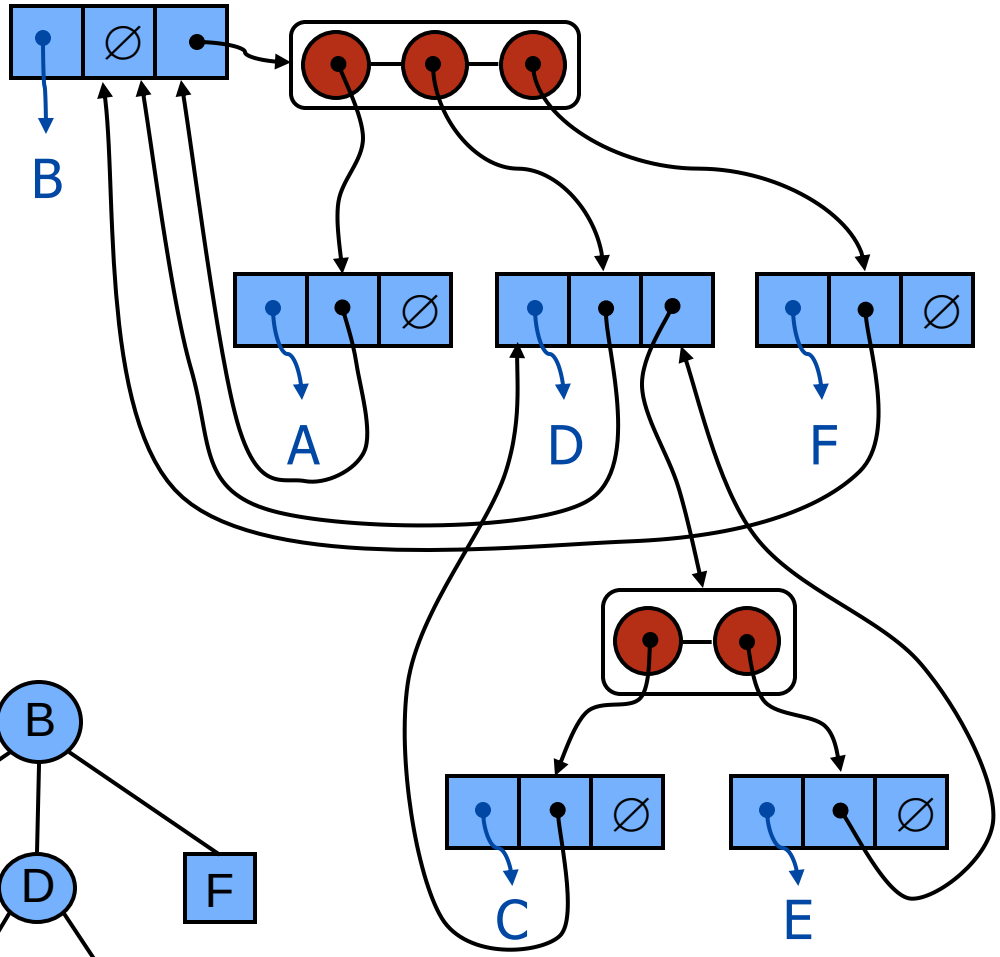
A node is represented by an object storing

- Element
- Parent node
- Sequence of children

Node objects implement the Position ADT

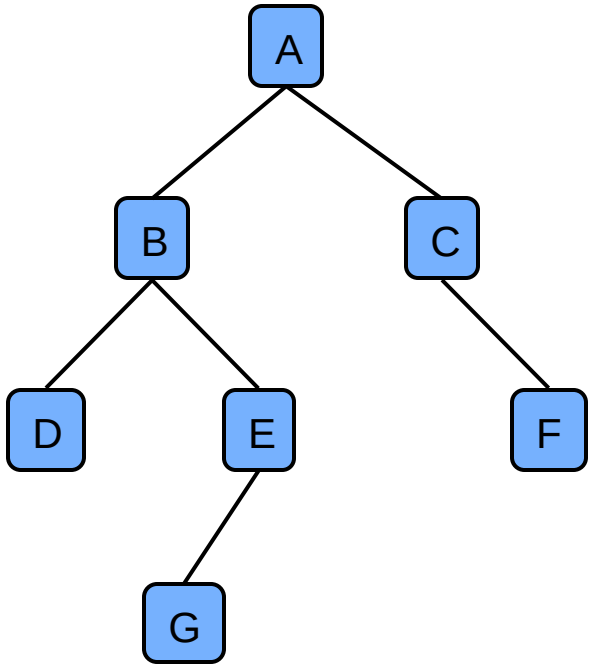






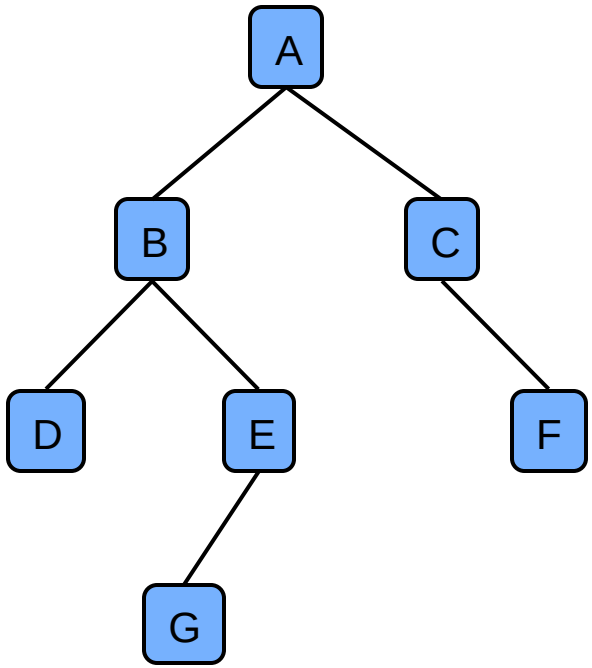
Examples of recursive code on trees

```
def depth(v)
  # compute the depth of v

    if v.parent = null then
      # root's depth is 0
        return 0
    else
        return depth(v.parent) + 1.
```



Examples of recursive code on trees

```
def height(v)
  # compute height of subtree at v

    if v.isExternal() then
      # a leave's height is 0
        return 0
    else
        h ← 0
        for each child w of v
            h ← max(h, height(w))
        return h + 1
```



Complexity analysis of recursive algorithms on trees

Sometimes, the method may call itself on all children

- In worst case, do a call on every node
- If the work done, *excluding the recursion*, is constant per call, then the total cost is linear in the number of nodes

Sometimes, the method calls itself on at most one child

- In worst case, do one call at each level of the tree
- If the work done, *excluding the recursion*, is constant per call, then the total cost is linear in the height of the tree