STAT5002 Weekly Independent Exercises - solution

Sheet 6 - Week 10

STAT5002

The author of White (2005) suspected that ravens (intelligent birds whose diet includes carrion) are attracted by the sound of gunshots, presumably in the hope of finding a dead animal to feed on. They conducted an experiment whereby they went to 12 locations in an established "hunting zone' ', counted how many ravens were visible in the vicinity 10 minutes before and 10 minutes after firing a rifle. The raw data is made available in the *ecostats* R package.

```
require(ecostats)
```

```
Loading required package: ecostats

Loading required package: mvabund
```

```
data(ravens)
ravens
```

```
   Before After delta    site treatment trees
1       0     2     2 pilgrim         1     1
2       0     1     1 pacific         1     1
3       0     4     4 uhl hil         1     1
4       0     1     1 wolff r         1     1
5       0     0     0 teton p         1     1
6       2     5     3 glacier         1     1
7       1     0    -1 ant fla         1     0
8       0     1     1 btb n           1     1
9       0     0     0 btb s           1     1
10      3     3     0 hay e           1     0
```

```
11       5      5      0 hay s              1      0
12       0      2      2 grove              1      1
13       0      0      0 pilgrim            2      1
14       1      0     -1 pacific            2      1
15       1      1      0 uhl hil            2      1
16       0      0      0 wolff r            2      1
17       0      1      1 teton p            2      1
18       1      1      0 glacier            2      1
19       6      3     -3 ant fla            2      0
20       3      2     -1 btb n              2      1
21       0      0      0 btb s              2      1
22       0      0      0 hay e              2      0
23       1      0     -1 hay s              2      0
24       0      0      0 grove              2      1
25       2      1     -1 pilgrim            3      1
26       1      0     -1 pacific            3      1
27       0      1      1 uhl hil            3      1
28       0      0      0 wolff r            3      1
29       3      6      3 teton p            3      1
30       1      1      0 glacier            3      1
31       1      1      0 ant fla            3      0
32       0      1      1 btb n              3      1
33       0      0      0 btb s              3      1
34       3      1     -2 hay e              3      0
35       3      4      1 hay s              3      0
36       1      0     -1 grove              3      1
37       0      0      0 pilgrim            4      1
38       1      0     -1 pacific            4      1
39       1      2      1 uhl hil            4      1
40       2      3      1 wolff r            4      1
41       4      5      1 teton p            4      1
42       1      0     -1 glacier            4      1
43       0      0      0 ant fla            4      0
44       0      0      0 btb n              4      1
45       0      1      1 btb s              4      1
46       2      0     -2 hay e              4      0
47       2      1     -1 hay s              4      0
48       0      0      0 grove              4      1
```

The author of the study also tried using other sounds (use `?ravens` to see all the details). The factor treatment indicates which sound: the value 1 corresponds to gunshot. We thus extract the first 12 rows.

```r
gunshot = ravens[1:12, ]
gunshot
```

```
   Before After delta    site treatment trees
1       0     2     2 pilgrim         1     1
2       0     1     1 pacific         1     1
3       0     4     4 uhl hil         1     1
4       0     1     1 wolff r         1     1
5       0     0     0 teton p         1     1
6       2     5     3 glacier         1     1
7       1     0    -1 ant fla         1     0
8       0     1     1 btb n           1     1
9       0     0     0 btb s           1     1
10      3     3     0 hay e           1     0
11      5     5     0 hay s           1     0
12      0     2     2 grove           1     1
```

The column headed `delta` gives `After-Before` differences.

1

If we were to use a T-test to analyse these observations, what kind would that be, and why?

Answer: Although we have two samples (`Before` and `After`), these are paired, since a pair of observations is made at each of the 12 sites. The two samples are not independent, so neither a Classical two-sample t-test nor a Welch test is appropriate. Rather we should take differences and perform a one-sample t-test on the differences.

2

Describe an appropriate statistical model for the data, including assumptions appropriate for the kind of T-test given in the previous part. Do the assumptions seem reasonable? Explain.

Answer: To perform a paired T-test, we assume that the differences are like a random sample $X_1, \dots, X_{12}$ from a box with mean μ, SD σ (both unknown) and *with a normal shape*. Since the 12 observations are integer-valued, with several observations equal to each other (which would not happen with a "proper'' normal sample), the normality assumption is questionable.

3

The T-statistic is the ratio of the mean difference and an estimate of the standard error of the mean difference. Determine the estimated standard error. Round to 3 decimal places if necessary. The R output below may be helpful.

```
apply(gunshot[, 1:3], 2, sd)
```

```
  Before    After    delta 
1.621354 1.858641 1.443376 
```

Answer: Since we are doing a paired T-test, we simply perform a one-sample T-test on the differences. Thus we use the sample SD of the differences to estimate σ giving $\hat{\sigma} = 1.443376$. The theoretical standard error of the mean difference is simply $SE(\bar{X}) = \sigma/\sqrt{12}$, so our estimated standard error is

$$\frac{\hat{\sigma}}{\sqrt{12}} \approx \frac{1.4434}{3.4641} \approx 0.417$$

4

Formally state the appropriate null and alternative hypotheses for this analysis.

Answer: We wish to test

- the null hypothesis $H_0 : \mu = 0$ against
- the **one-sided** alternative $H_0 : \mu > 0$.

A one-sided alternative is appropriate here because the investigator anticipated a positive effect (i.e. $\mu > 0$) before the data was collected.

5

Describe the test statistic and state its distribution when the null hypothesis is true (and the assumptions underlying the procedure are reasonable).

Answer: The test statistic we use is the one-sample T-statistic based on the (`After-Before`) differences $X_1, \dots, X_{12}$, i.e.

$$T = \frac{\bar{X}}{\frac{\hat{\sigma}}{\sqrt{12}}}.$$

If H_0 is true (and the assumptions underlying the T-test are reasonable) then T should be distributed as Student's t-distribution with $n - 1 = 11$ degrees of freedom.

6

Using the R output below, state what kind of value the test statistic should take so that we reject the null hypothesis at

- the 5% level of significance;
- the 1% level of significance.

Note: the R function `outer(u, v, fun)` returns a matrix whose (`i`,`j`-th element is `fun(u[i],v[j])`).

```
qt.values = outer(c(0.95, 0.975, 0.99, 0.995), c(11, 12, 22, 24), qt)
rownames(qt.values) = c("95%", "97.5%", "99%", "99.5%")
colnames(qt.values) = c("11 df", "12 df", "22 df", "24 df")
qt.values
```

```
         11 df    12 df    22 df    24 df
95%   1.795885 1.782288 1.717144 1.710882
97.5% 2.200985 2.178813 2.073873 2.063899
99%   2.718079 2.680998 2.508325 2.492159
99.5% 3.105807 3.054540 2.818756 2.796940
```

Answer: Since the test is one-sided, we shall declare the result significant at the 5% level if it takes a value exceeding the upper 5% point of the t_{11} distribution, which according to the output above is **1.796**.

We shall declare the result significant at the 1% level if it takes a value exceeding the upper 1% point of the t_{11} distribution, which according to the output above is **2.718**.

7

Determine the value taken by the test statistic. Is the result significant at the 5% level? What about at the 1% level?

Answer: The value taken by the test statistic is

$$\frac{1.0833333}{0.417} \approx 2.6 \,.$$

Therefore the result **is** significant at the 5% level but **is not** significant at the 1% level.

8

Consider the simulation code and output given below.

```r
set.seed(23)
dif = gunshot$delta
dif
```

```
 [1]  2  1  4  1  0  3 -1  1  0  0  0  2
```

```r
t.test(dif)
```

```

	One Sample t-test

data:  dif
t = 2.6, df = 11, p-value = 0.02469
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.1662562 2.0004105
sample estimates:
mean of x
 1.083333
```

```r
stat = t.test(dif)$stat
stat
```

```
  t
2.6
```

```r
sim.stat = 0
n = length(dif)
for (i in 1:10000) {
    samp = sample(dif, size = n, replace = T)
    sim.stat[i] = t.test(samp, mu = mean(dif))$stat
}
sum(abs(sim.stat) >= abs(stat))
```

```
[1] 352
```

```r
sum(sim.stat >= stat)
```

```
[1] 74
```

Using the simulation output instead of appealing to Student's theory, is the result signif- icant at the 5% level? What about the 1% level?

Answer: The idea is to use the simulated statistics to approximate the distribution of the test statistic when the null hypothesis is true (hence we use `mu=mean(dif)` as the hypothesised mean inside `t.test()` in the loop).

Since the appropriate test is one-sided, rejecting for large values of the test statistic, the simulation-based P-value is given by the `proportion` of simulated statistics exceeding the observed value of `stat`, i.e. 2.6. This is given by the `number` of simulated statistics exceeding 2.6, divided by 10,000, which is

$$\frac{74}{10000} = 0.0074\,.$$

Since this is less than both 0.05 and 0.01, the result is significant at both the 5% and 1% levels.

References

Crow White. Hunters ring dinner bell for ravens: Experimental evidence of a unique foraging strategy. Ecology (Durham), 86(4):1057–1060, 2005.