# Assignment2025

SID 540798904

1.

(a)

Let μ = the population mean of test A - the population mean of test B; which means that if μ > 0, the test A is better than test B ( new better than old)

Given that we want to determine whether the newer is better than older, we can make hypotheses that :

H0 : μ <= 0 H1 : μ > 0

(b)

Given that we got 2 groups of sample from 2 category, and we don’t know any statistics from the population, we need to stimulate the population mean by the sample. The t-test can do it, so we can use T-test or T-statistics to evaluate the hypotheses. Also, the T-statistics is suitable for the small number of sample. Given that we need to judge whether μ <= 0, we need to use the one side T-statistics.

(c)

In T-statistics, we need to plot the qq plot (Quantile-Quantile Plot). If the distribution is near a line, the μ in population is near a normal distribution. Then we can use T-statistics for μ.

In R: we can use

diff = test.A - test.B

qqnorm(diff)

qqline(diff, col = "red")

The result is

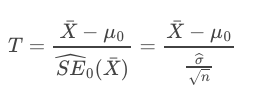
图表

AI 生成的内容可能不正确。

Which the distribution is near a line, so it can use T-statistics.

(d)

The formula of T-statistics



Where u0 = 0, so we can calculate the mean of μ and the standard deviation of μ, then calculate the T-score, where due to the hypothesis, we need to calculate P(T > t\_diff).

Using R :

mean\_diff = mean(diff)

sd\_diff = sd(diff)

t\_diff = (mean\_diff - 0) / (sd\_diff/ sqrt(20))

t\_diff

Which result in 2.096674.

图形用户界面, 文本, 应用程序

AI 生成的内容可能不正确。

P-value :

p\_value = pt(t\_diff,df = 19, lower.tail = FALSE)

p\_value

Which result in 0.02482011.

图形用户界面, 文本

AI 生成的内容可能不正确。

In this case, we need to test μ <= 0, so it is one side T-test. After calculating the T-score, using pt() to stimulate the population and the freedom is n – 1 which is 19 and due to the one side T-test, we use the lower.tail = False to get the p-value, rather than \* 2.

(e)

if use 5%, which means that 0.05 significance level and 95% confidence interval. The p-value 0.02482001is lower than the given significance level, so we reject H0 and conclude that the new tire significantly improves breaking deceleration.

(f)

Using the following code

set.seed(1)

time = 10000

n = length(diff)

T.stats.sim <- numeric(time)

for (i in 1 : time) {

samp = sample(diff, size = n, replace = TRUE)

T.stats.sim[i] = (mean(samp) - mean(diff)) / (sd(samp) / sqrt(n))

}

hist(T.stats.sim, breaks = 50, prob = TRUE,

main = "Bootstrap t-statistics vs Theoretical t-distribution",

xlab = "t-statistic", col = "lightblue", border = "white")

curve(dt(x, df = n - 1), add = TRUE, lty = 2, col = "red", lwd = 2)

Result :

图片包含 文本

AI 生成的内容可能不正确。

Using 10000 times replacemental draws to stimulate the t distribution graph, and draw the t distribution curve on this graph, we can see that it is quite close. Then, we calculate the percentage of the T score of the given sample at, then calculate the P-value.

(g)

In R:

mean(T.stats.sim > t\_diff)

Result : 0.0199

图形用户界面, 应用程序

AI 生成的内容可能不正确。

The code find the percentage of data which is higher than the T score, and in this case, it is the P-value. The P-value is 0.0199, which is quite closed to the P-value using T-statistics, still smaller than the 5%, so we can also reject the H0.

2.

(a)

Let xa = mean of population in online tutoring, xb mean of population in in-person tutoring. So when it is no effect, xa\_bar should equal xb\_bar.

H0 : xa = xb H1 : xa != xb

(b)

We need to check the 2 sample is followed by normal shaped. Using QQ Plot can do that.

In R:

qqnorm(group.A)

qqline(group.A)

qqnorm(group.B)

qqline(group.B)

Result

图表, 散点图

AI 生成的内容可能不正确。

图表, 散点图

AI 生成的内容可能不正确。

We can see that this 2 sample all fit in a line, so we can say that it follow a normal shape.

Also, we can check the standard deviation

图形用户界面, 应用程序

AI 生成的内容可能不正确。

Which is similar.

(c)

We can use the following formula to calculate the T score. The standard deviation of groupA and B is similar, so we can use classic T-test.

First, the total formula is :

In which xa\_bar, xb\_bar means the sample means of 2 sample set, σpbar means the weight average standard deviation of 2 sample set, m and n is the set size of 2 sample set.

The σp\_bar can be calculated by that:

Where σxa\_bar is the standard deviation of sample a and σxb\_bar is the standard deviation of sample b.

We can use R to calculate the at first, which is

m = length(group.A)

n = length(group.B)

sd\_a = sd(group.A)

sd\_b = sd(group.B)

mean\_a = mean(group.A)

mean\_b = mean(group.B)

sigma\_p\_bar = round(sqrt( (((m - 1) \* sd\_a^2) + ((n - 1) \* sd\_b^2)) / (m + n - 2) ), 3)

sigma\_p\_bar

Result:

文本

AI 生成的内容可能不正确。

Then we can calculate the T score

t\_score = round((mean\_a - mean\_b) / (sigma\_p\_bar \* sqrt(1/m + 1/n)),3)

t\_score

Result:

图形用户界面, 文本, 应用程序

AI 生成的内容可能不正确。

(d)

This case is a 2 side T test because we want to determine whether the 2 sample set have a similar means. Given that the critical region of rejection at the 5%, we need to find the 97.5% of t distribution, we can use qt() to do that.

qt(0.975, df = m + n - 2)

Result:



We can see that the t\_score is out of the confident interval area (-2.024394, 2.024394), so we should reject H0 and accept H1, which means that there is effect on this 2 in-person tutoring.

Or we can caculate P-value by

p\_value\_2 = 2 \* pt(t\_score,df = m + n - 2,lower.tail = FALSE)

p\_value\_2

Which result to :

图形用户界面, 文本, 应用程序

AI 生成的内容可能不正确。

Which is smaller than the given 5% level of significance. So we reject the H0.

(e)

Using code below to procedure the Welch Two Sample t-test

t.test(group.A, group.B, var.equal = FALSE)

Result:

文本

AI 生成的内容可能不正确。

We can see that the T score is quite close, and df is a little smaller than the m+n-2 = 38. P-value is 0.03453, and quite close to the result of classical two-sample t-test, still smaller than 5% so we can also reject H0, accept H1.

3.

(a)

H0 : device preference is independent of age.

H1 : device preference is not independent of age.

If we reject H0, then we have H1, which means that device preference is associated with age.

(b)

Expect e\_ij = row\_total\_i \* column\_total\_j / total

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Laptop | Desktop | Tablet | Total |
| Under 18 | 11.54 | 10.15 | 8.31 | 30 |
| 18-29 | 11.54 | 10.15 | 8.31 | 30 |
| 30-49 | 15.39 | 13.54 | 11.08 | 40 |
| 50+ | 11.54 | 10.15 | 8.31 | 30 |
| Total | 50 | 44 | 36 | 130 |

(c)

The chi-squared test should satisfy :

* Sample size n is large.
* All the categories have large probabilities.

The n is 130 in this situation and all the categories Ej is bigger than 5, so according to lecture content, it is a good practice to use chi-squared test.

(d)

We need to calculate T and then use chi-squared distribution about that.

In R, we can use that

* First input all the data

under\_18 = c(12,6,12)

one8\_29 = c(14,10,6)

three0\_49 = c(16,12,12)

five0\_ = c(8,16,6)

Oij = rbind(under\_18, one8\_29, three0\_49, five0\_)

rownames(Oij) = c("Under 18", "18–29", "30–49", "50+")

colnames(Oij) = c("Laptop","Desktop","Tablet")

# or easily use that

Eij = matrix(c(

11.54, 10.15, 8.31,

11.54, 10.15, 8.31,

15.38, 13.54, 11.08,

11.54, 10.15, 8.31

), nrow = 4, byrow = TRUE)

rownames(Eij) = c("Under 18", "18–29", "30–49", "50+")

colnames(Eij) = c("Laptop", "Desktop", "Tablet")

* Then calculate the T

T = sum((Oij - Eij)^2 / Eij)

T

Result

文本

AI 生成的内容可能不正确。

(e)

First we need to calculate the df or freedoom degree, which is

In R:

df = (nrow(Oij) - 1) \* (ncol(Oij) - 1)

p\_value = pchisq(T, df, lower.tail = FALSE)

p\_value

Result:

图形用户界面, 应用程序, Word

AI 生成的内容可能不正确。

Which is bigger than 5% significance, so we can accpet H0, which means that the device preference is independent of age.